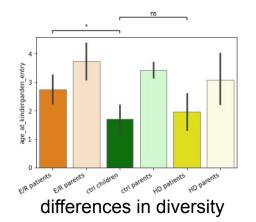
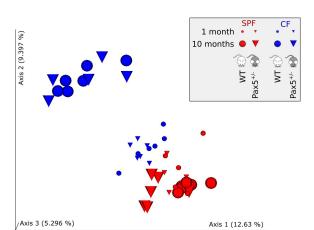




#### **Contents**

- 1. Alpha Diversity
- 2. Beta Diversity
- 3. Dimensionality Reduction (PCoA)





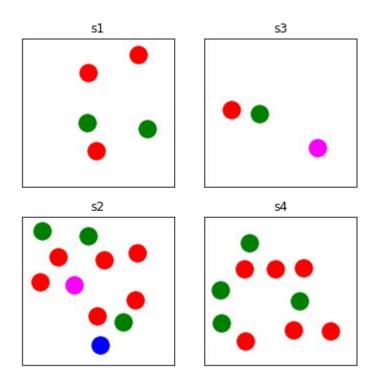
# 1. Alpha Diversity

- a. observed features
- b. Chao1
- c. Faith's PD



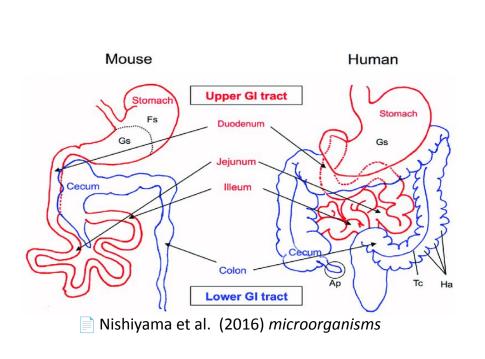
## 2. Alpha Diversity

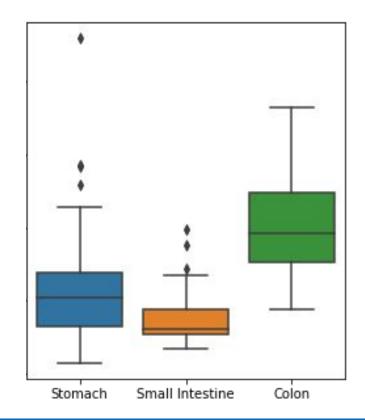
- one sample at a time
- "how complex is one sample?"
- alternative perspectives
  - a. richness: observed features
  - b. richness: Chao1
  - c. evenness: Shannon
  - d. phylogenetic diversity: Faith's PD





## Alpha Diversity: mouse gut microbiome

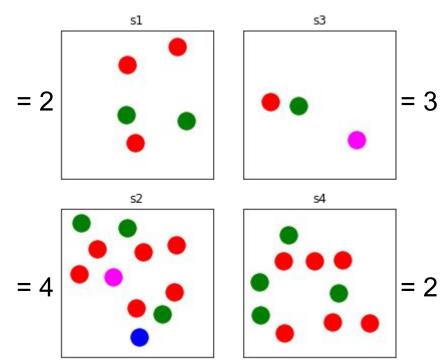






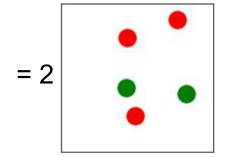
## 2. Alpha Diversity - a: <u>observed features</u>

- richness
- simple: count number of features

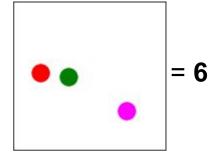


## 2. Alpha Diversity - b: Chao1

- richness
- simple: count number of features
- + emphasize singletons



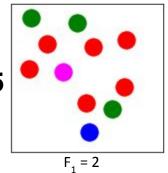
 $F_1 = 3$   $F_2 = 0$   $S_{obs} = 3$ s3



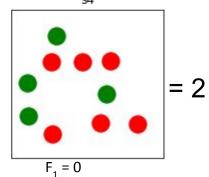
$$chao1 = S_{obs} + rac{F_1(F_1-1)}{2(F_2+1)}$$
 =



