Ripples of the QCD Critical Point

Shi Yin

15.05.2024

Based on:

[1] W.-j. Fu, X.-f. Luo, J. M. Pawlowski, F. Rennecke, R. Wen, S. Yin; Phys. Rev. D.104 (2021), 094047.
[2] W.-j. Fu, X.-f. Luo, J. M. Pawlowski, F. Rennecke, S. Yin; arXiv: 2308.15508.

Outline

- Self introduction
- Introduction
- Looking for ripples
- Summary

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Self introduction

Name: Shi Yin (尹诗)

Education: Dalian University of Technology, Theoretical Physics

Research area: QCD phase structure, Relativistic heavy ion collision phenomenology

PostDoc: JLU, supported by Alexander von Humboldt foundation

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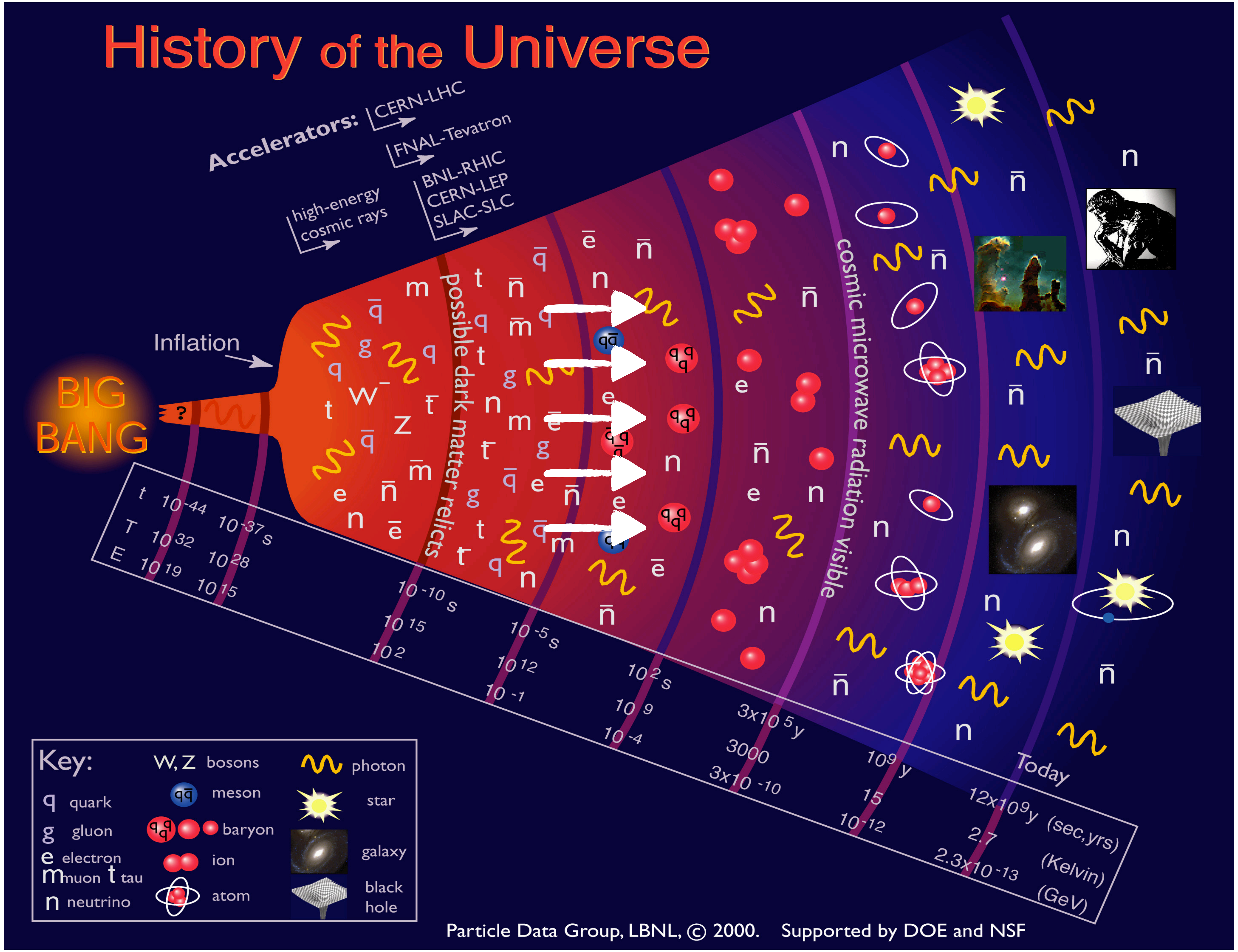
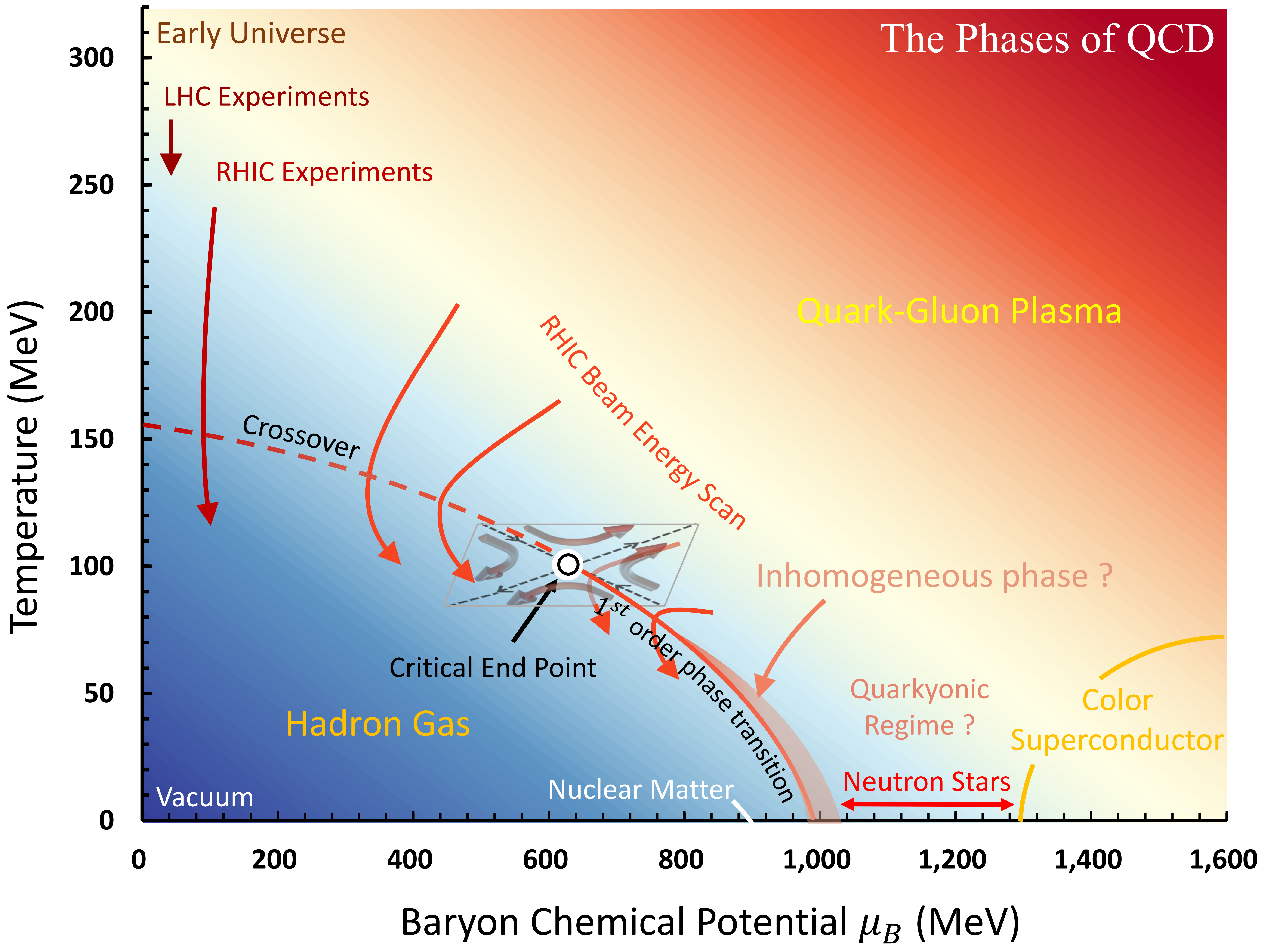
Introduction

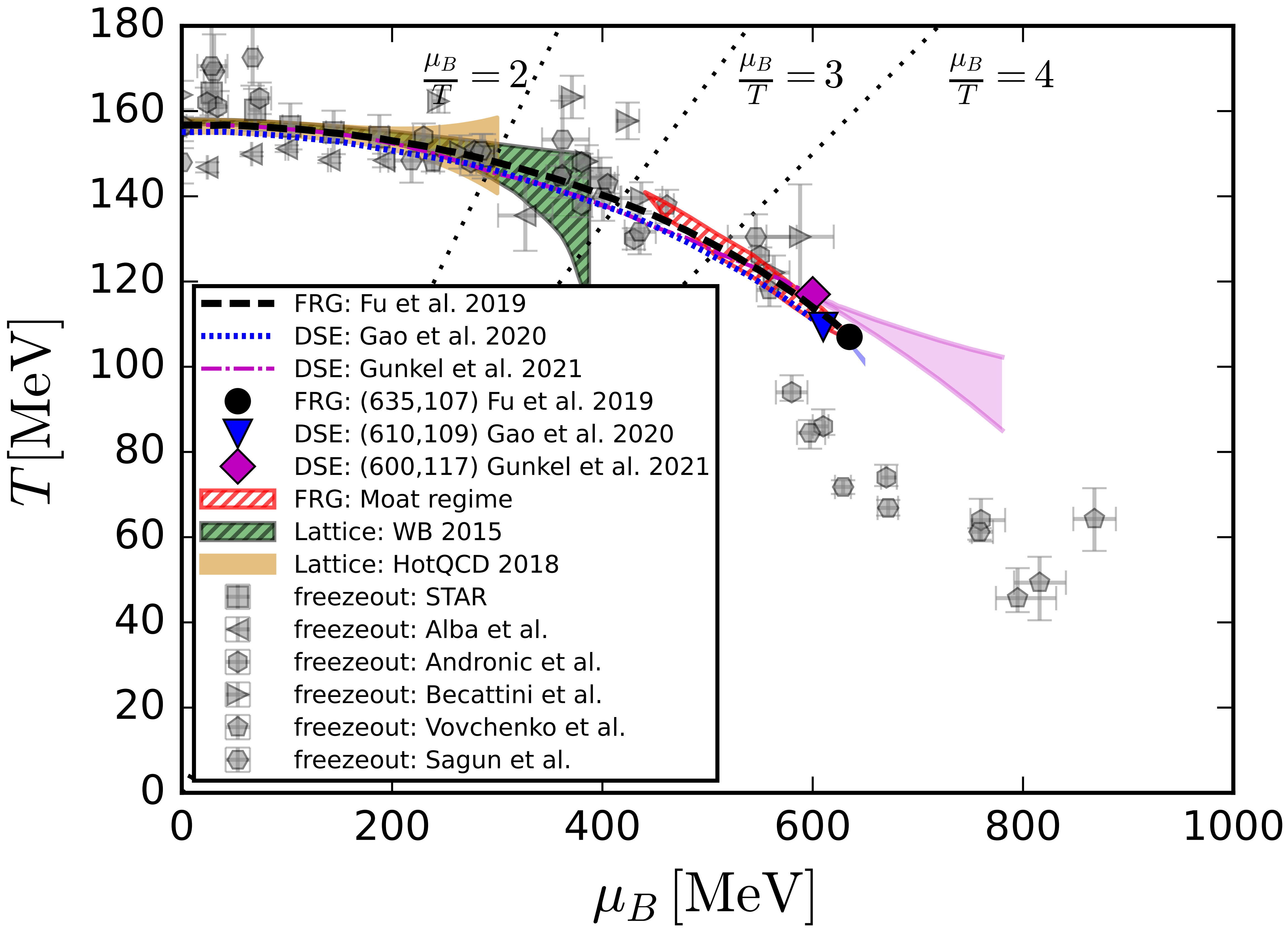


Phase transition

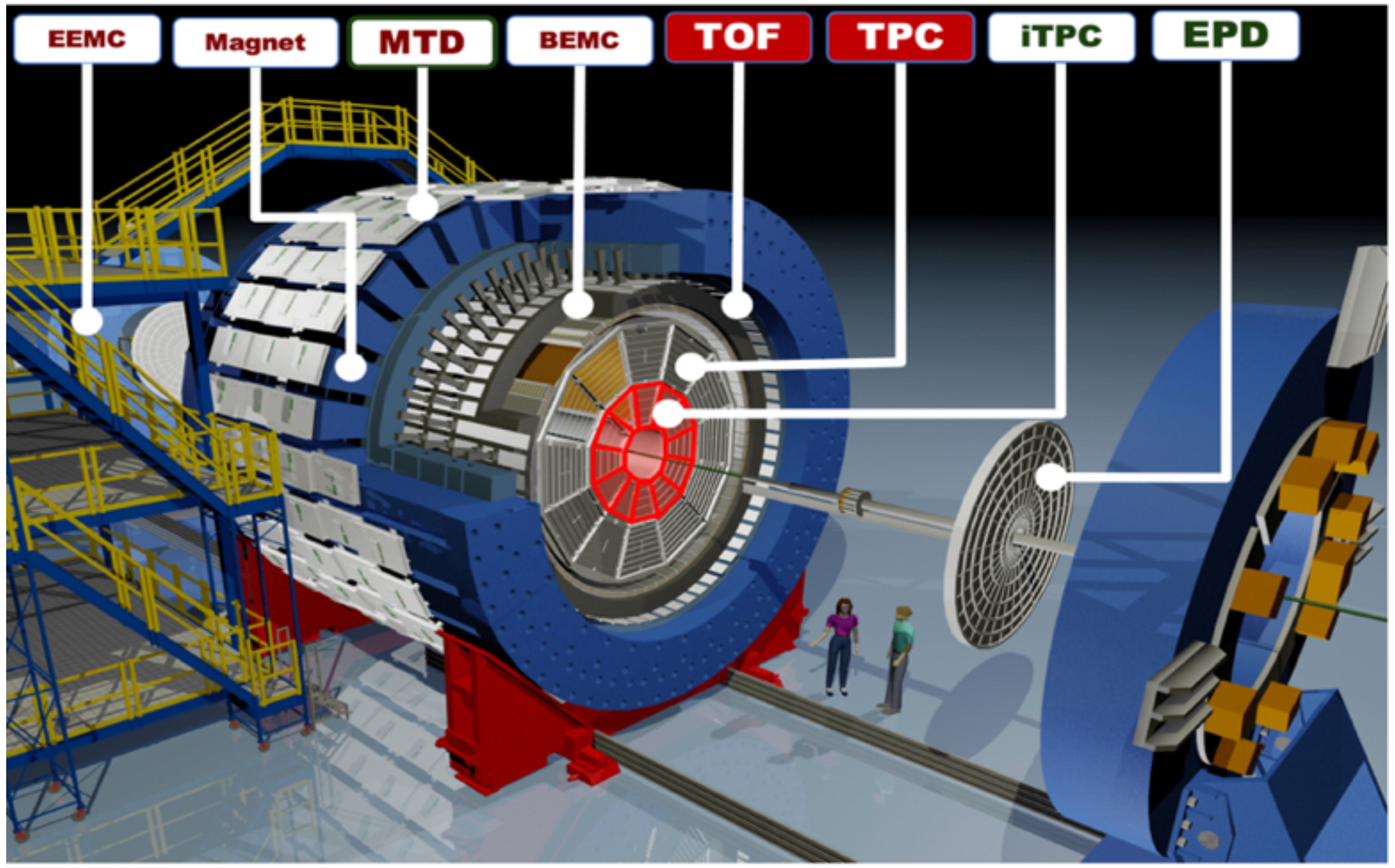


Critical phenomena and functional renormalization group,
S. Yin, Y.-y. Tan and W.-j. Fu
DOI: 10.11889/j.0253-3219.2023.hjs.46.040002