F<sub>1</sub> = features with count 1 F<sub>2</sub> = features with count 2 S<sub>obs</sub> = observed features







 $F_2 = 0$   $S_{obs} = 2$ 

## 2. Alpha Diversity - b: Chao1

- richness
- simple: count number of features
- + emphasize singletons

$$chao1 = S_{obs} + \overbrace{rac{F_1(F_1-1)}{2(F_2+1)}}$$

Anne Chao



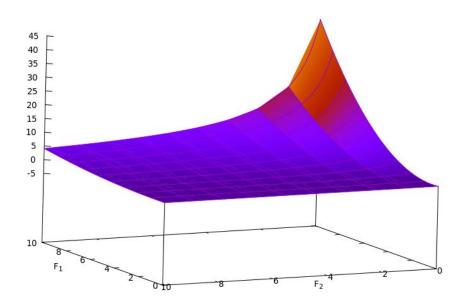
 $F_1$  = features with count 1

F<sub>2</sub> = features with count 2

S<sub>obs</sub> = observed features

Anne Chao (1984) Scand J Statist "Nonparametric Estimation of the Number of Classes in a Population"

"when most of the information is concentrated on the low order occupancy numbers"



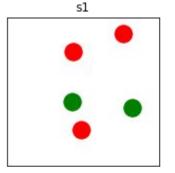
## 2. Alpha Diversity - c: Shannon

evenness

$$H = -\sum_{i=1}^s \left(p_i \log_2 p_i
ight)$$

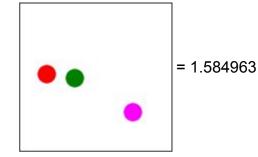
= 1.617190

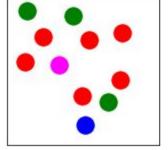
= 0.970951

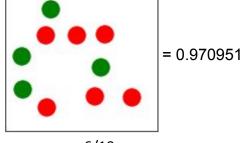


 $p_{red} = 3/5$  $p_{green} = 2/5$   $p_{red} = 1/3$   $p_{magenta} = 1/3$   $p_{green} = 1/3$ 

s3







 $p_{red} = 6/11$   $p_{magenta} = 1/11$  $p_{green} = 3/11$   $p_{blue} = 1/11$ 

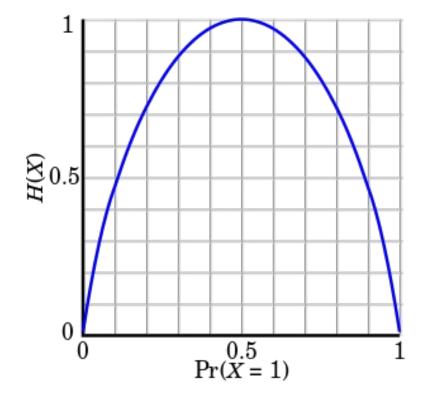
 $p_{red} = 6/10$ p<sub>green</sub> = 4/10

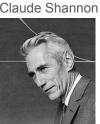
Claude Shannon

## 2. Alpha Diversity - c: Shannon

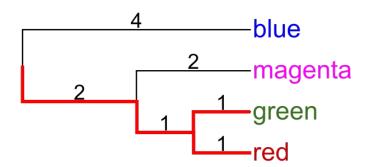
evenness

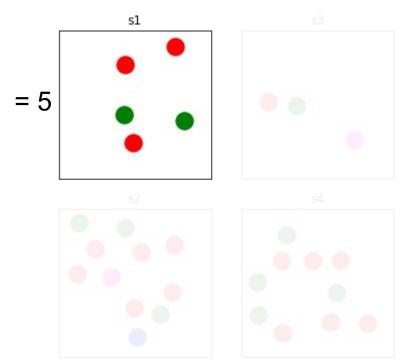
$$H = -\sum_{i=1}^s \left(p_i \log_2 p_i
ight)$$
 § 0.5





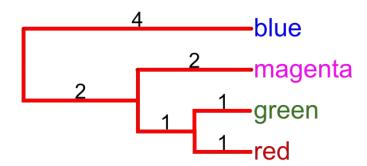
- captured evolutionary diversity
- needs a phylogenetic tree!

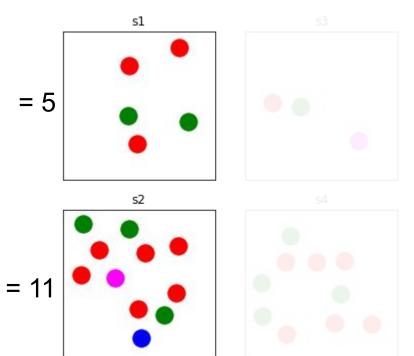




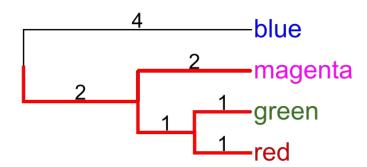


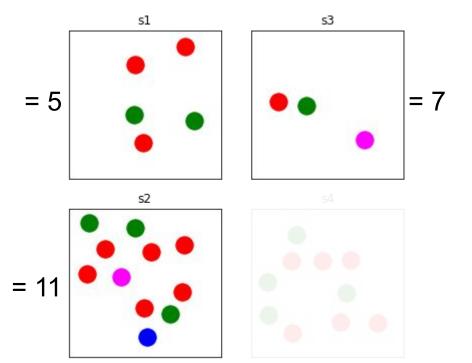
- captured evolutionary diversity
- needs a phylogenetic tree!



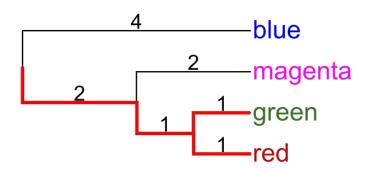


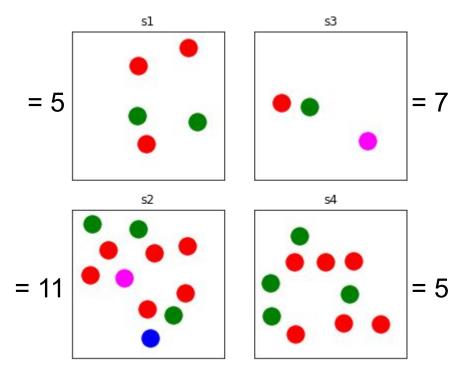
- captured evolutionary diversity
- needs a phylogenetic tree!





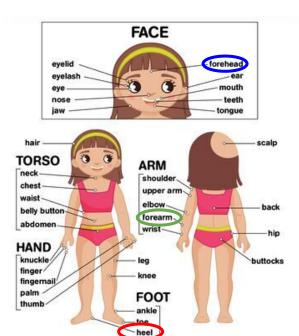
- captured evolutionary diversity
- needs a phylogenetic tree!



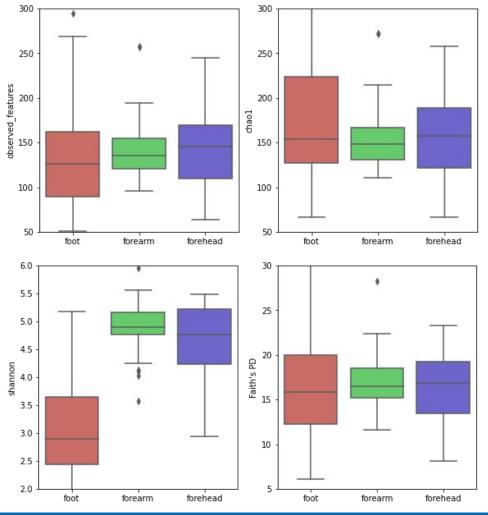




# Alpha Diversity: skin microbiome



https://reference.yourdictionary.com/resources/parts-of-the-body-for-kids-names-and-basic-functions.html