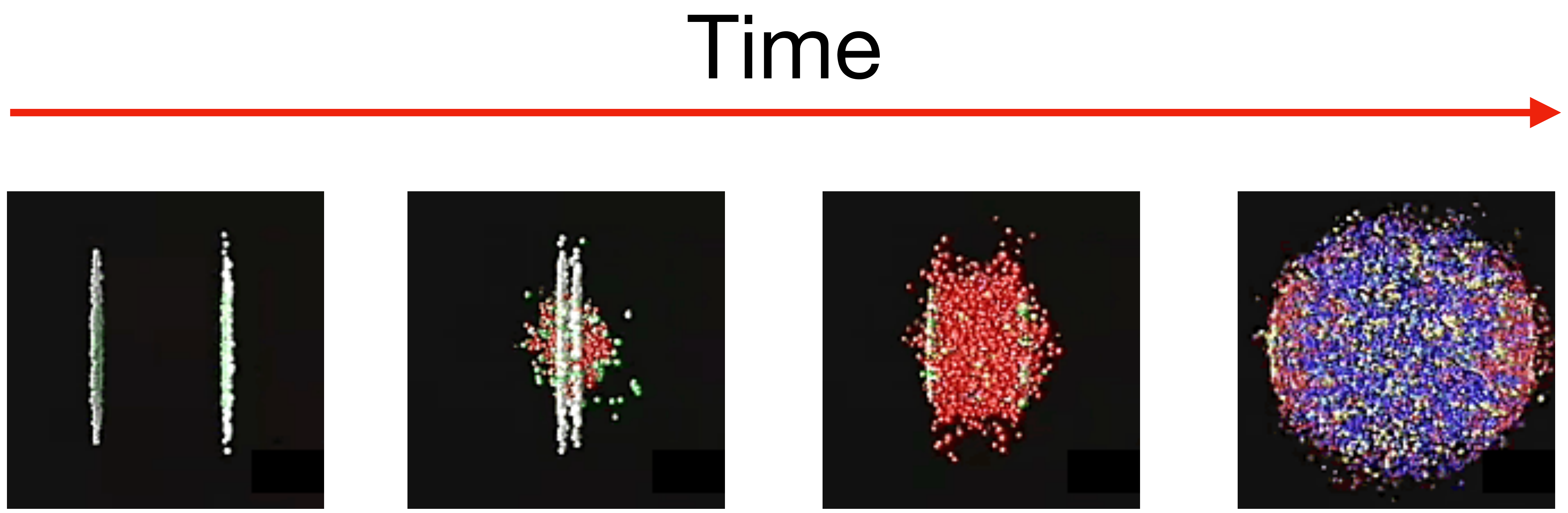
Collection of phase boundaries

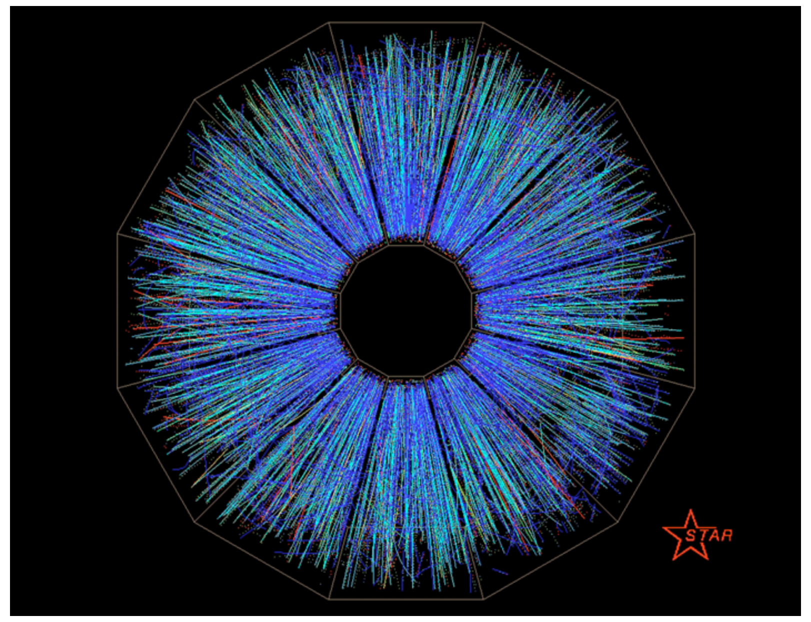


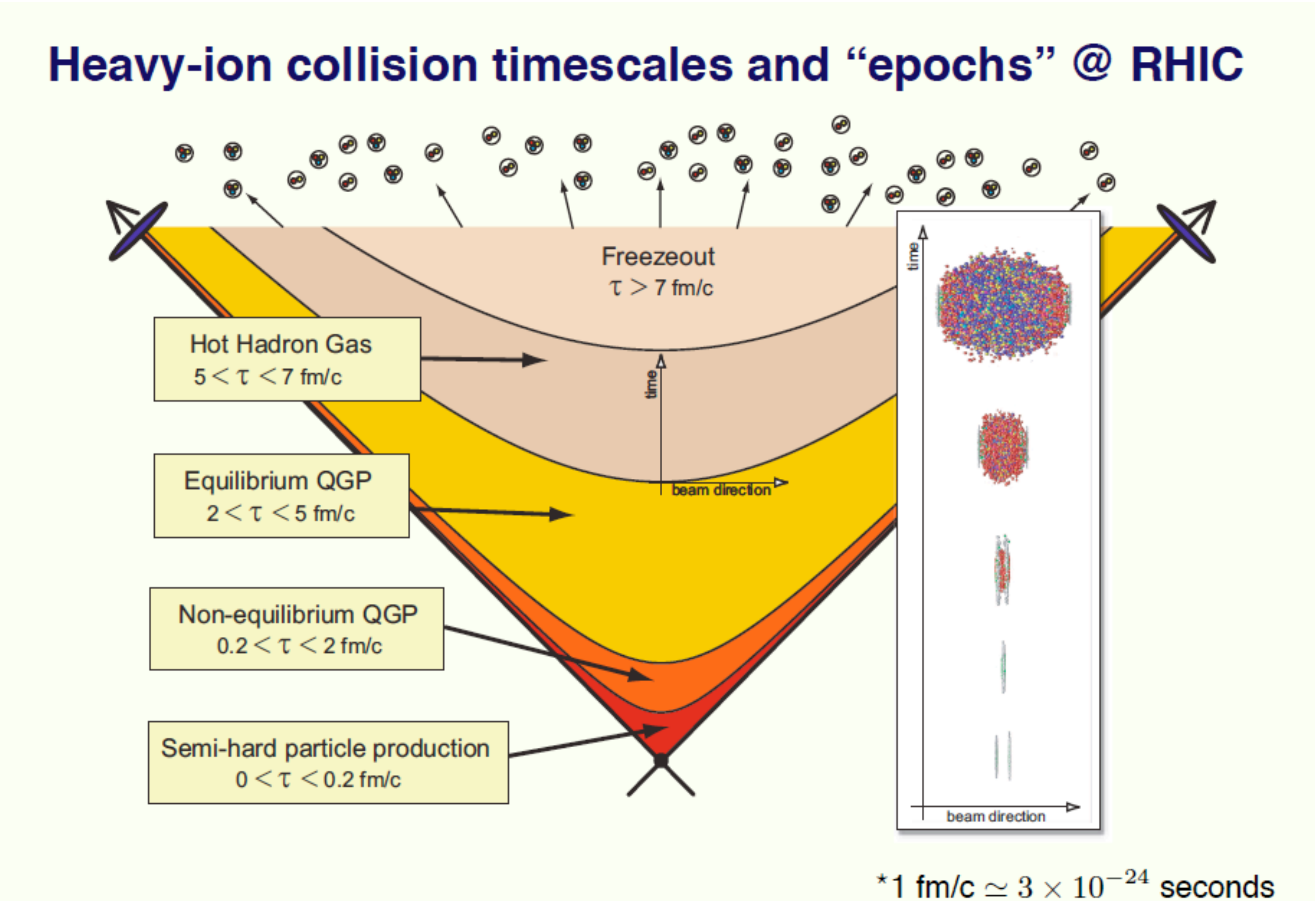
Heavy Ion Collision



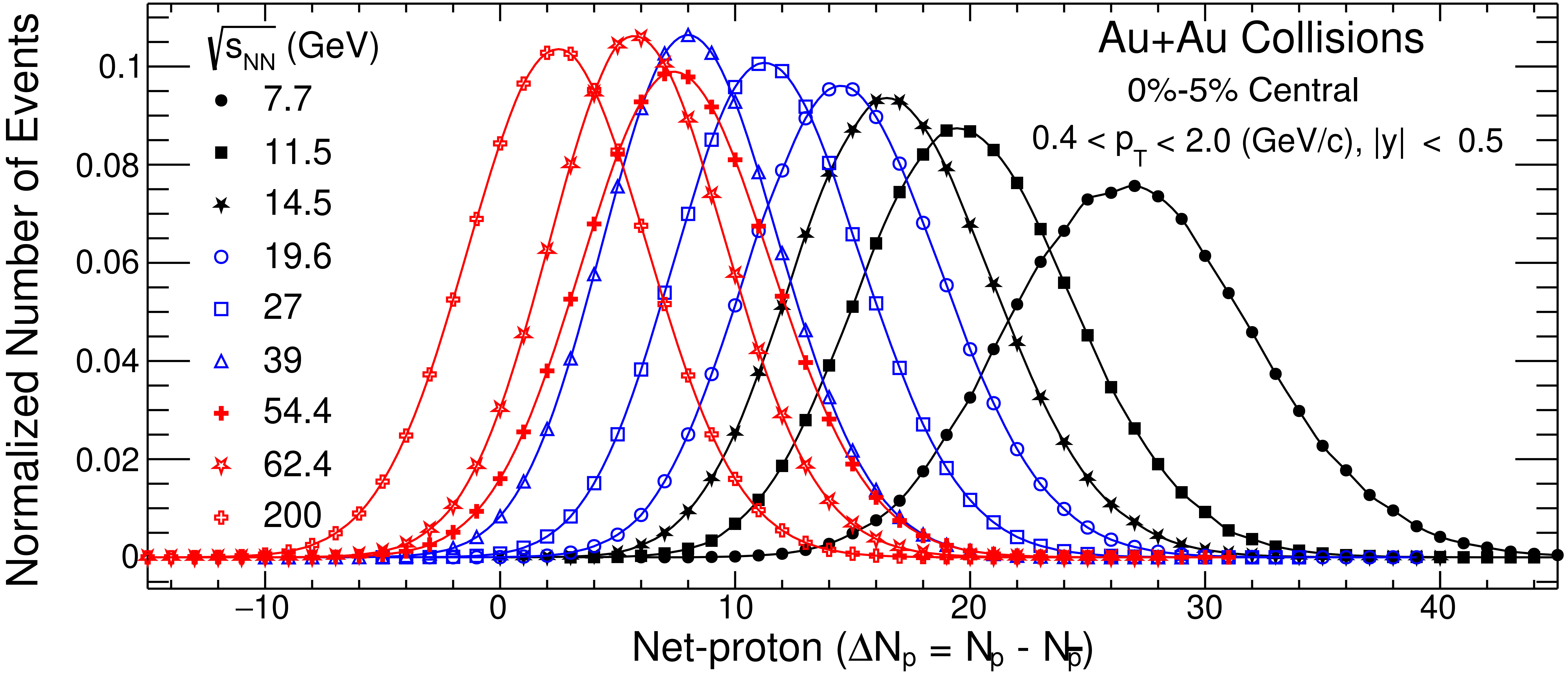
STAR Collaboration, K. H. Ackermann et. al., STAR detector overview, Nucl.Instrum.Meth.A 499 (2003) 624-632







Baryon number distribution



J. Adam et. Al., *Phys.Rev.Lett.* 126 (2021) 9, 092301

Baryon number fluctuations

$$\chi_1^B = \frac{1}{VT^3}\langle N_B \rangle,$$
$$\chi_2^B = \frac{1}{VT^3}\langle (\delta N_B)^2 \rangle,$$
$$\chi_3^B = \frac{1}{VT^3}\langle (\delta N_B)^3 \rangle,$$
$$\chi_4^B = \frac{1}{VT^3}\left(\langle (\delta N_B)^4 \rangle - 3\langle (\delta N_B)^2 \rangle^2\right), \cdots$$

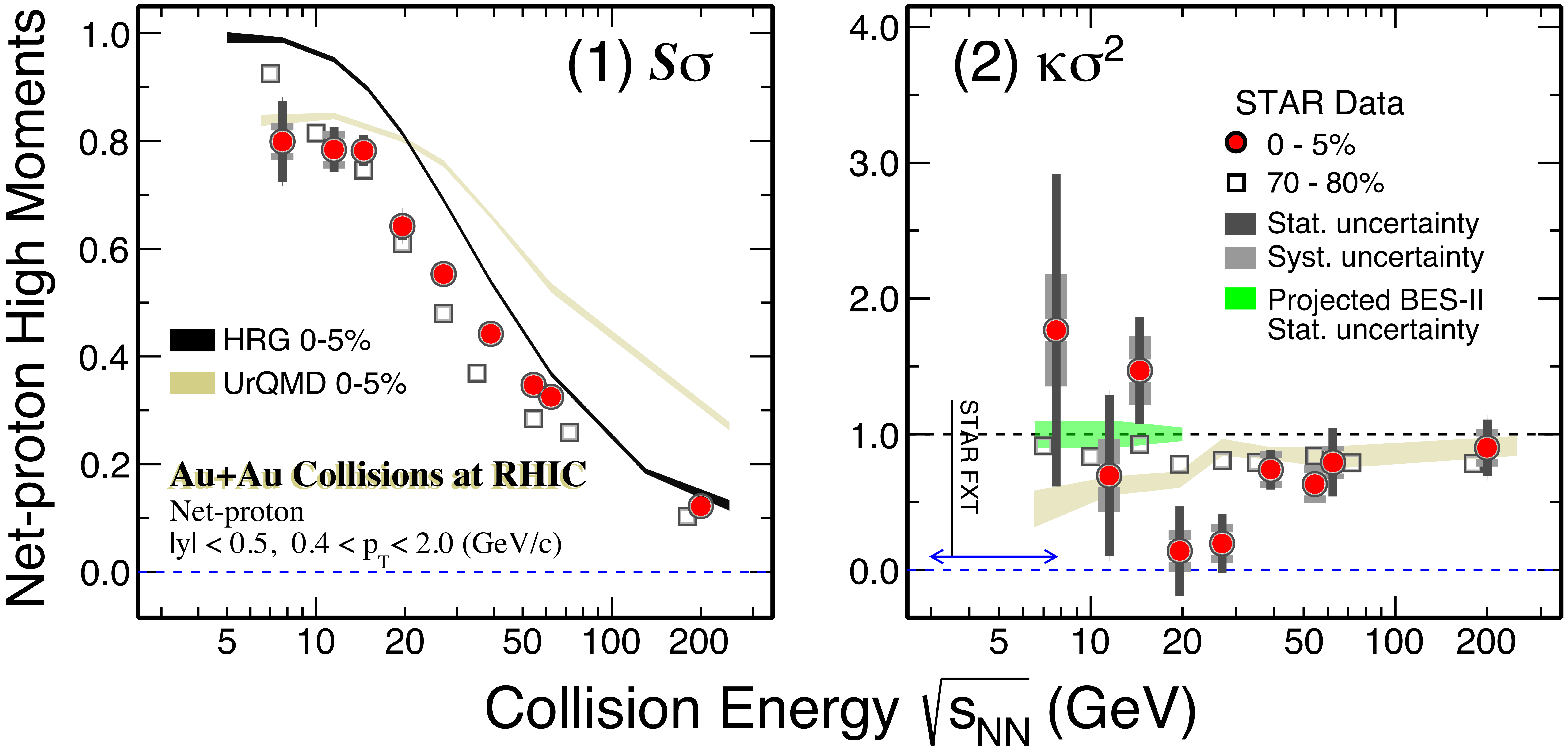
$$\delta N_B = N_B - \langle N_B \rangle$$

Mean value $M = VT^3\chi_1^B$

Variance $\sigma^2 = VT^3\chi_2^B$

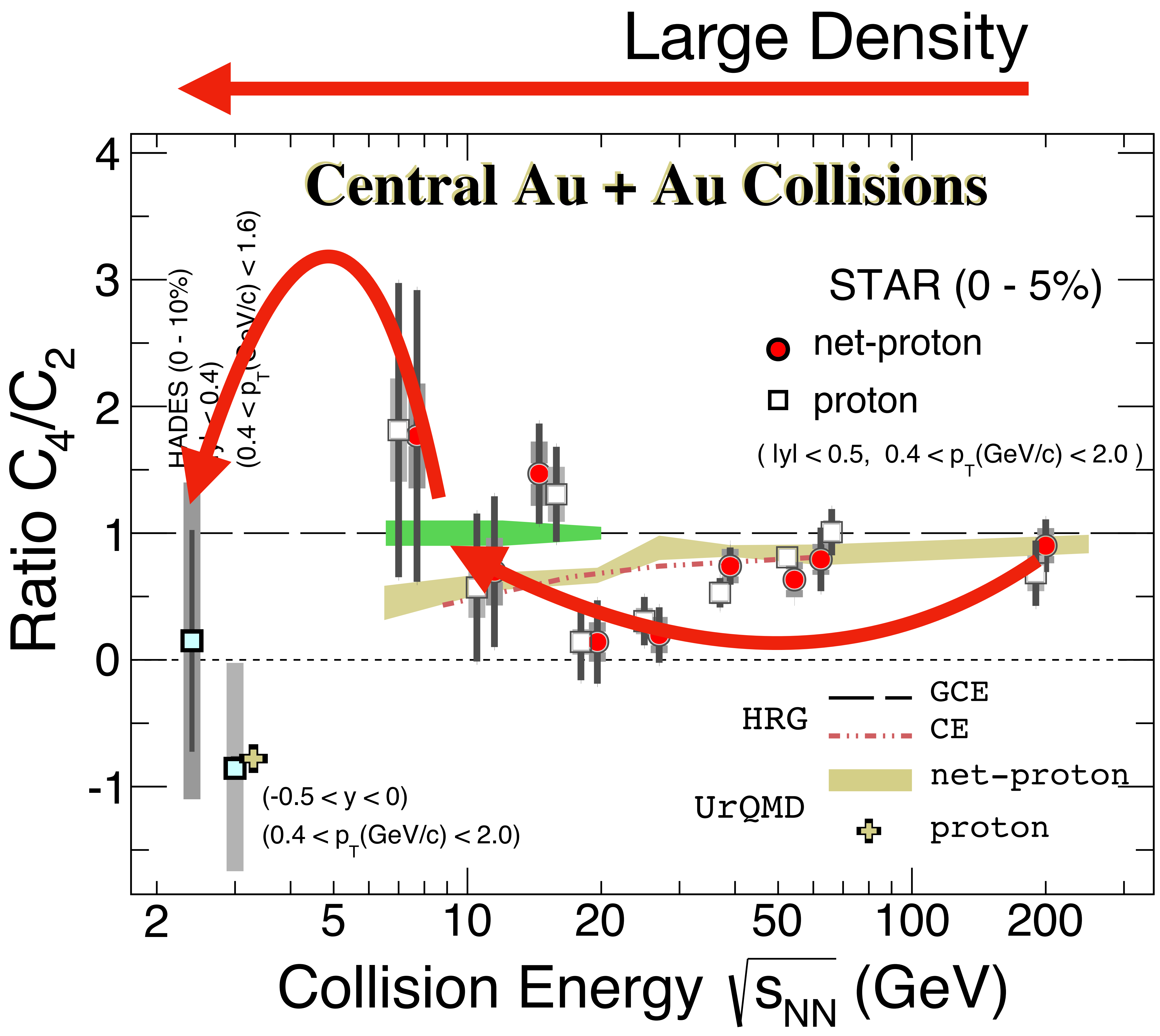
Skewness $S = \frac{\chi_3^B}{\chi_2^B \sigma}$

Kurtosis $\kappa = \frac{\chi_4^B}{\chi_2^B \sigma^2}$

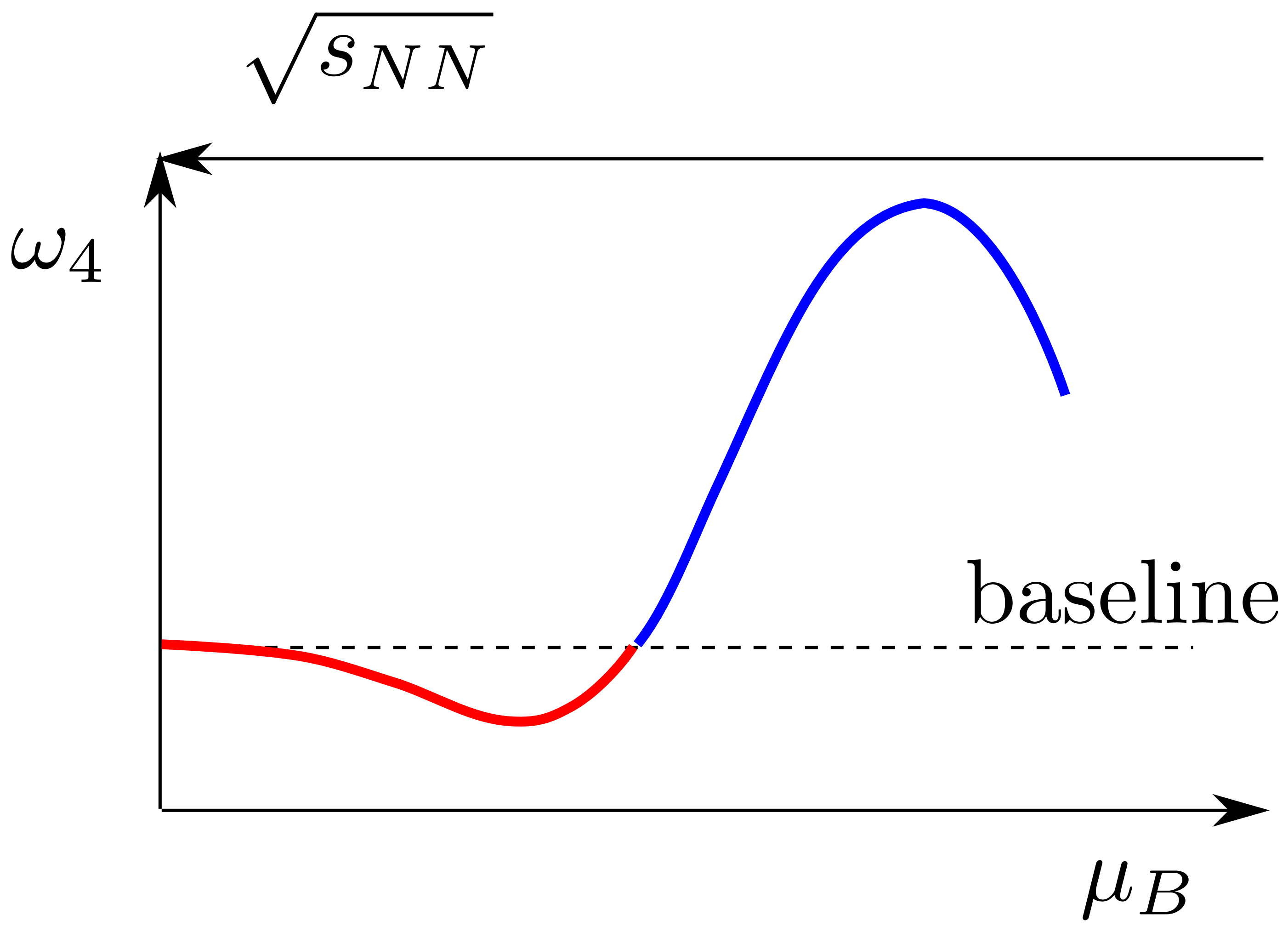


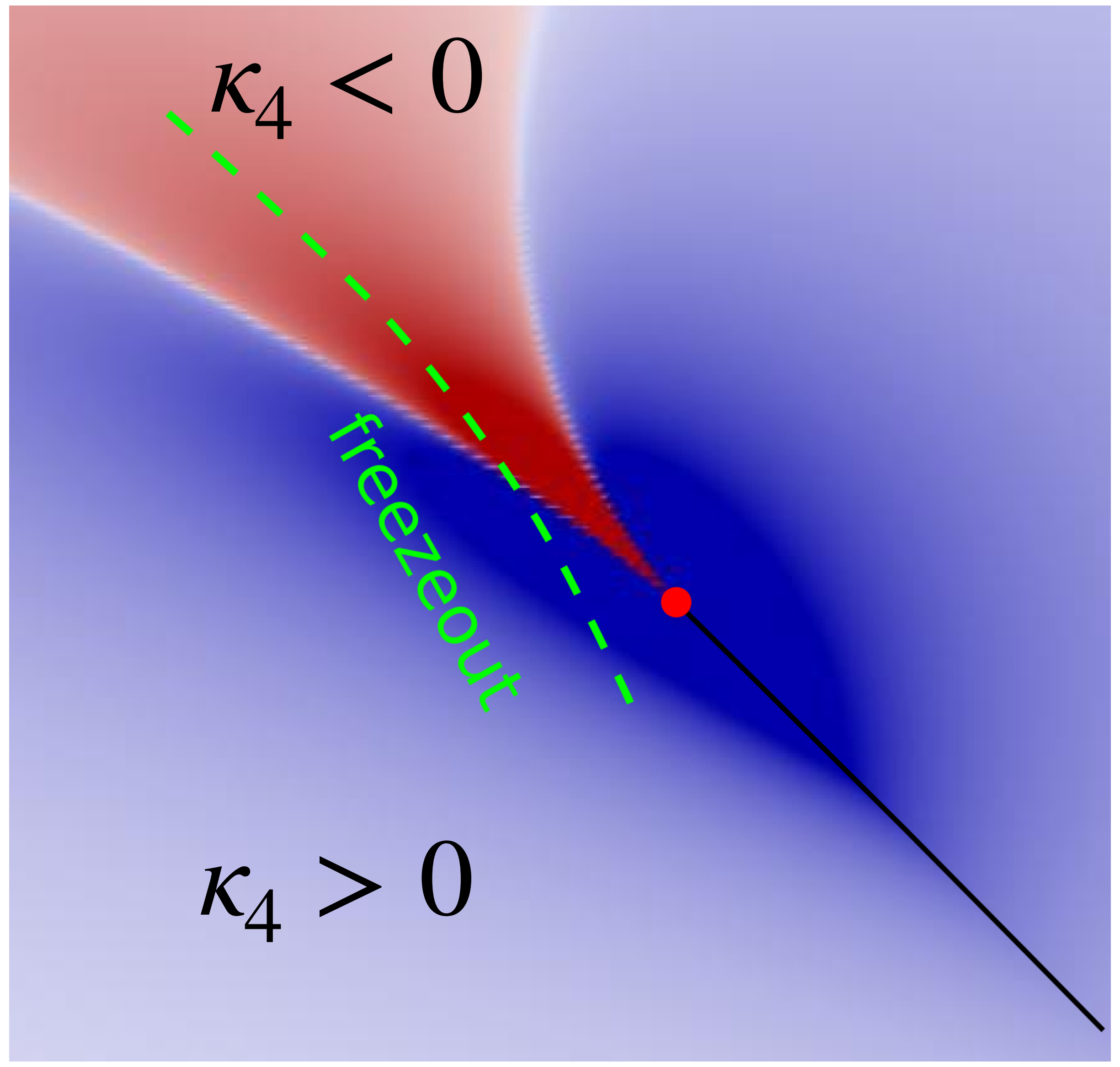
J. Adam *et al.* (STAR), arXiv: 2001.02852