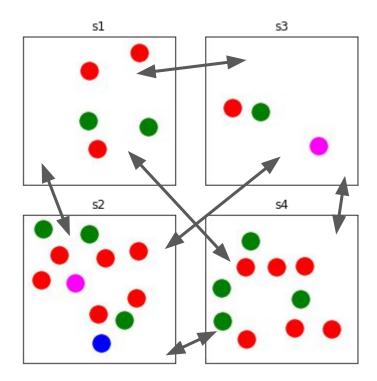
# 3. Beta Diversity

- a. Jaccard
- b. Bray-Curtis
- c. UniFrac



### 3. Beta Diversity

- pair of samples at a time
- "how different are two samples?"
- alternative perspectives
  - a. Jaccard
  - b. Bray-Curtis
  - c. UniFrac
- a "Distance"



#### **Distance Axioms**

For all  $x_i$ ,  $x_i$  and  $x_k$ 

- 1.  $d(x_i, x_j) \geq 0$ .
- 2.  $d(x_i, x_j) = 0$ , iff  $x_i$  is equal to  $x_j$ .
- 3.  $d(x_i, x_j) = d(x_j, x_i)$ .
- 4.  $d(x_i, x_j) \leq d(x_i, x_k) + d(x_k, x_j)$ .

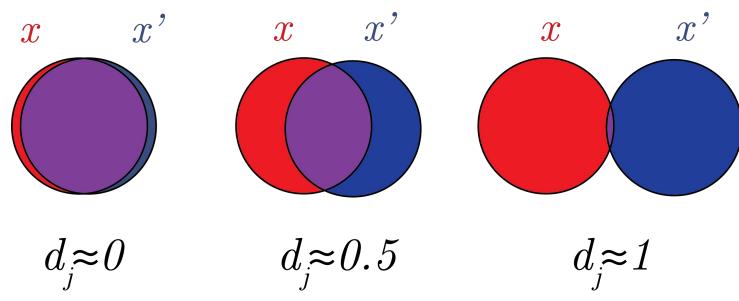
#### **Distance Axioms**

For all  $x_i$ ,  $x_i$  and  $x_k$ 

- 1.  $d(x_i, x_i) \geq 0$ . non-negativity
- 2.  $d(x_i, x_j) = 0$ , iff  $x_i$  is equal to  $x_j$ . identity
- 3.  $d(x_i, x_i) = d(x_i, x_i)$ . symmetry
- 4.  $d(x_i, x_i) \leq d(x_i, x_k) + d(x_k, x_i)$ . no wormholes

## 3. Beta Diversity - a: Jaccard

Fraction of unique features, regardless of abundance.



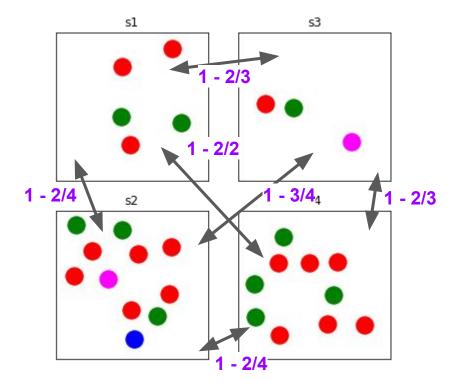
📄 Jaccard, Paul. Nouvelles recherches sur la distribution florale. 1908.

Paul Jaccard

## 3. Beta Diversity - a: <u>Jaccard</u>

Fraction of unique features, regardless of abundance.

$$J(A, B) = 1 - \frac{A \cap B}{A \cup B}$$



Paul Jaccard

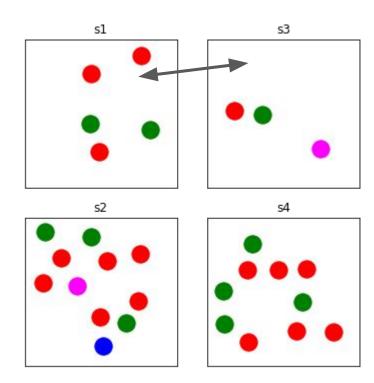


Jaccard, Paul. Nouvelles recherches sur la distribution florale. 1908.