Baryon number fluctuations



M. S. Abdallah et. al., *Phys.Rev.Lett.* 128 (2022) 20, 202303





A.Aprahamian, et al., Reaching for the horizon: The 2015 long range plan for nuclear science(2015).

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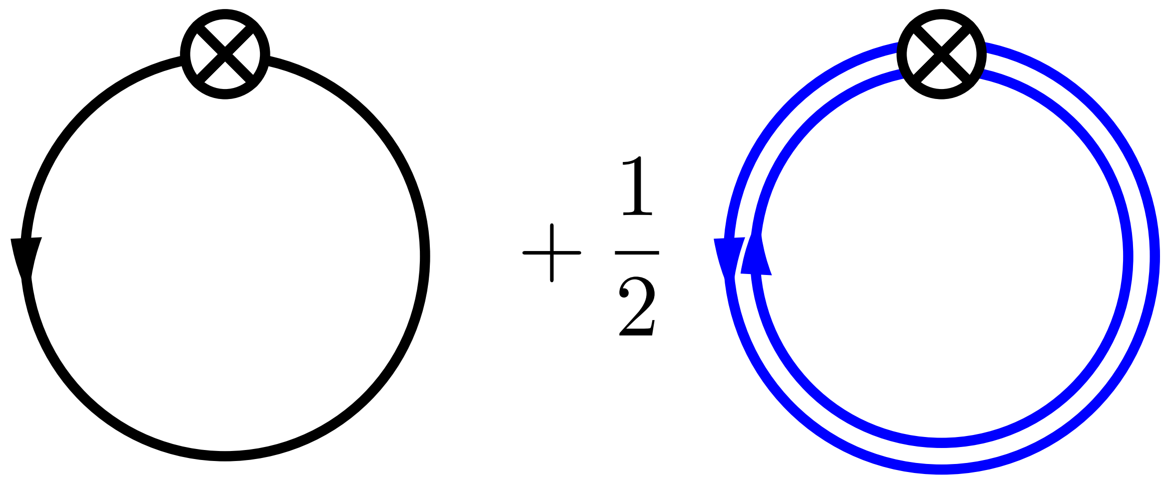
Looking for ripples

First Step: QCD-assisted LEFT within fRG

$$N_f = 2$$

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}\Big[\gamma_\mu\partial_\mu - \gamma_0(\mu + igA_0)\Big]q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2 + h_k\bar{q}(\tau^0\sigma + \vec{\tau}\cdot\vec{\pi})q + V_k(\rho, A_0) - c\sigma \Big\}$$

$$\partial_t\Gamma_k[\Phi] = - \quad + \frac{1}{2}$$



$$\Omega[T,\mu_B] = V_{\text{glue}}(L,\bar{L}) + V_{\text{matter}}(\rho, L, \bar{L}) - c\sigma$$

$$p = -\Omega[T,\mu_B]$$

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$$

$$R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

Only Grand Canonical Ensemble !!!

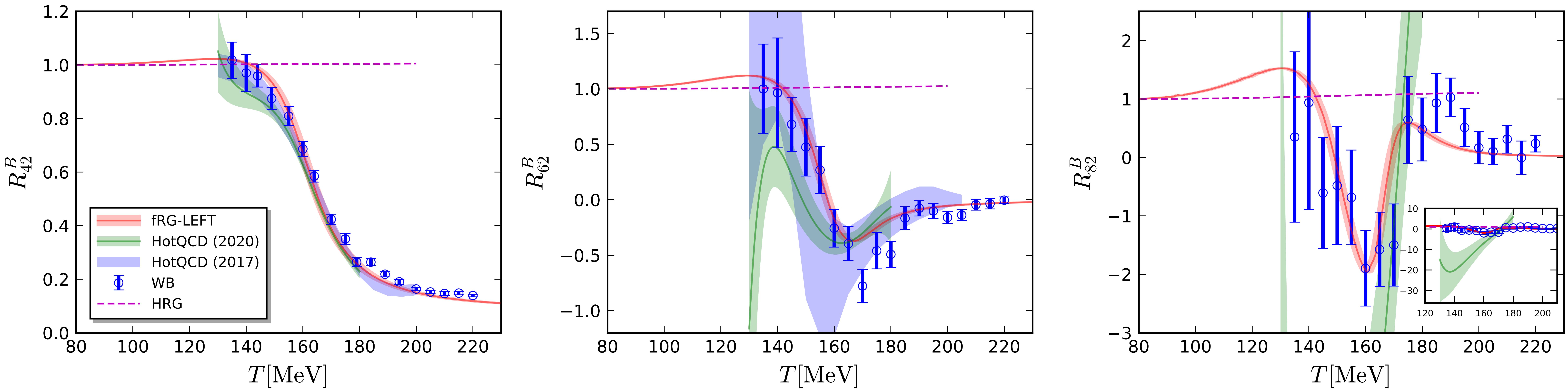
QCD-assisted LEFT

Benchmarked by linear scale-matching

$$T^{\mathrm{LEFT}} = c_T \; T^{\mathrm{QCD}}$$

$$\mu_B^{\mathrm{LEFT}} = c_\mu \; \mu_B^{\mathrm{QCD}}$$

- c_T is determined by matching the pseudo-critical temperatures
- c_μ is determined by matching the curvature of the phase boundaries



fRG-LEFT: Fu, Luo, Pawlowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

HotQCD: A. Bazavov *et al.*, *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

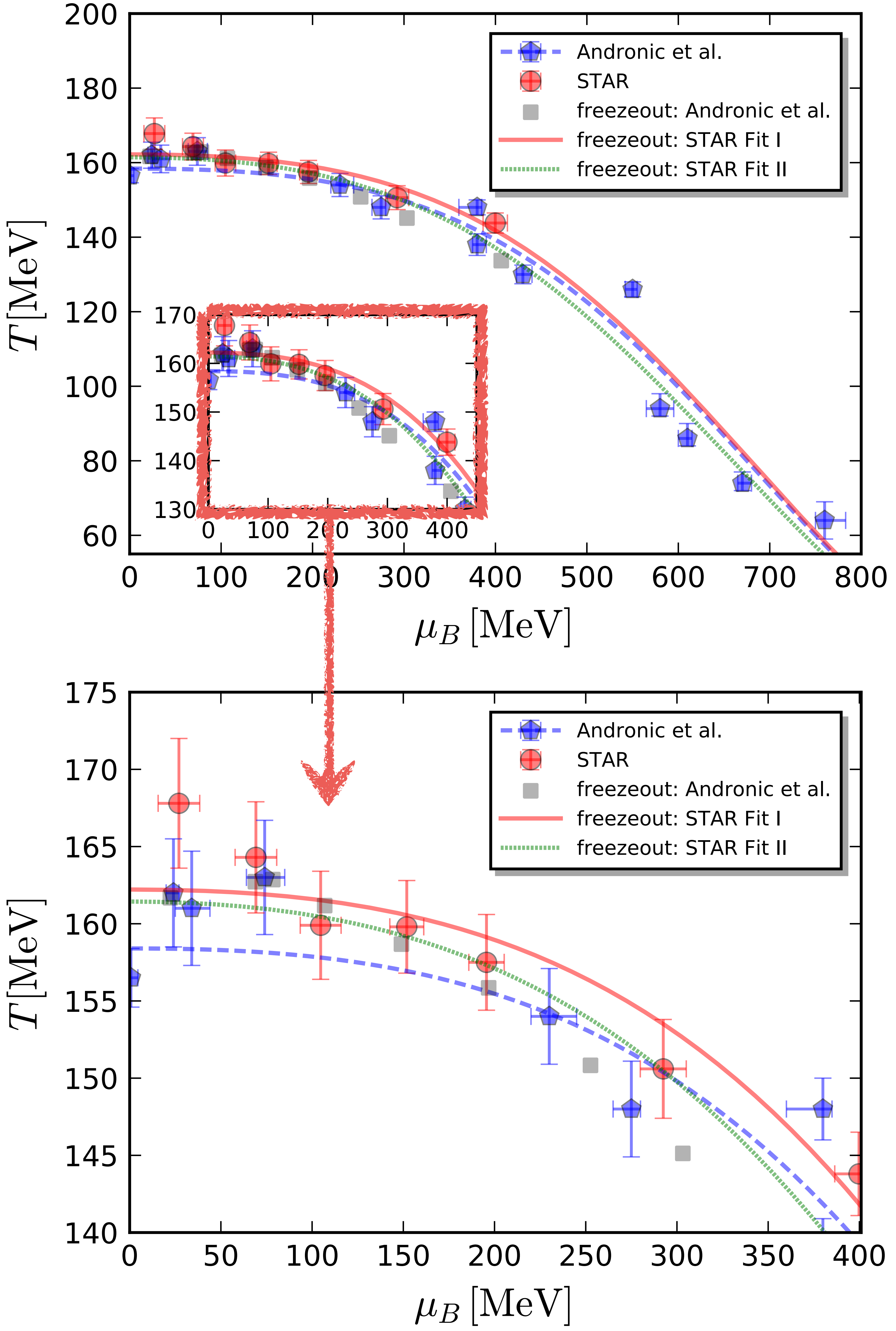
WB: S. Borsanyi *et al.*, *JHEP* 10 (2018) 205

QCD-assisted LEFT

Finite density

Fu, Luo, Pawlowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

Freeze-out curves



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich,
Nature 561 (2018) 7723, 321

$$\mu_{B\,\mathrm{CF}} = \frac{a}{1 + 0.288\sqrt{s_{\mathrm{NN}}}}\,,$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

all data points

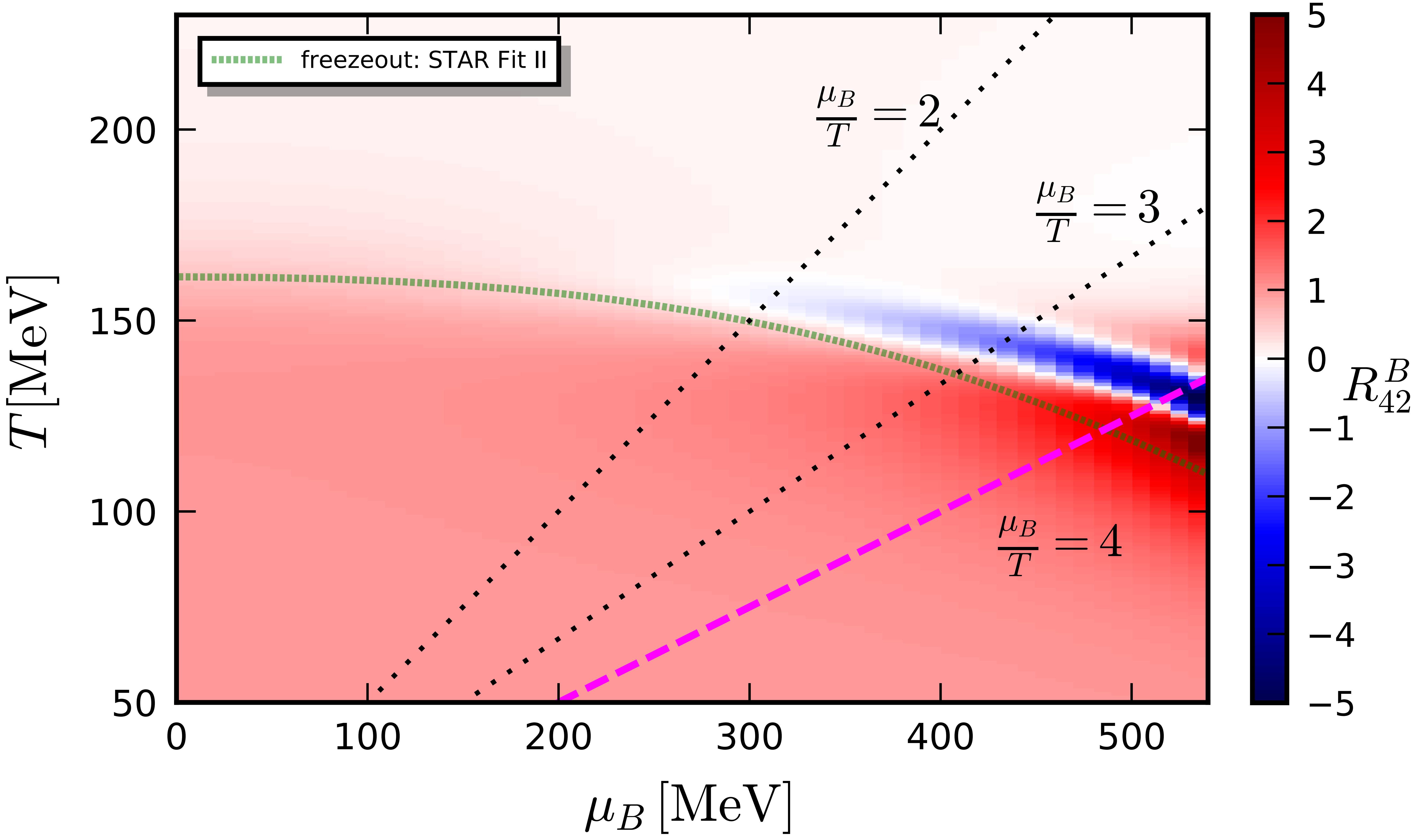
3. freeze-out: STAR Fit II

neglecting first two at low μ_B and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

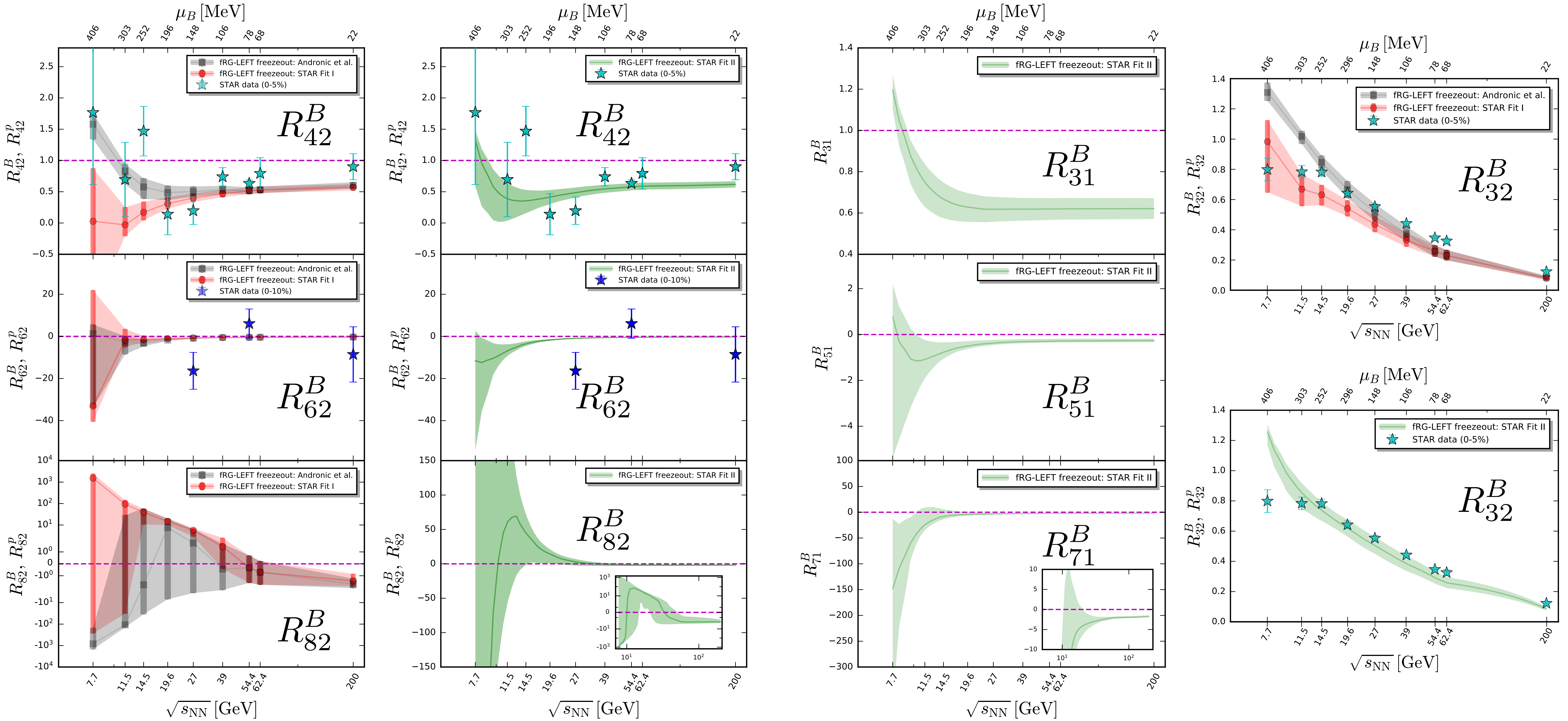
Fu, Luo, Pawlowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

Freeze-out curves



Fu, Luo, Pawlowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

Baryon number fluctuations on Freeze-out curves



Fu, Luo, Pawlowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

Summary of our first step

1. We use a QCD-assisted LEFT to compute baryon number fluctuations.
2. Fluctuations have a non-monotonic energy dependence.
3. Non-monotonic behavior can arise with the sharper crossover.

Shortcoming

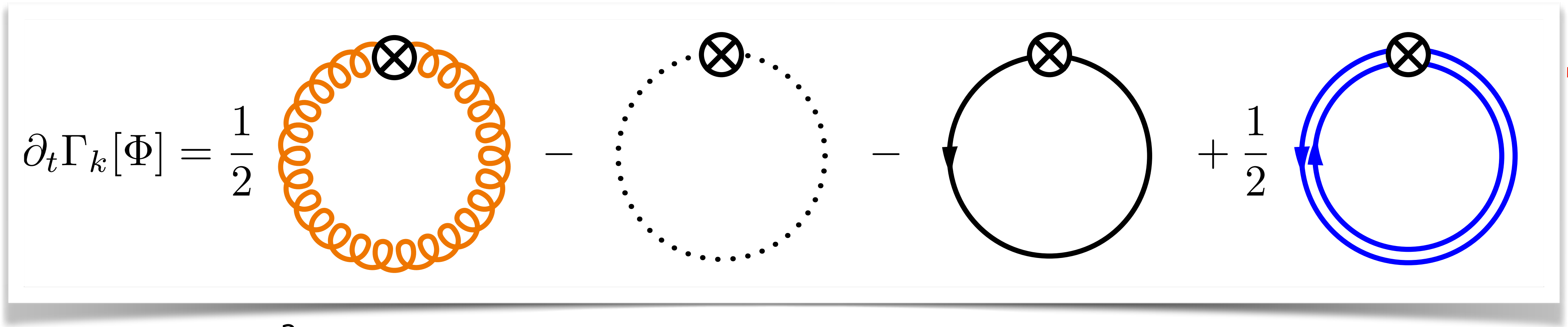
A. Reliable only in low-density areas
B. Only Grand Canonical Ensemble results

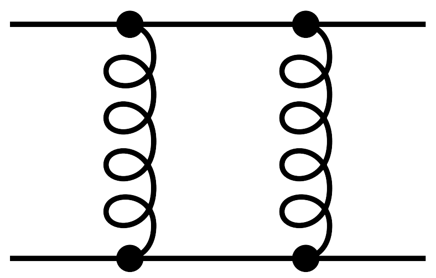
Looking for ripples

Second Step: Updated QCD-assisted LEFT

Call support from fRG-QCD

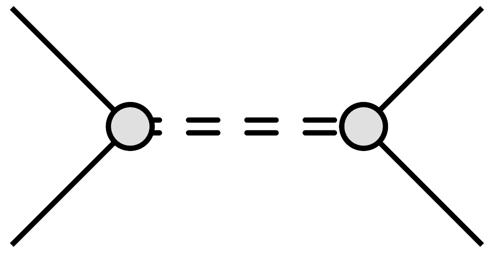
First Principle QCD flow equation:





Four-quark interaction

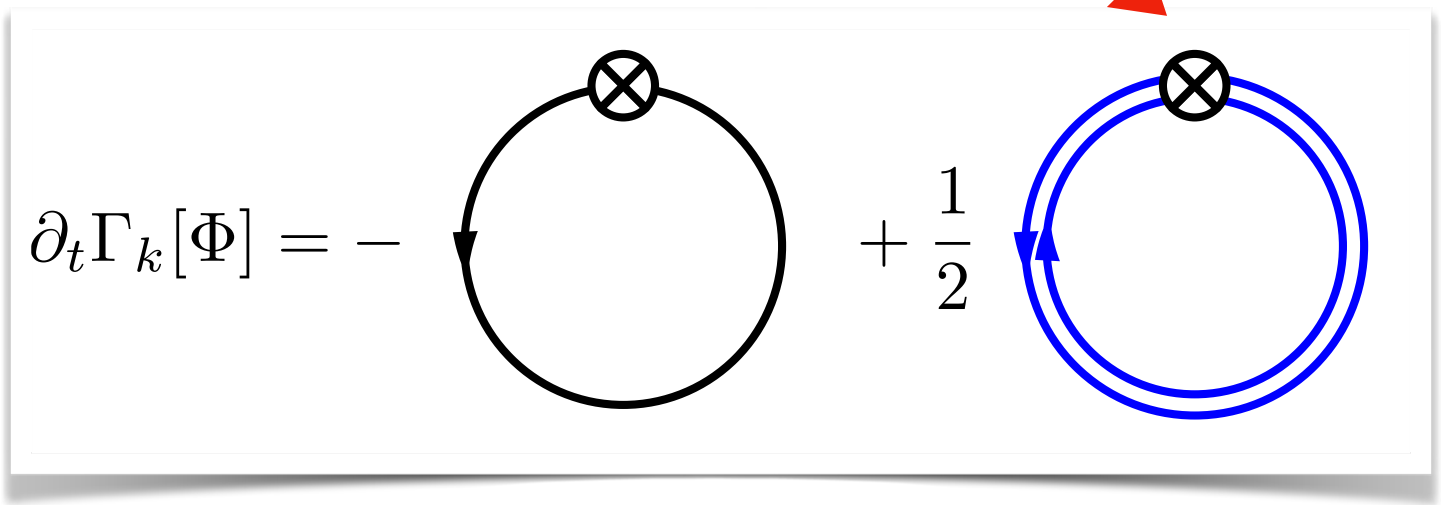
Dynamical hadronization

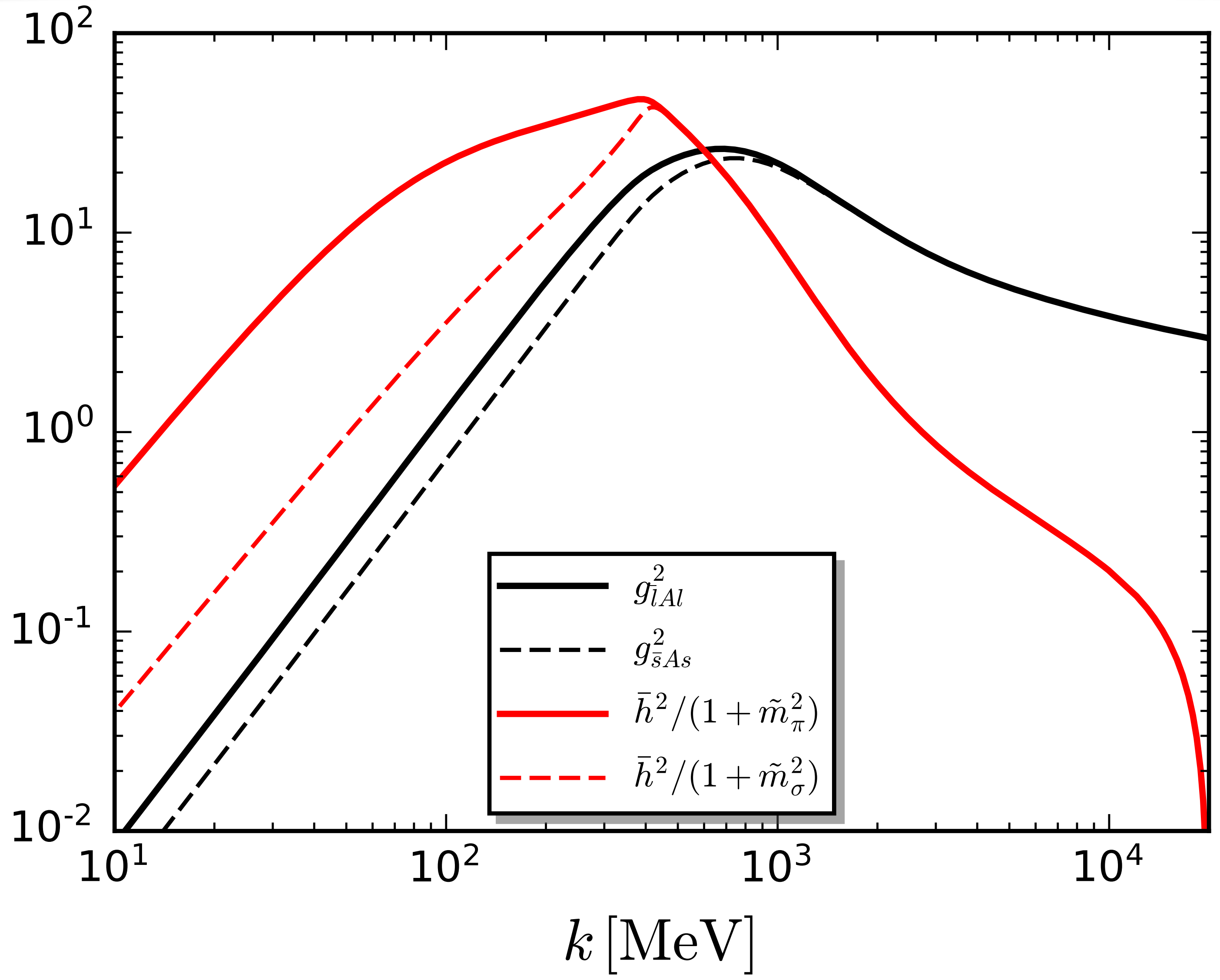


Yukawa coupling

$$\partial_t \bar{h} = \left(\frac{1}{2}\eta_\phi + \eta_q\right)\bar{h} - \bar{m}_\pi^2 \dot{\bar{A}} + \overline{\mathrm{Flow}}^{(3)}_{(\bar{q}\vec{\tau}q)\vec{\pi}}$$

LEFT flow equation:





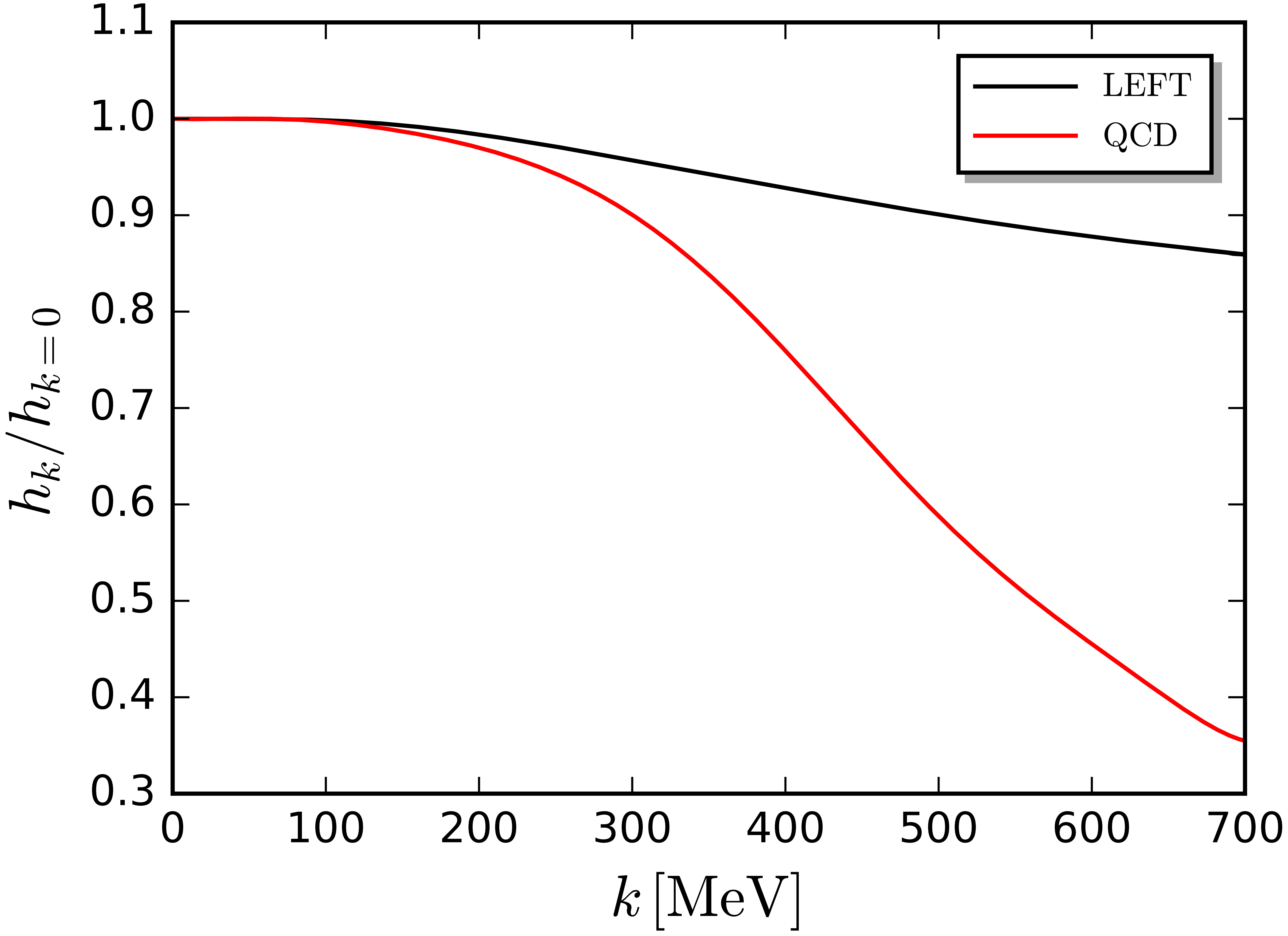
Fu, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

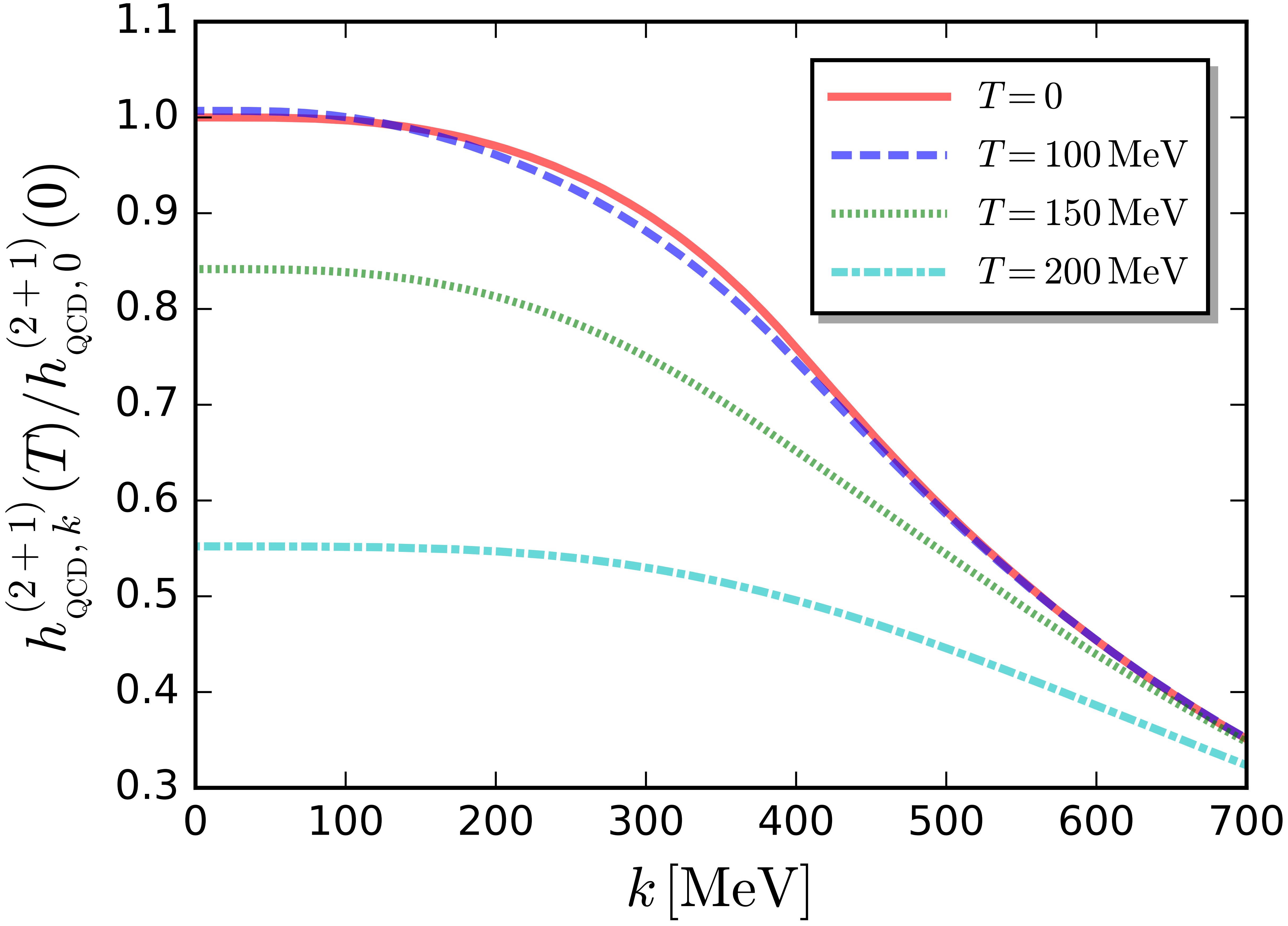
Exchange couplings

- **fRG-LEFT have been improved.**
- **Dynamics of first-principle fRG-QCD has been encoded in the LEFT via the Yukawa coupling.**