compares relative abundances

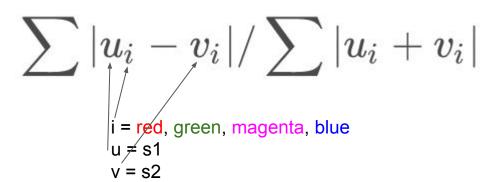
$$\sum |u_i-v_i|/\sum |u_i+v_i|$$

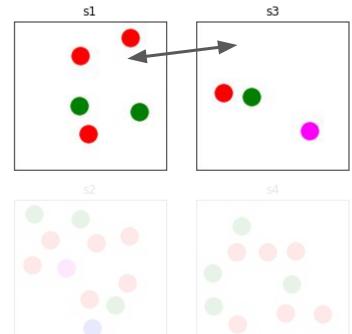


J. Roger Bray & John T. Curtis



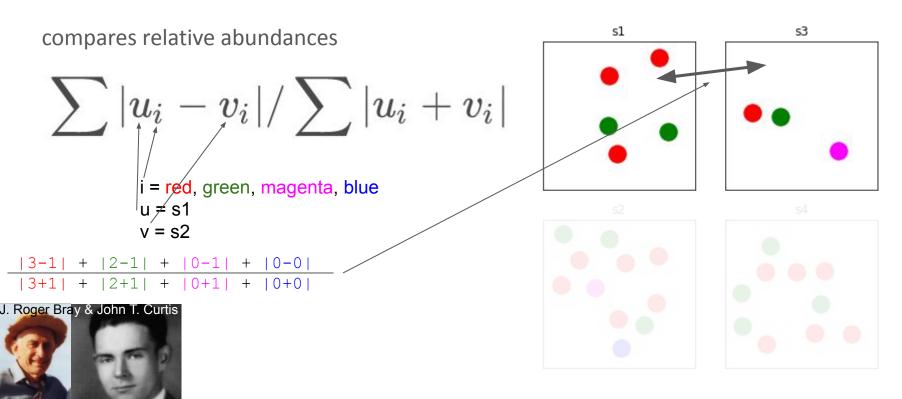
compares relative abundances



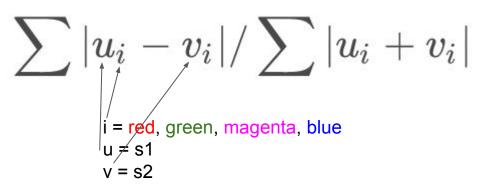


J. Roger Bray & John T. Curtis



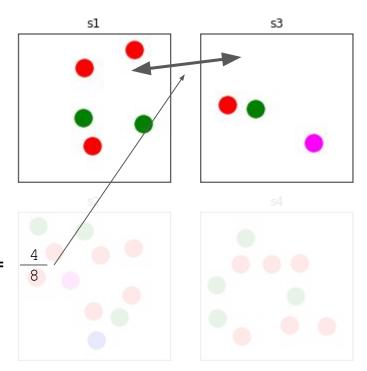


compares relative abundances



$$\frac{|3-1| + |2-1| + |0-1| + |0-0|}{|3+1| + |2+1| + |0+1| + |0+0|} = \frac{2 + 1 + 1 + 0}{4 + 3 + 1 + 0} = \frac{2 + 1 + 1 + 0}{4 + 3 + 1 + 0}$$





compares relative abundances

$$\sum |u_i - v_i| / \sum |u_i + v_i|$$

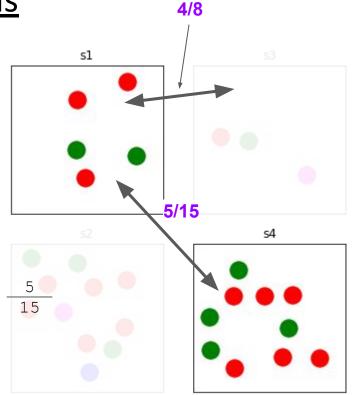
$$|i| = \text{red}, \text{ green, magenta, blue}$$

$$|u| \neq \text{s1}$$

$$|v| = \text{s4}$$

$$\frac{|3-6| + |2-4| + |0-0| + |0-0|}{|3+6| + |2+4| + |0+0| + |0+0|} = \frac{3 + 2 + 0 + 0}{9 + 6 + 0 + 0} = \frac{3 + 2 + 0 + 0}{9 + 6 + 0 + 0}$$