Updated QCD-assisted LEFT

Yukawa coupling

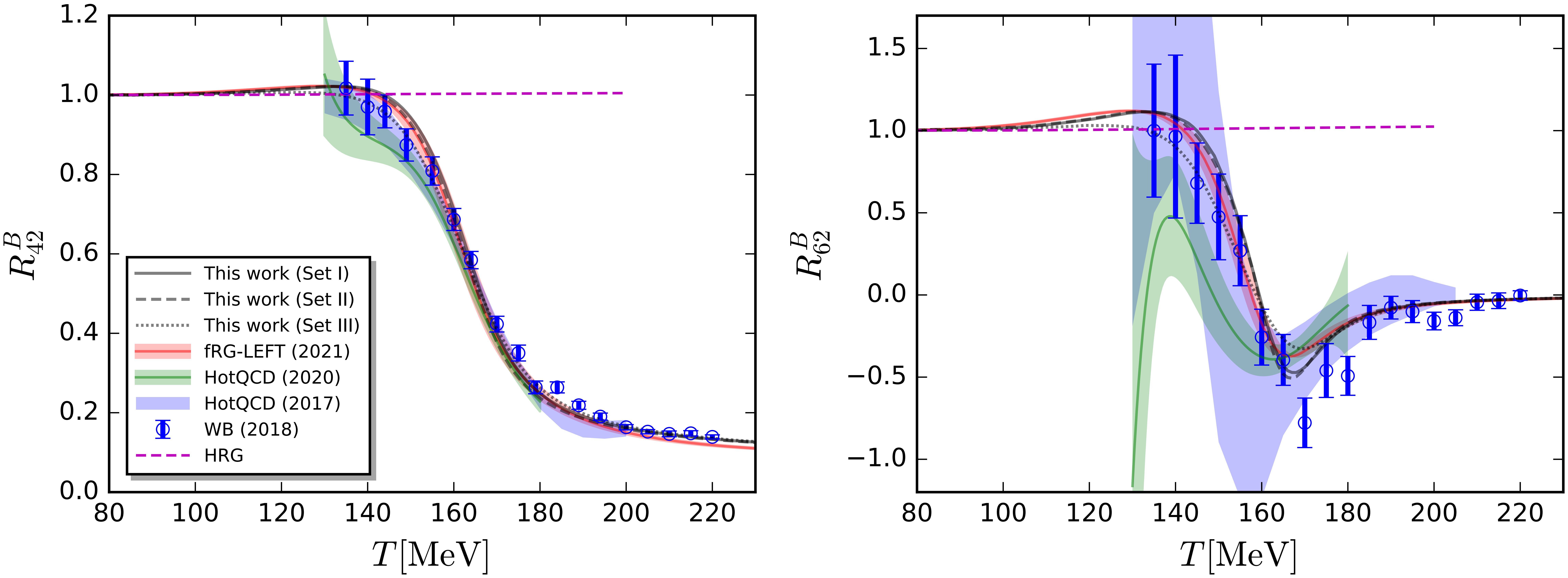




Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

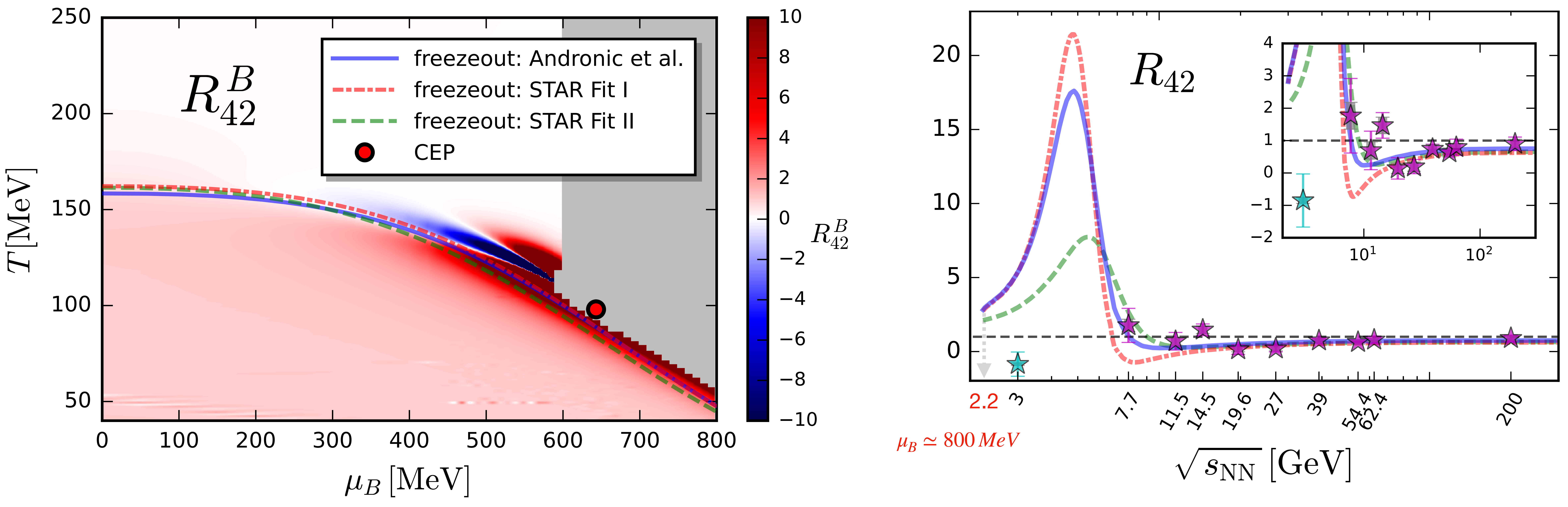
Updated QCD-assisted LEFT

$\mu_B = 0$



Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

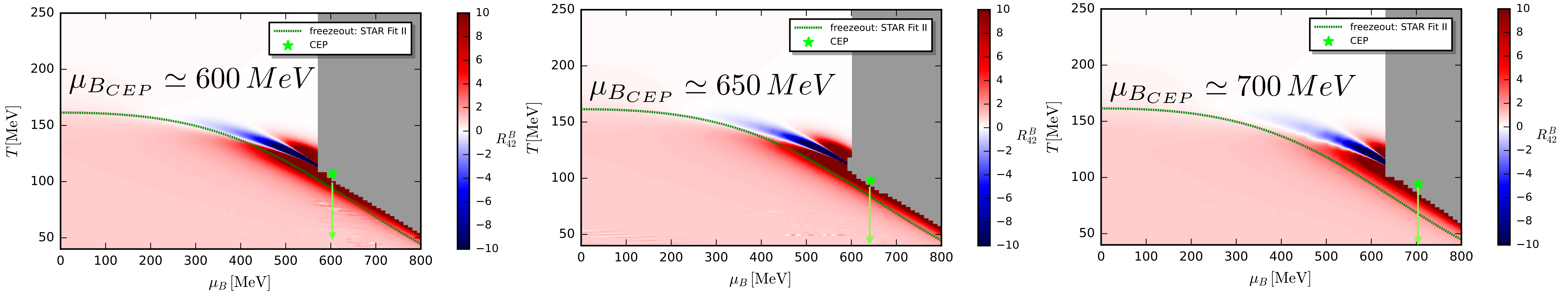
Fluctuations Phase diagram

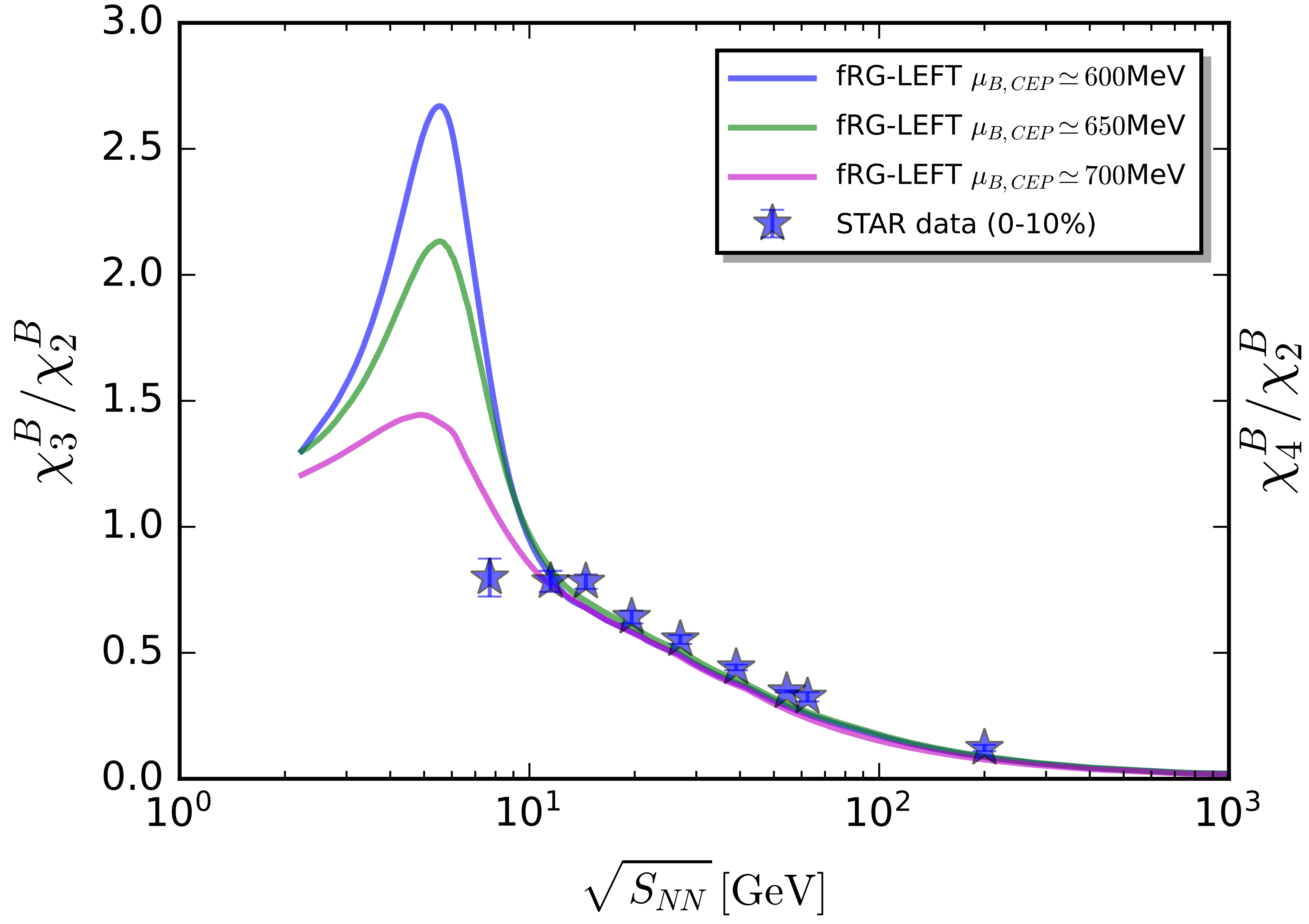


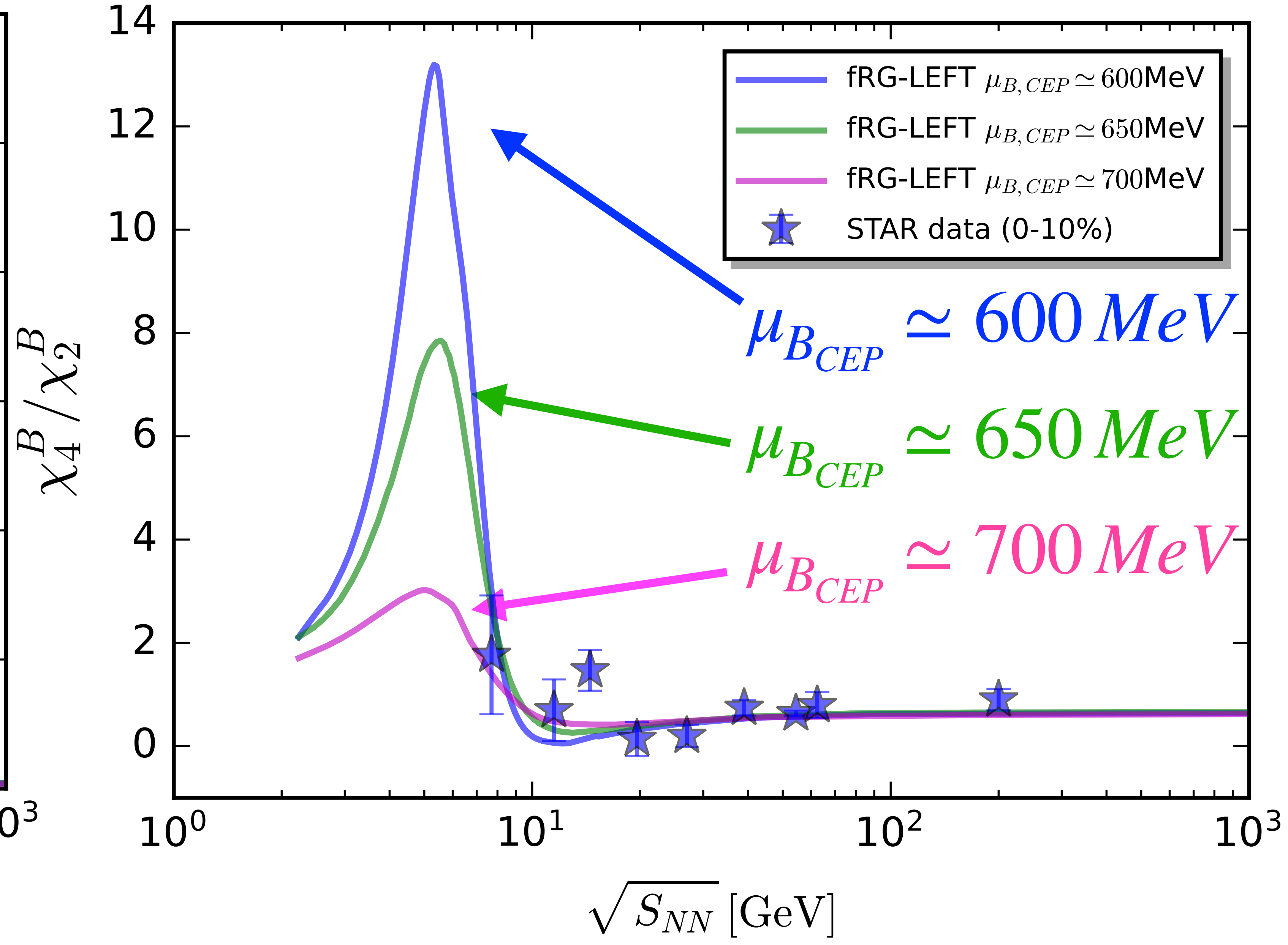
- Calculations have been extended from $\mu_B \sim$ 500 to 800 MeV in the improved fRG-LEFT.
- A "peak" structure is found in the regime of low collision energy.

Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv*:2308.15508

Different location of CEP



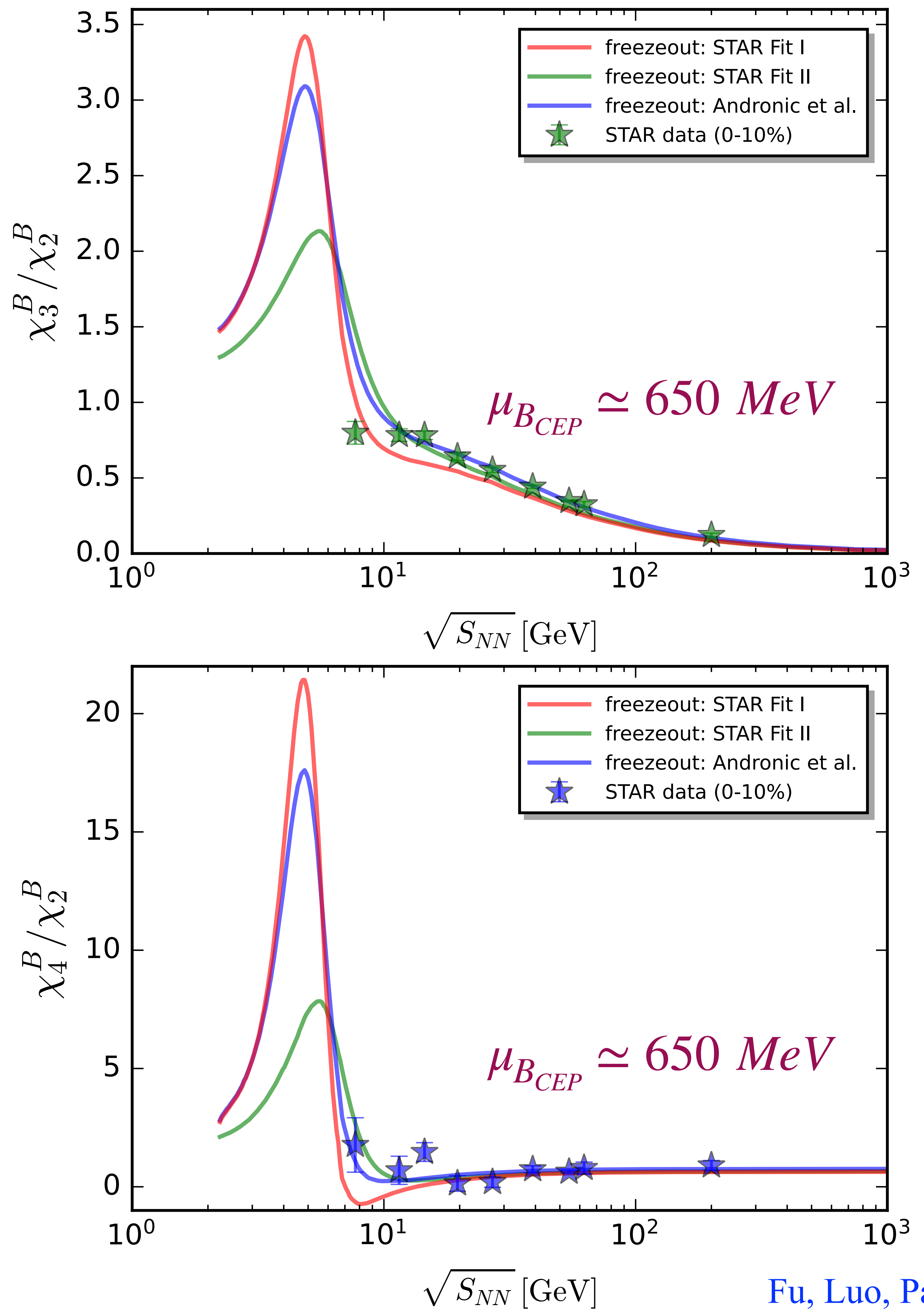


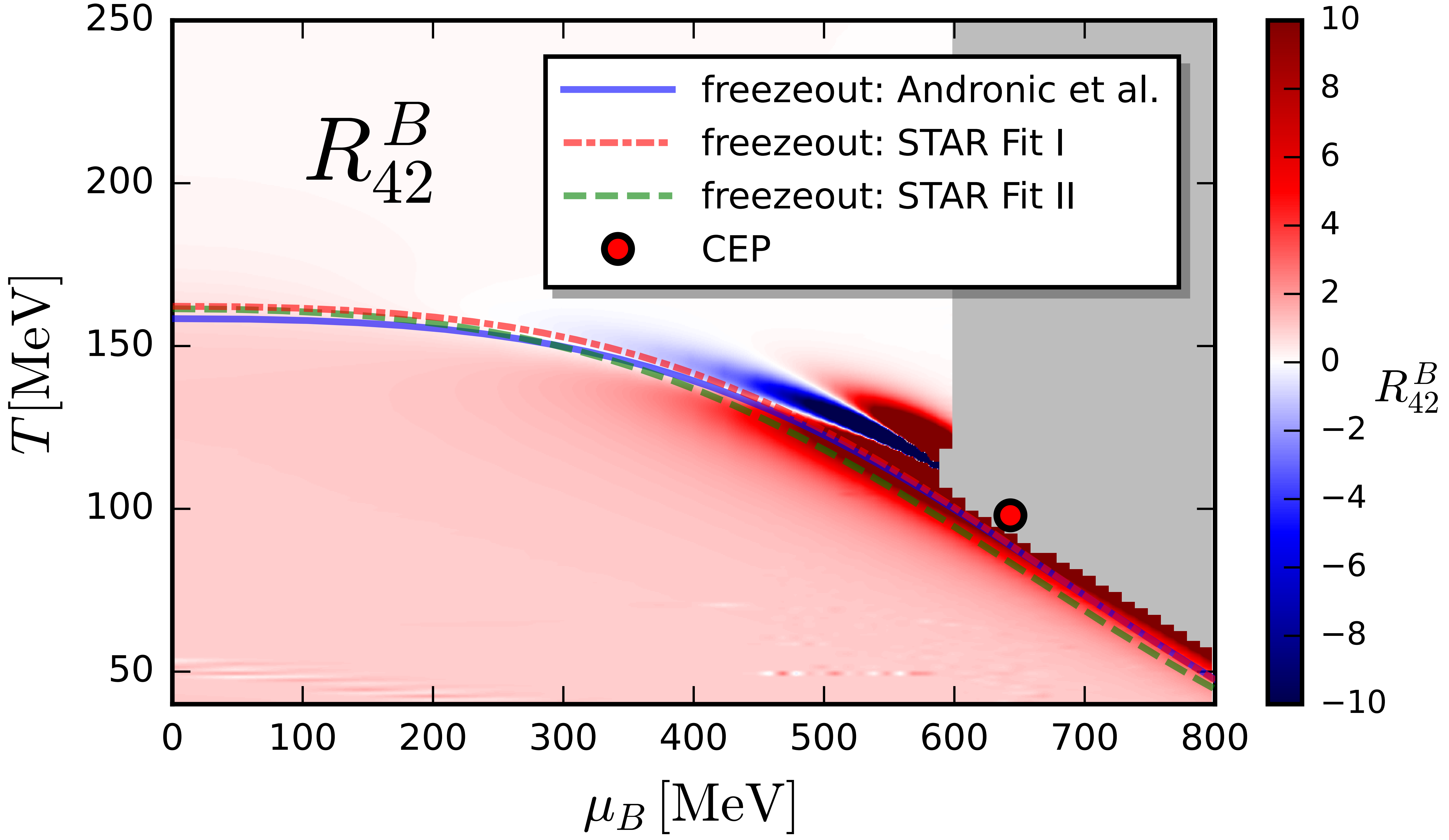


- Location of the peak, i.e., the corresponding collision energy shows very mild dependence on the location of CEP in the range $\mu_{B_{\rm CEP}} \simeq (600 \sim 700)$ MeV.
- While the height of the peak shows significant dependence.

Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv*:2308.15508

Different freeze-out curves

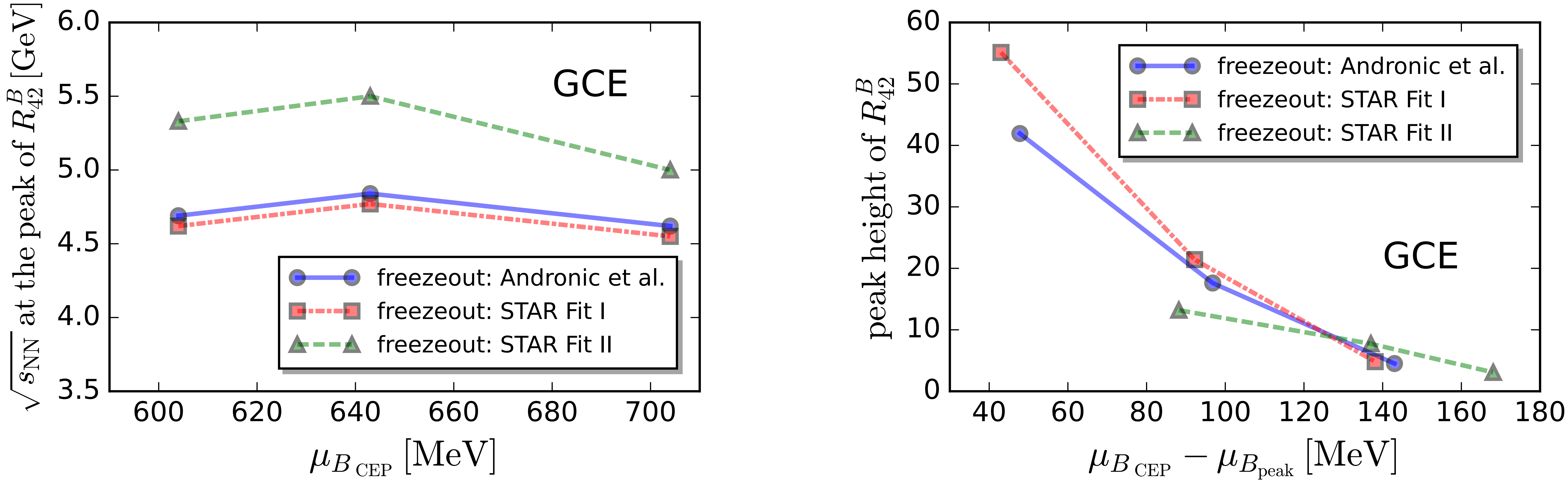




- Height and location of the peak are influenced by different freeze-out curves, especially for the height.
- Closer the freeze-out curve is to the phase boundary, higher the peak.

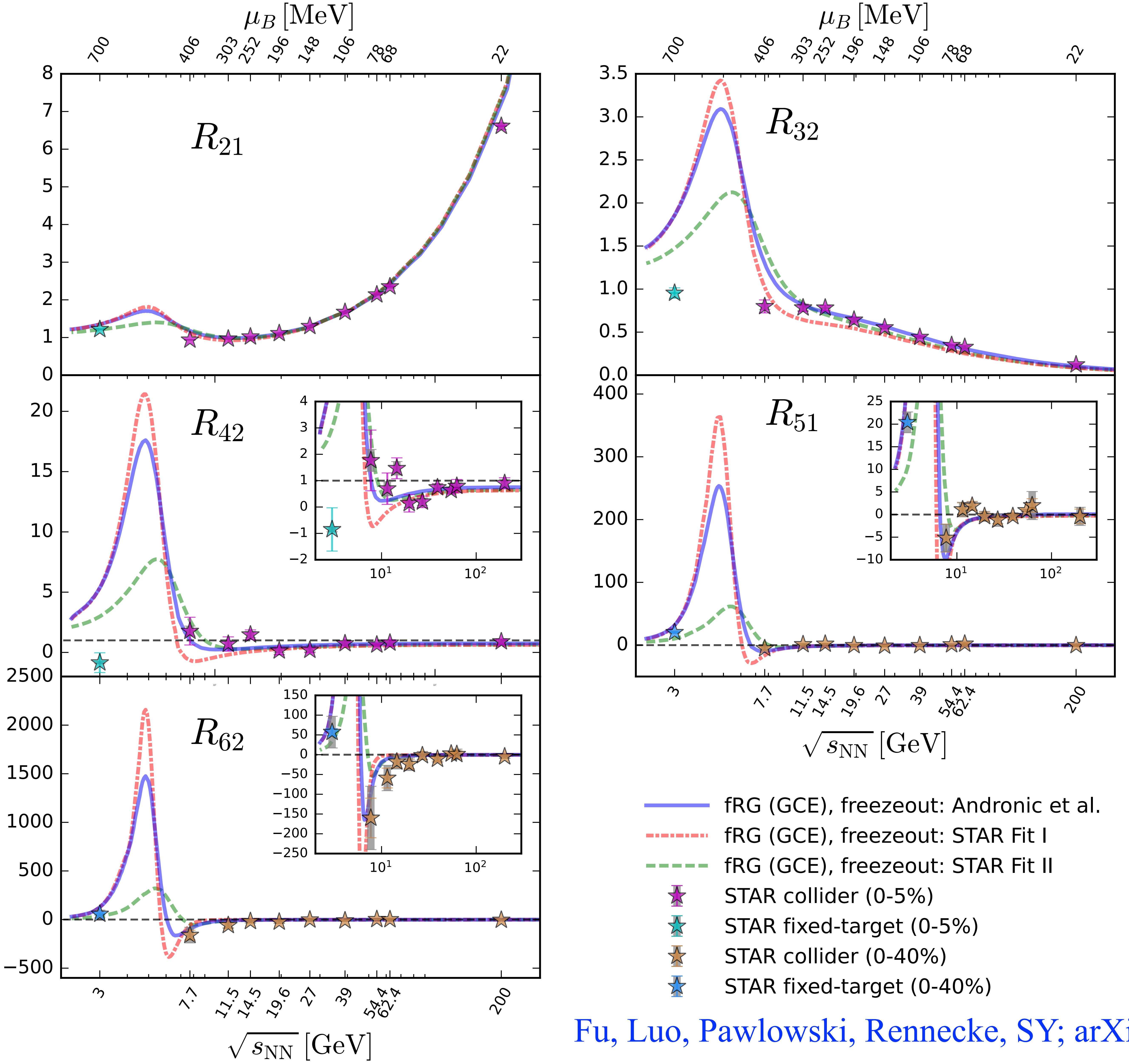
Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

Different location of CEP & freez-eout curves



Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

Fluctuations within GCE



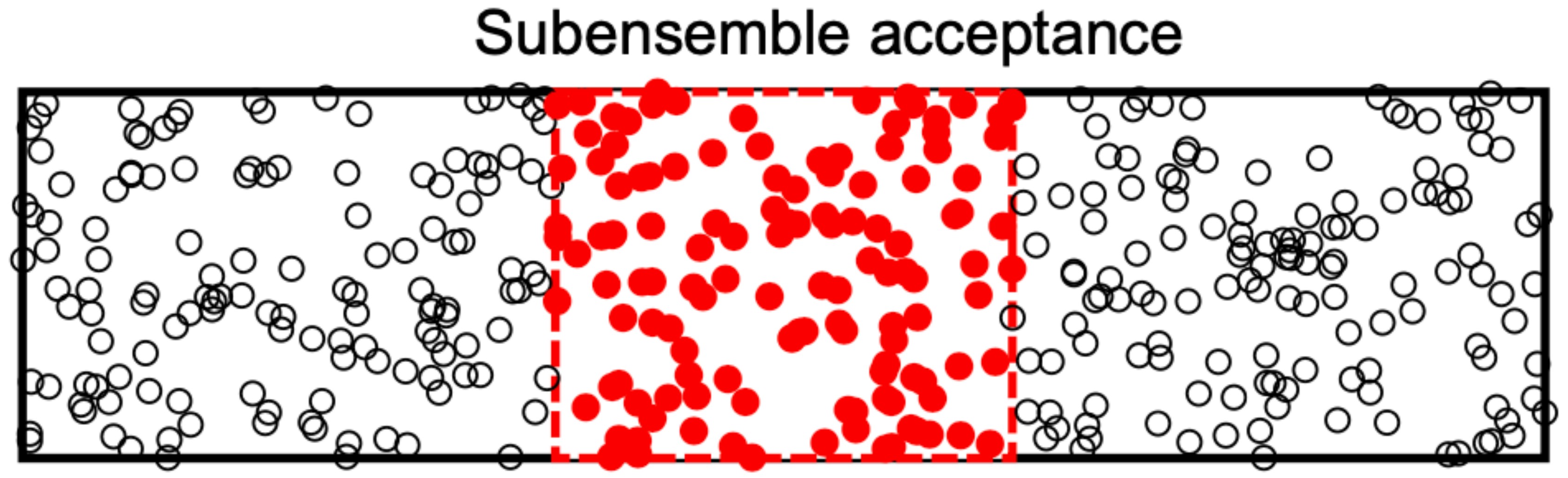
Fu, Luo, Pawlowski, Rennecke, SY; arXiv: 2308.15508

From GCE to CE

- Subensemble acceptance method

$V_1 = \alpha V$ α Proportion of subsystems to total system

$\beta = 1 - \alpha$ β Proportion of remaining systems

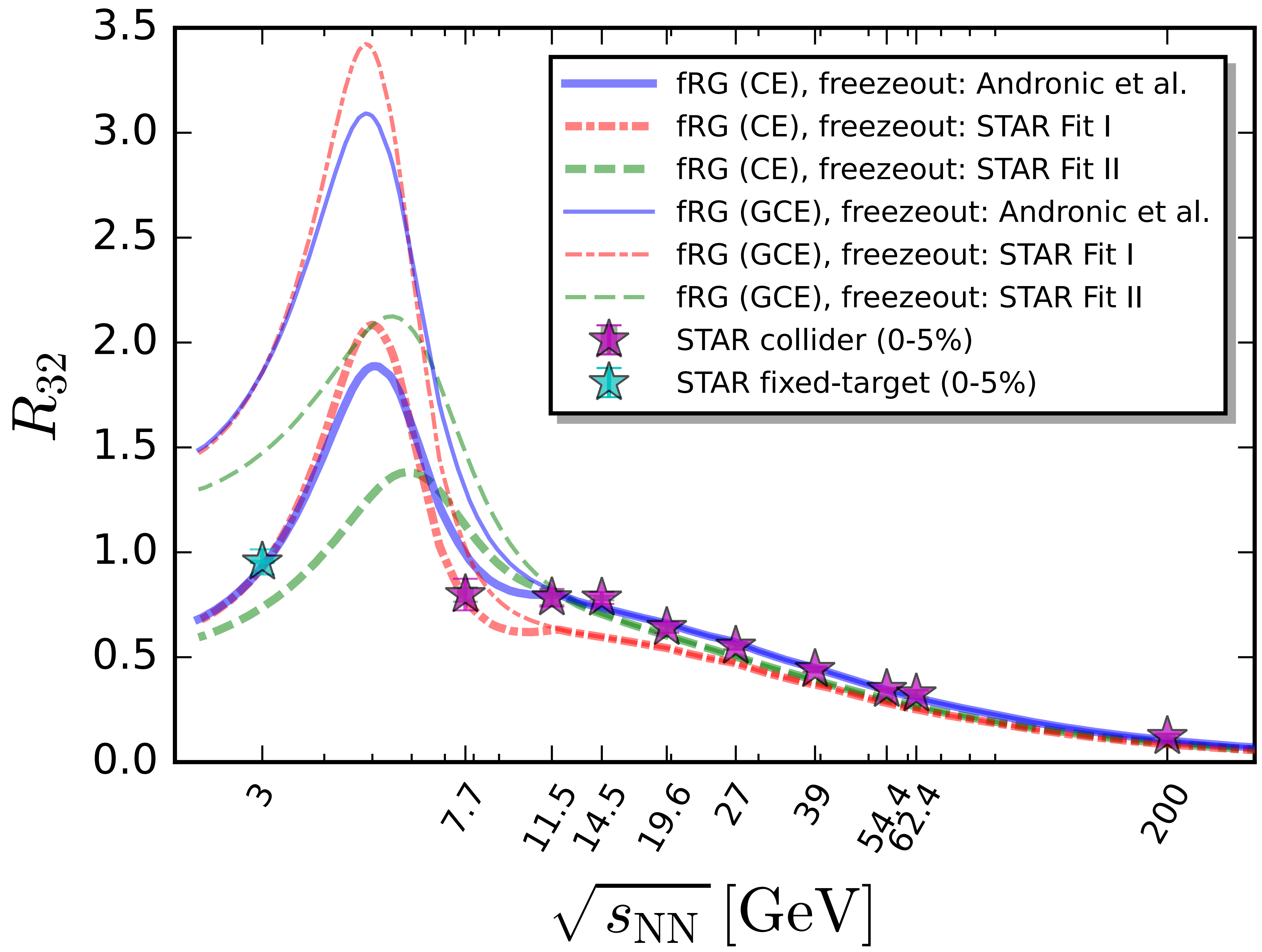


$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

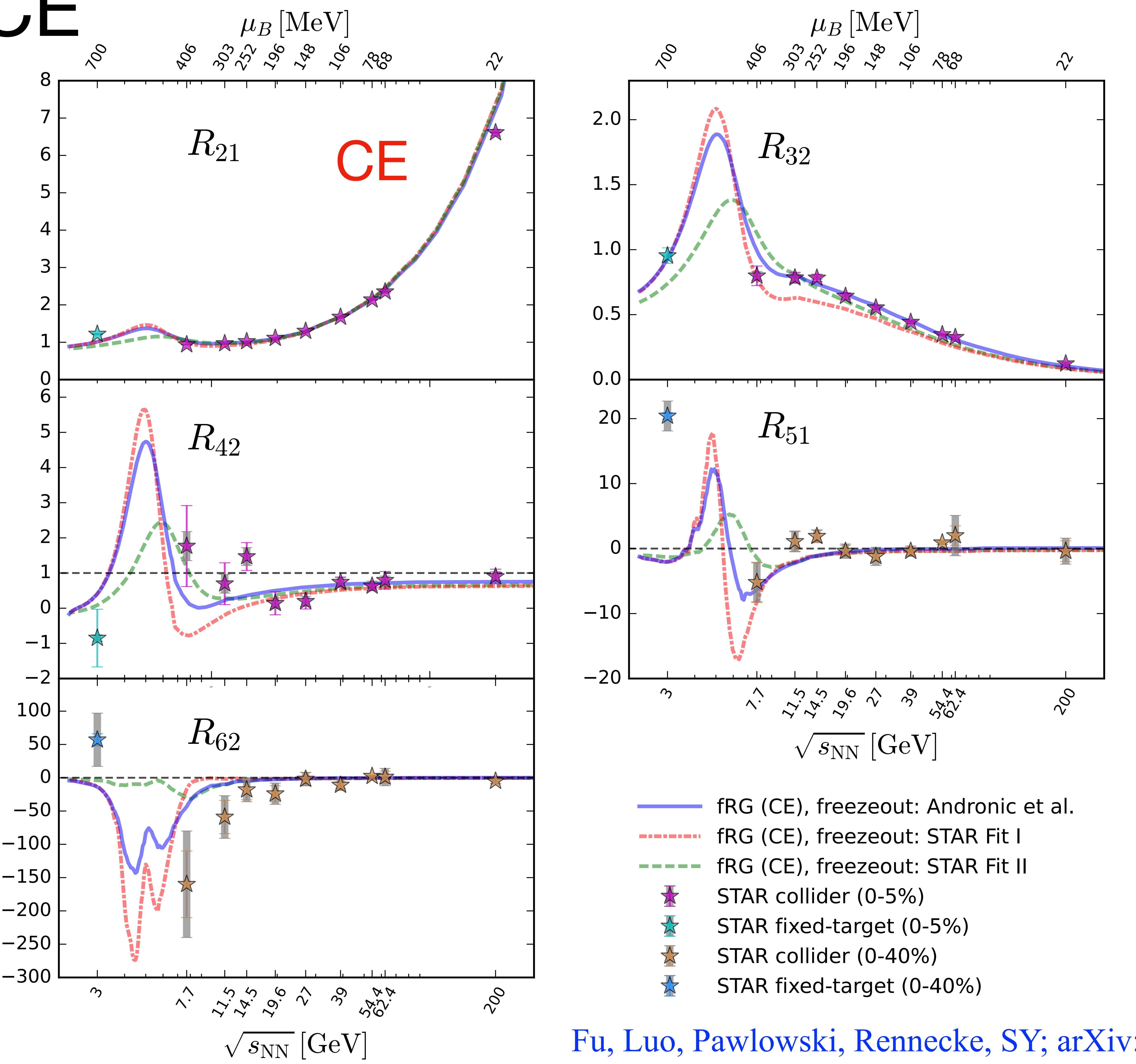
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2$$

V. Vovchenko et al., *Phys.Lett.B* 811 (2020) 135868



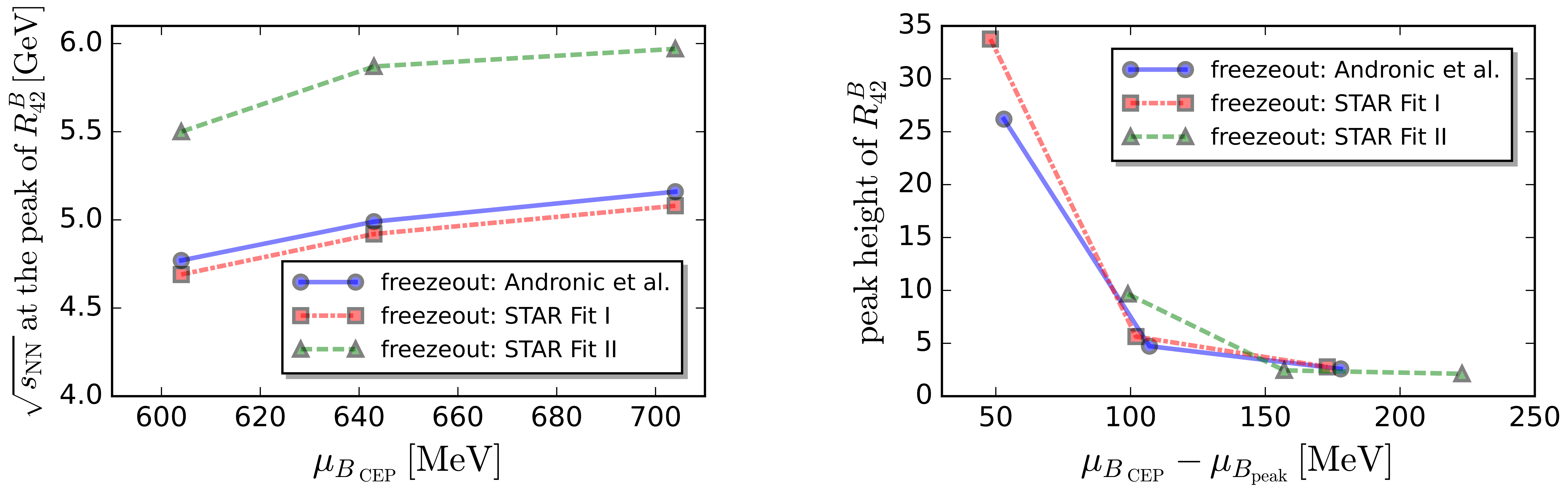
Fu, Luo, Pawlowski, Rennecke, SY; arXiv: 2308.15508

From GCE to CE



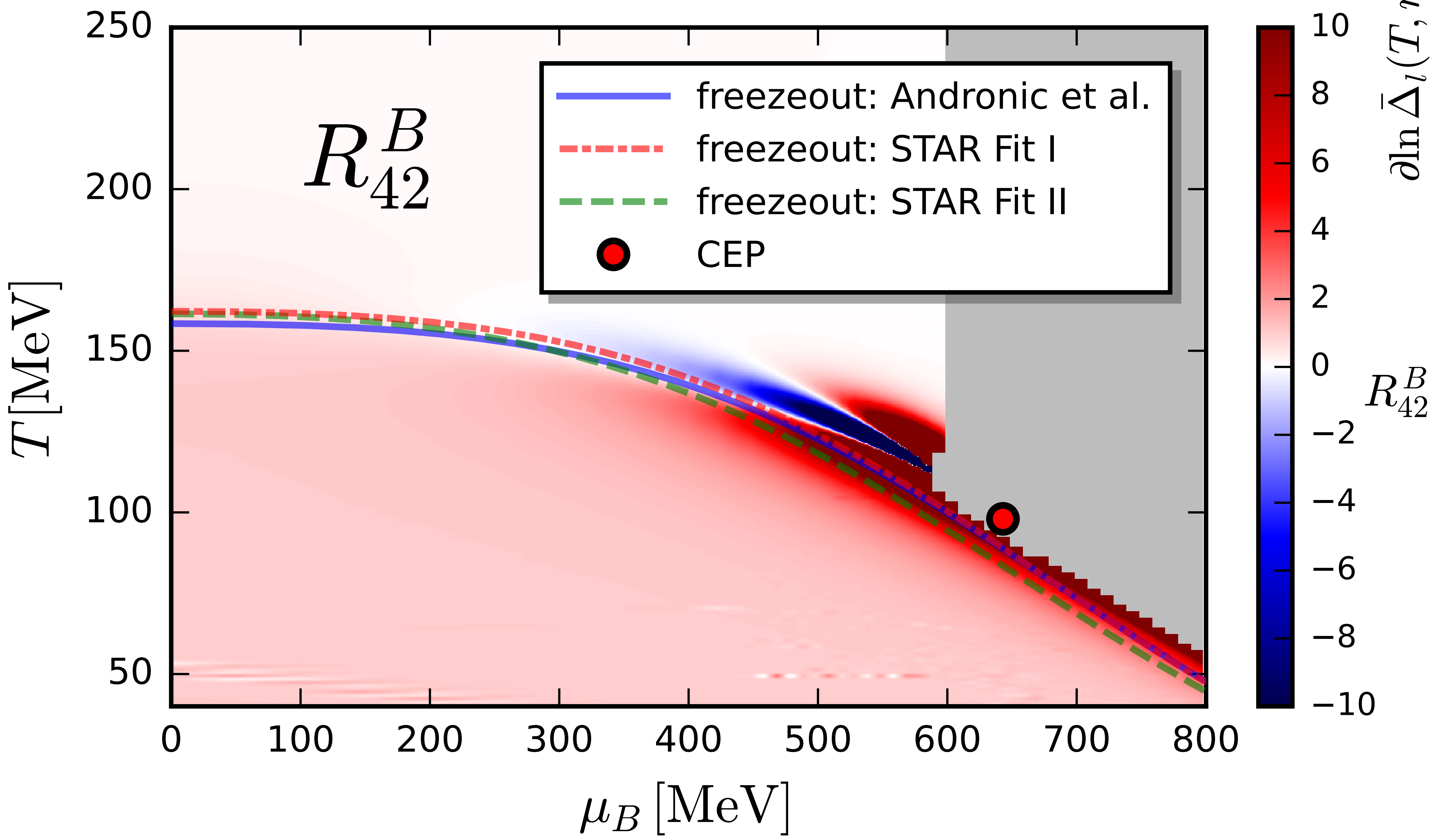
Fu, Luo, Pawlowski, Rennecke, SY; arXiv: 2308.15508

Different location of CEP & freez-eout curves



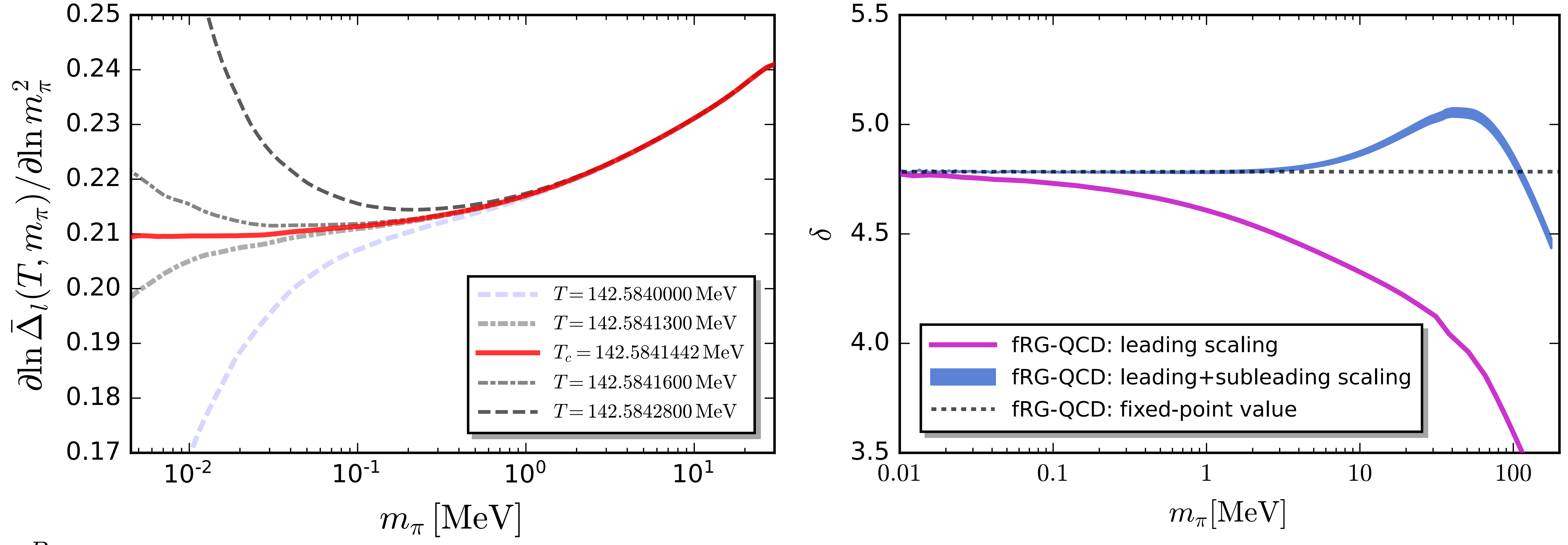
Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

Non-monotonic energy dependence



Fu, Luo, Pawlowski, Rennecke, Wen, SY, *arXiv:*2308.15508

Jens Braun et.al.; arXiv: 2310.19853

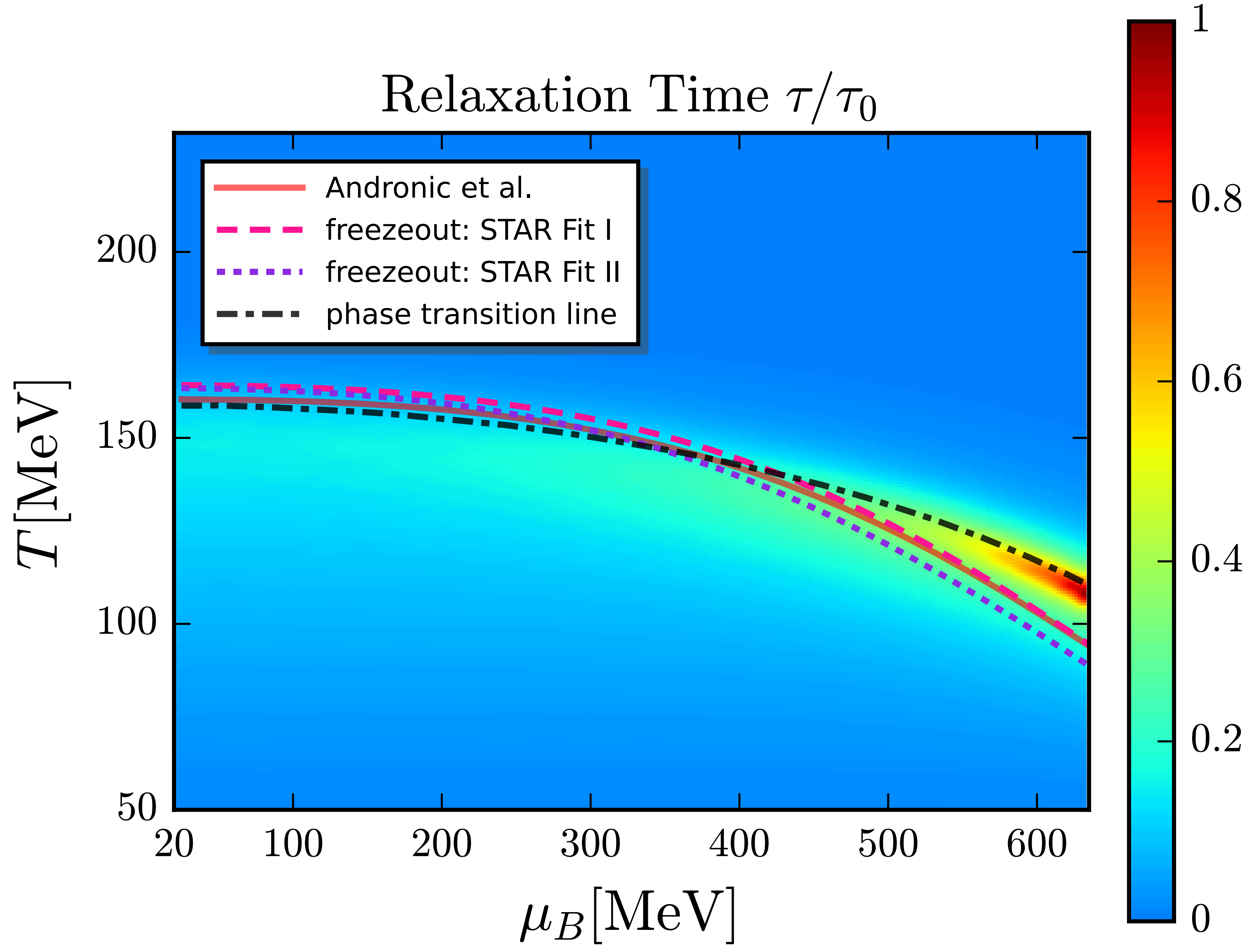


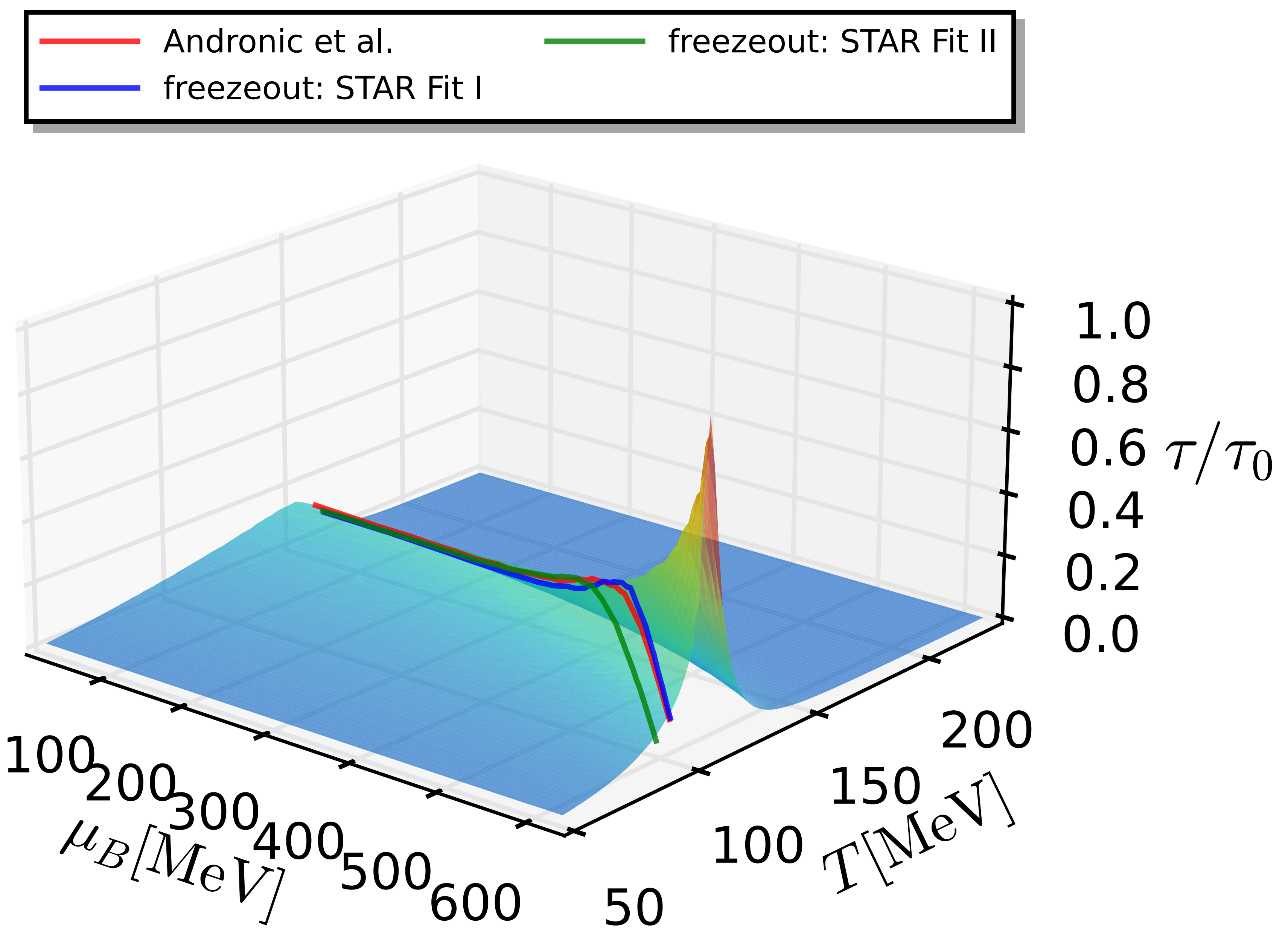
$$\bar{\Delta}_l^{\rm crit}(m_\pi) = B_c m_\pi^{2/\delta}(1 + a_m m_\pi^{2\theta_H})$$

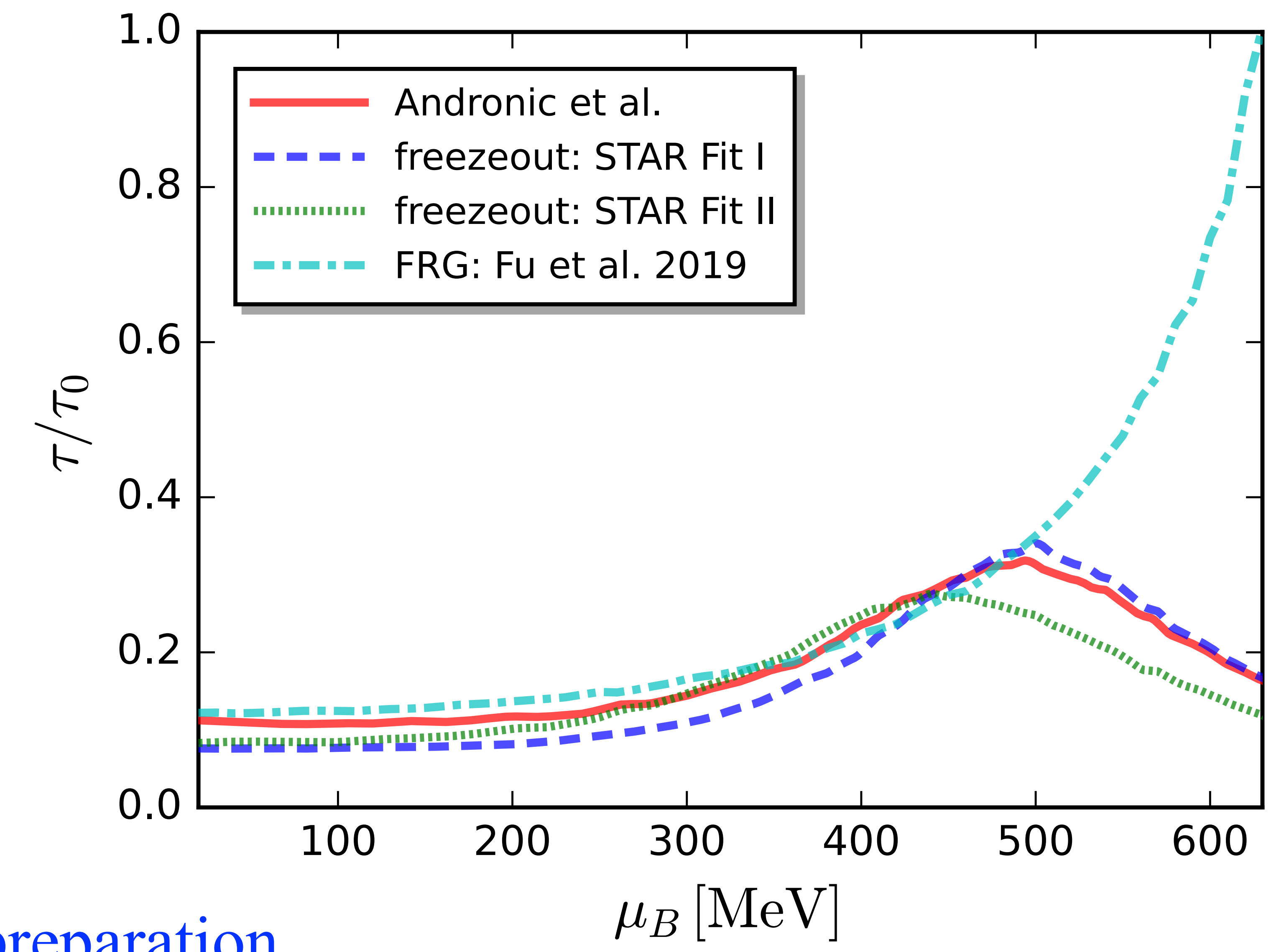
- ⭐ Non-monotonic energy dependence comes from gradually sharp crossover
- ⭐ Freeze-out curves are outside the critical region

Non-equilibrium effect

Obtained by Langevin equation with fRG-QCD data







Fu, Pawlowski, Rennecke, Tan, SY; in preparation

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Summary

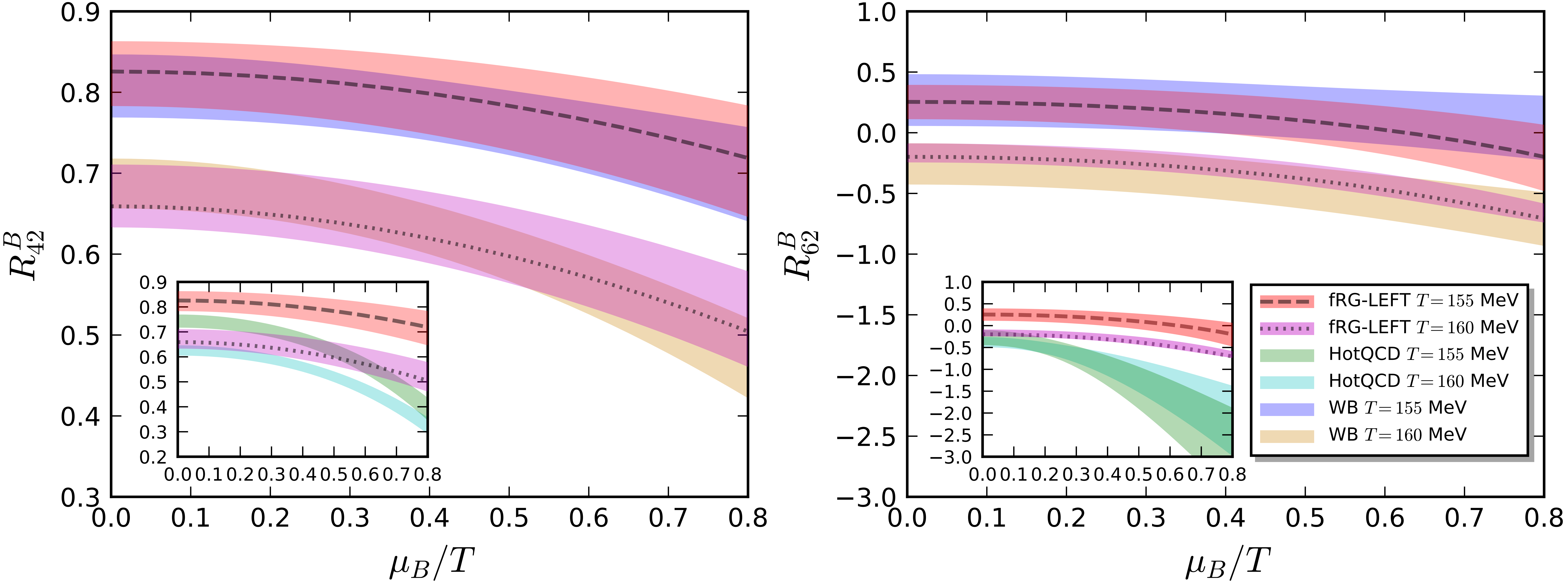
- Two steps toward quantitative calculation of baryon number fluctuations
    - QCD-assisted LEFT
    - Updated QCD-assisted LEFT
- Included global charge conservation effect
    - Subensemble acceptance method

Outlooks

- The third step: Fluctuations from full QCD system
- Include non-equilibrium effect
- Looking forward to more accurate freeze-out curve