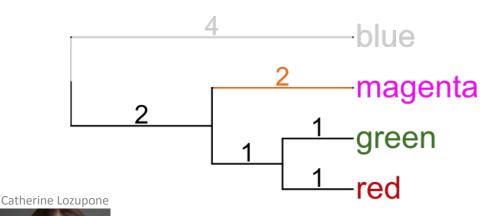


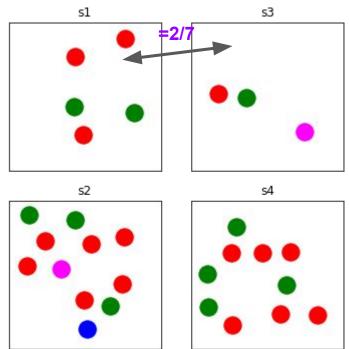




## 3. Beta Diversity - c: <u>UniFrac</u>

- Measures the fraction of <u>unique</u> branch length
- needs a phylogenetic tree!



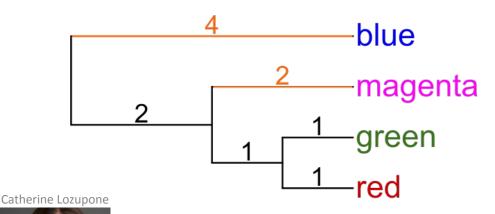


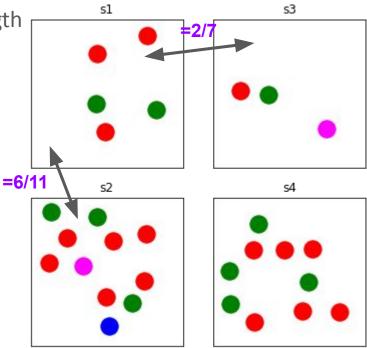


## 3. Beta Diversity - c: <u>UniFrac</u>

Measures the fraction of <u>unique</u> branch length

needs a phylogenetic tree!



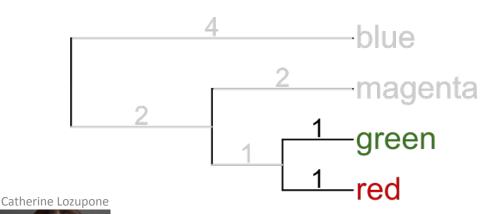


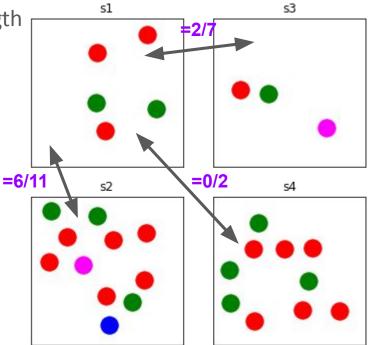


## 3. Beta Diversity - c: <u>UniFrac</u>

Measures the fraction of <u>unique</u> branch length

needs a phylogenetic tree!