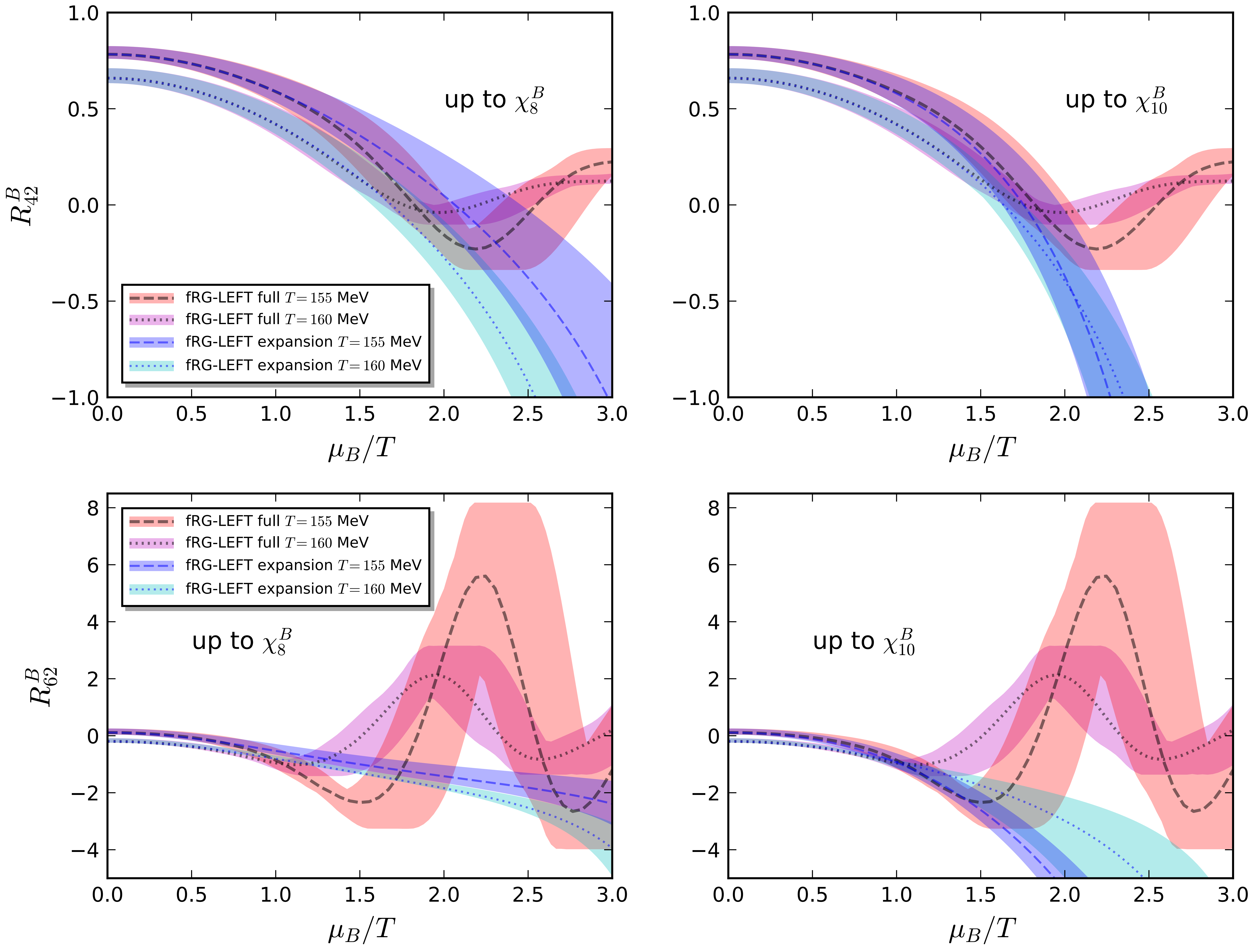
Thank you very much !!!!

Backup



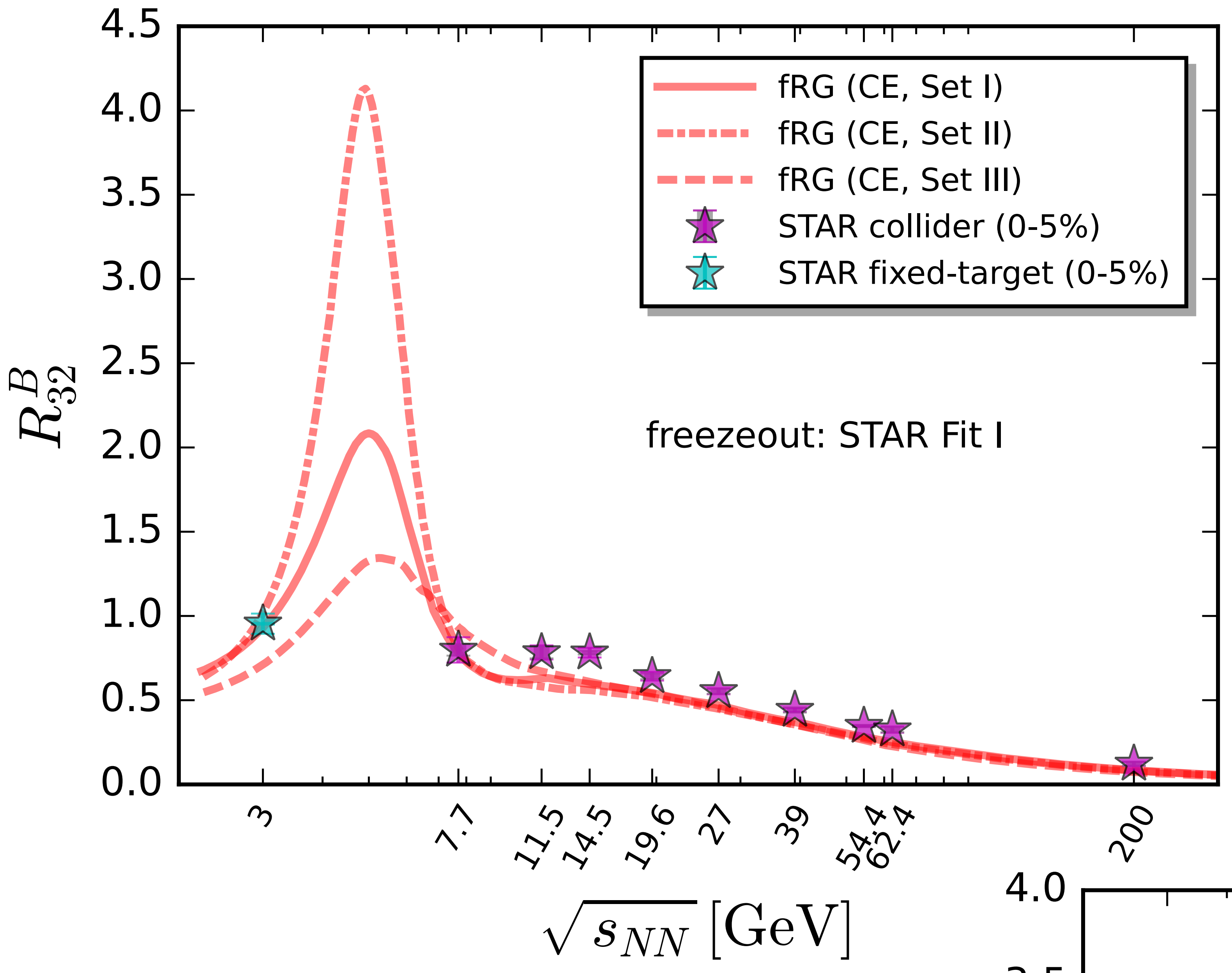
0.9
0.8
0.7
0.6
0.5
0.4
0.3
R^B_{42}
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
μ_B/T
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
R^B_{62}
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
μ_B/T
fRG-LEFT $T=155$ MeV
fRG-LEFT $T=160$ MeV
HotQCD $T=155$ MeV
HotQCD $T=160$ MeV
WB $T=155$ MeV
WB $T=160$ MeV

Backup

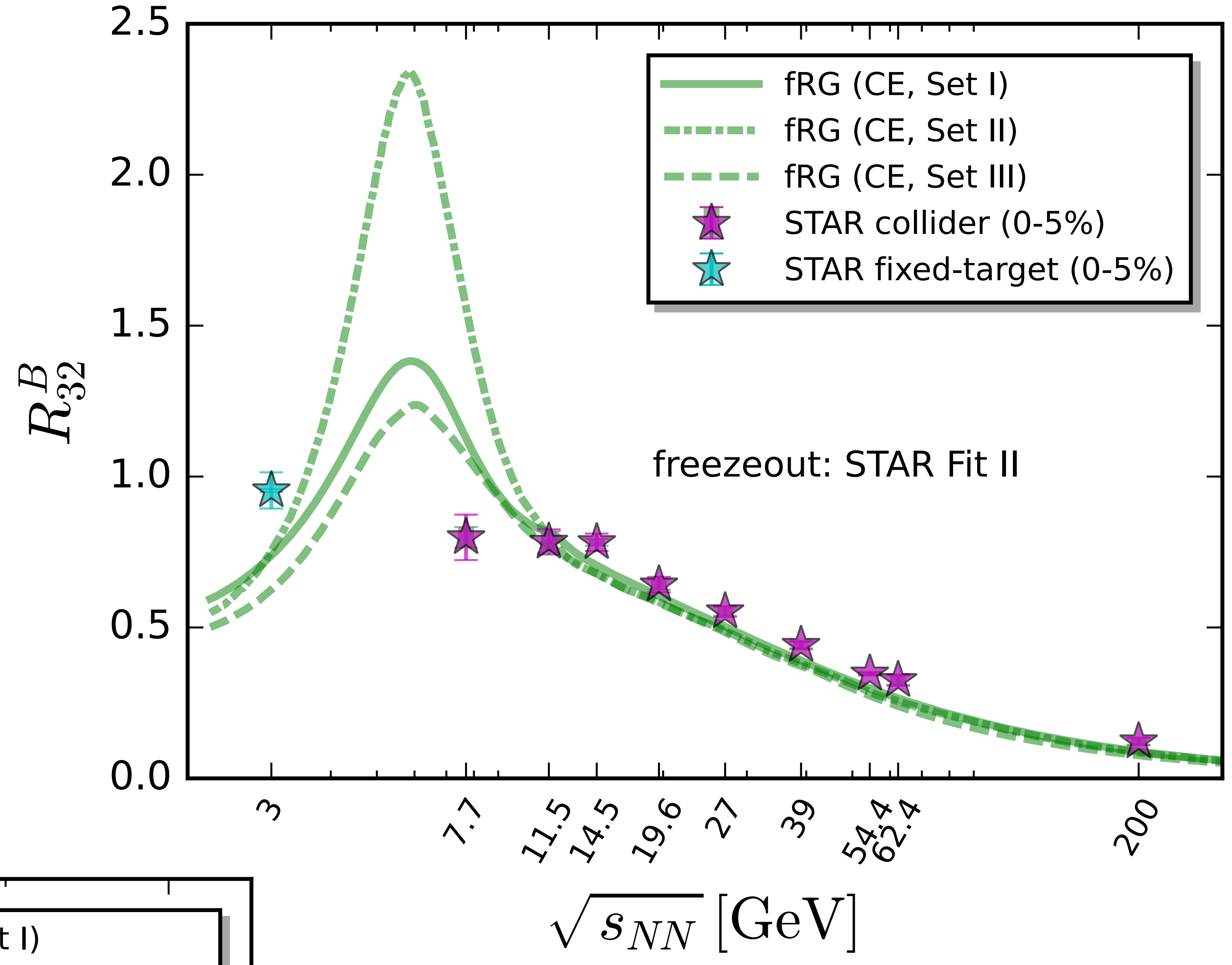


up to χ_8^B
up to χ_{10}^B
up to χ_8^B
up to χ_{10}^B
R_{42}^B
R_{62}^B
μ_B/T
fRG-LEFT full $T = 155$ MeV
fRG-LEFT full $T = 160$ MeV
fRG-LEFT expansion $T = 155$ MeV
fRG-LEFT expansion $T = 160$ MeV

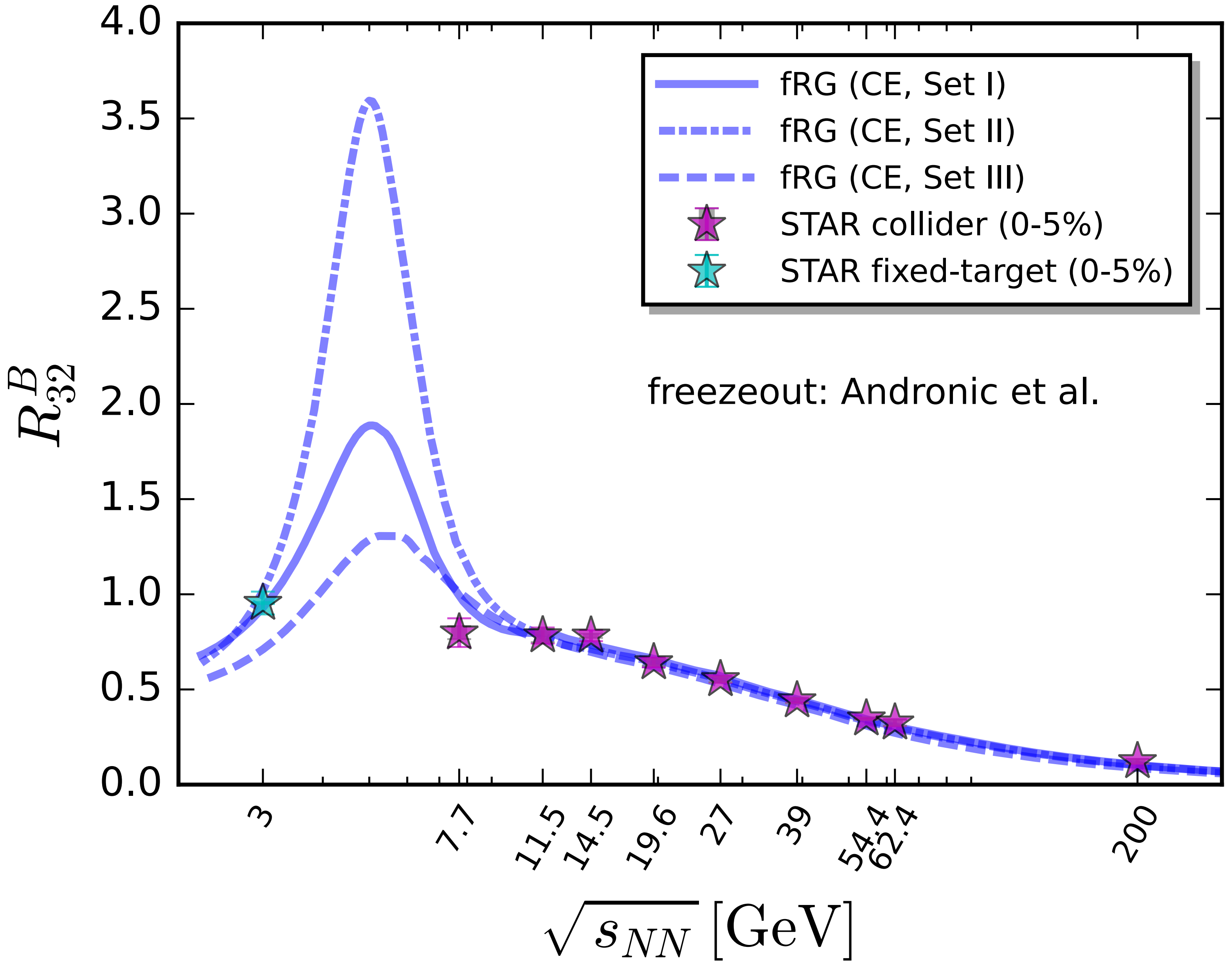
Backup



fRG (CE, Set I)
fRG (CE, Set II)
fRG (CE, Set III)
STAR collider (0-5%)
STAR fixed-target (0-5%)
freezeout: STAR Fit I
R_{32}^B
$\sqrt{s_{NN}}$ [GeV]
3
7.7
11.5
14.5
19.6
27
39
54.4
62.4
200

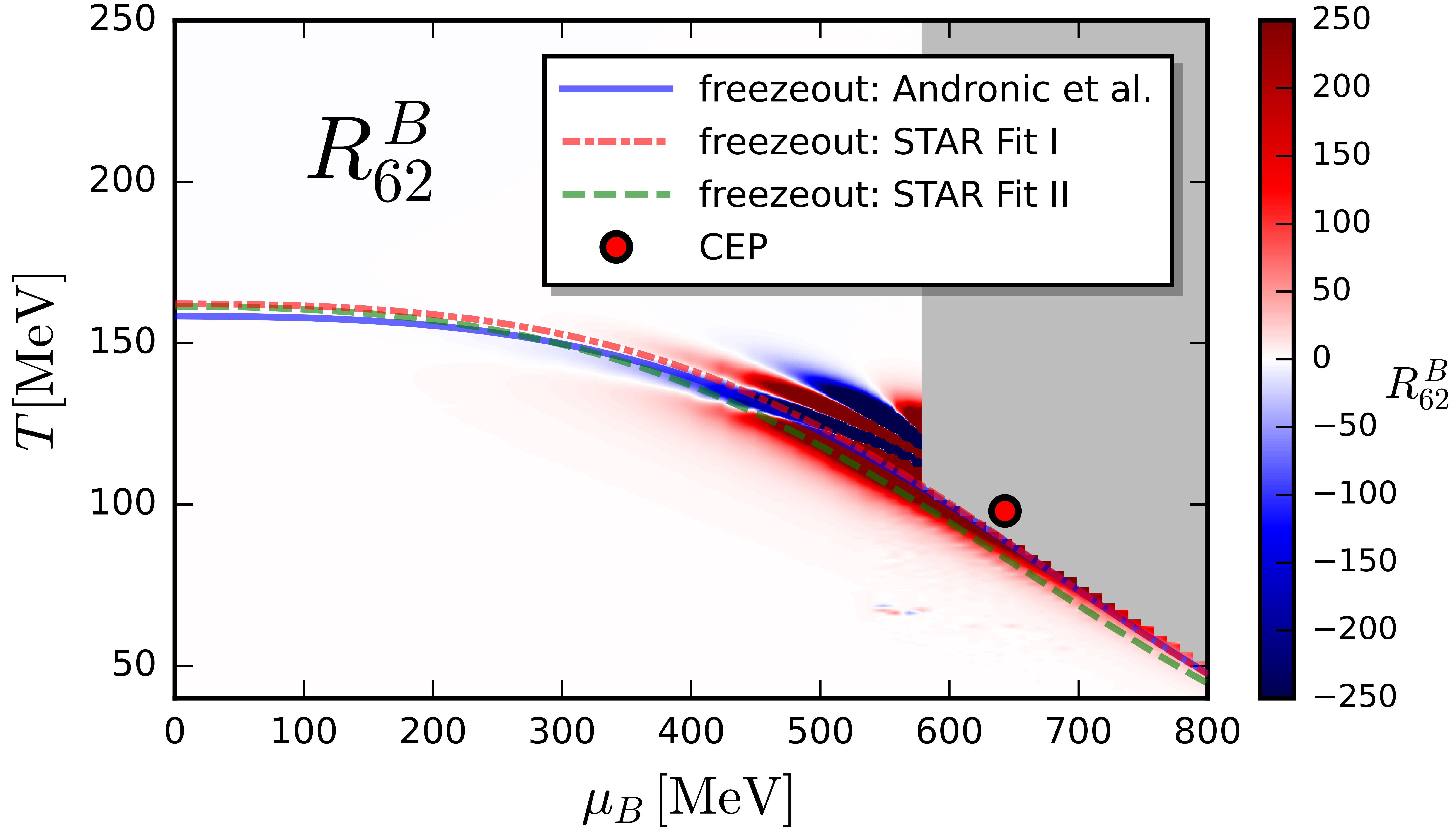


fRG (CE, Set I)
fRG (CE, Set II)
fRG (CE, Set III)
STAR collider (0-5%)
STAR fixed-target (0-5%)
freezeout: STAR Fit II
R_{32}^B
$\sqrt{s_{NN}}$ [GeV]
3
7.7
11.5
14.5
19.6
27
39
54.4
62.4
200



fRG (CE, Set I)
fRG (CE, Set II)
fRG (CE, Set III)
STAR collider (0-5%)
STAR fixed-target (0-5%)
freezeout: Andronic et al.
R_{32}^B
$\sqrt{s_{NN}}$ [GeV]
3
7.7
11.5
14.5
19.6
27
39
54.4
62.4
200

Backup



R_{62}^{B}
freezeout: Andronic et al.
freezeout: STAR Fit I
freezeout: STAR Fit II
CEP
250
200
150
100
50
0
100
200
300
400
500
600
700
800
$T\,[\mathrm{MeV}]$
$\mu_B\,[\mathrm{MeV}]$
250
200
150
100
50
0
−50
−100
−150
−200
−250
R_{62}^{B}