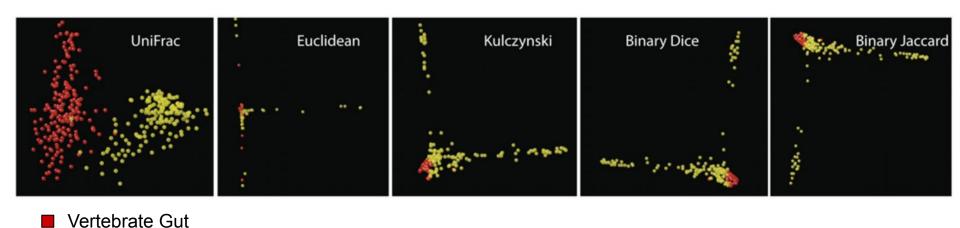








## Selecting a metric

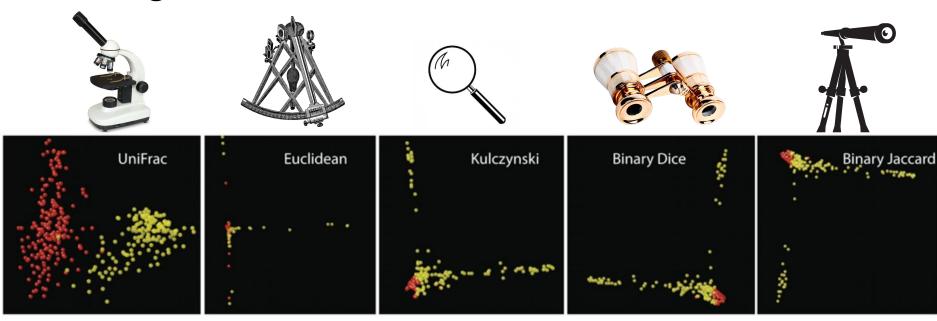


Hamady, Micah, and Rob Knight. "Microbial community profiling for human microbiome projects: Tools, techniques, and challenges." Genome research 19.7 (2009): 1141-1152.

Free living



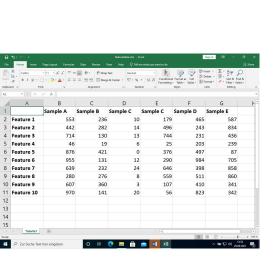
## Selecting a metric

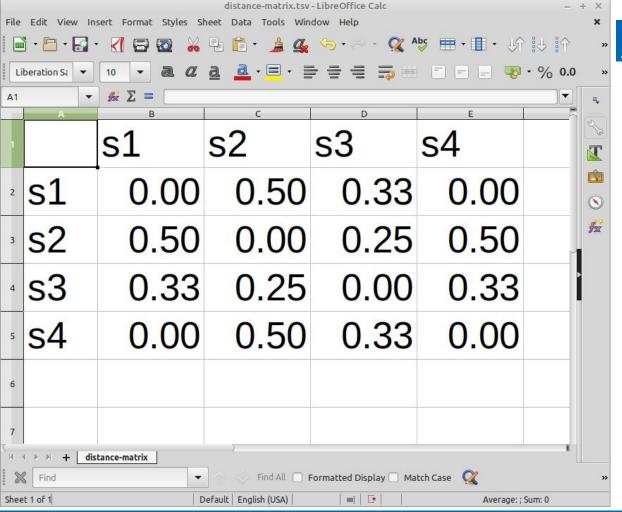


- Vertebrate Gut
- Free living

Hamady, Micah, and Rob Knight. "Microbial community profiling for human microbiome projects: Tools, techniques, and challenges." Genome research 19.7 (2009): 1141-1152.

## 4. Dimensionality Reduction (PCoA)

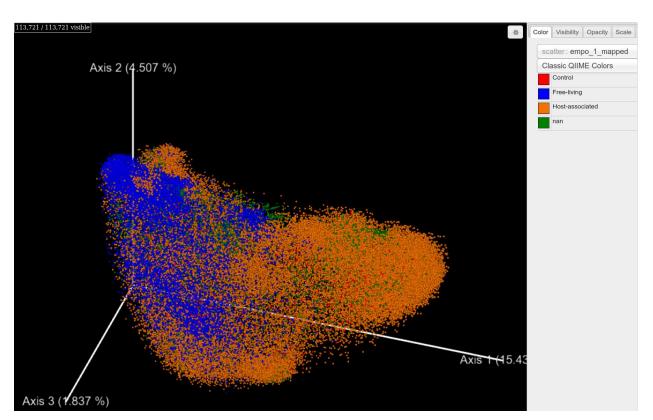






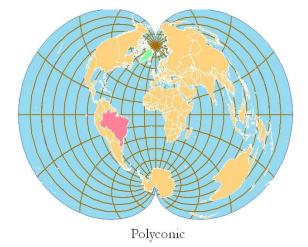
## https://bit.ly/2LHMDFC







Gall Stereographic



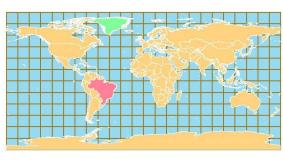


Plate Carree

area in km<sup>2</sup>

Brazil: 8,515,767

Greenland: 2,166,086

https://en.wikipedia.org/wiki/List\_of\_c ountries\_and\_dependencies\_by\_area

Principal Scale 1:375,000,000 Created by Matt Sandee 02/10 Data Source: ESRI 2008