Properties of Heavy-Flavour Four-Quark states from Functional Methods

Joshua Hoffer

Supervisor: Christian Fischer

Institut für Theoretische Physik
Justus-Liebig-Universität Gießen

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

HFHF
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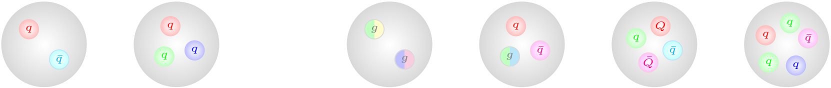
Motivation

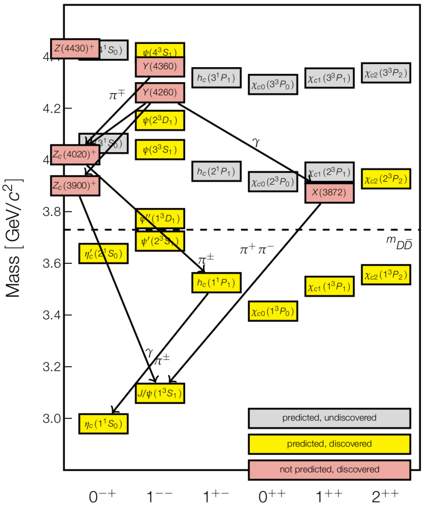
Conventional Hadrons: Exotic Hadrons:



Mesons | Baryons | Glueballs | Hybrids | Four-quark states | Pentaquarks

- Powerful toolkit to classify conventional hadrons: Quark Model (QM).
- Lot of particles measured that do not fit into the QM picture, i.e., exotic hadrons.
- A few examples:
  - Light scalar mesons: σ, κ, a_0, f_0
  - Exotic XYZ-states: $X(3872)$, $X(3915)$, $Z_c(3900)$, $Z_c(4430)$, $\psi(4230)$
  - $T_{cc}(3875)^+, T_{csJ}(2900)^0$ ...

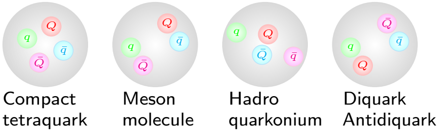
Tetraquark candidates in the charmonium region



Wolfgang Gradl, BESIII, St Goar 2015

Many unexpected states found by Belle II, BABAR, BES III, LHCb, ...

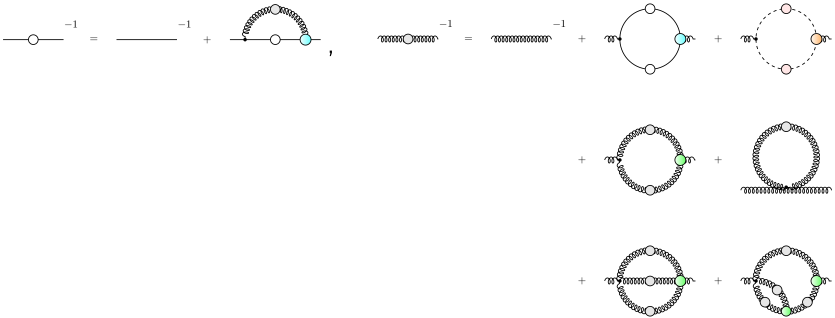
Internal structure??



Related to details of underlying QCD forces between quarks

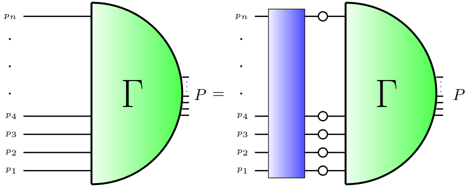
Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,



Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,
  - Hadronic bound state equations: BSEs, Faddeev eqs. .



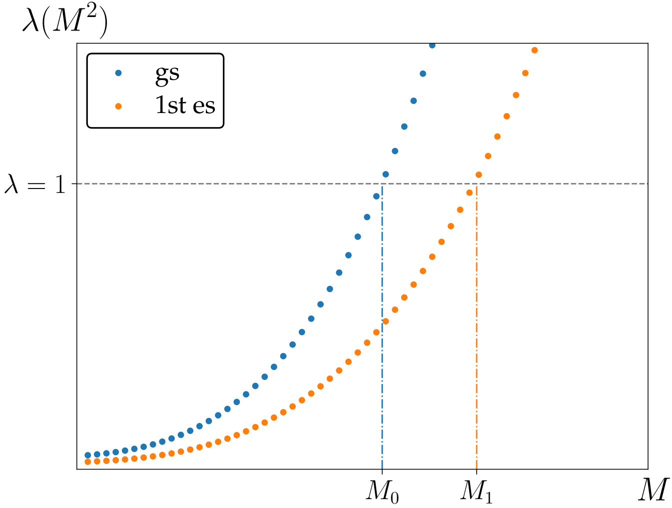
Eigenvalue equation

$$\lambda_i(P^2)\,\Gamma^{(n)} = K^{(n)} G^{(n)}\,\Gamma^{(n)}$$

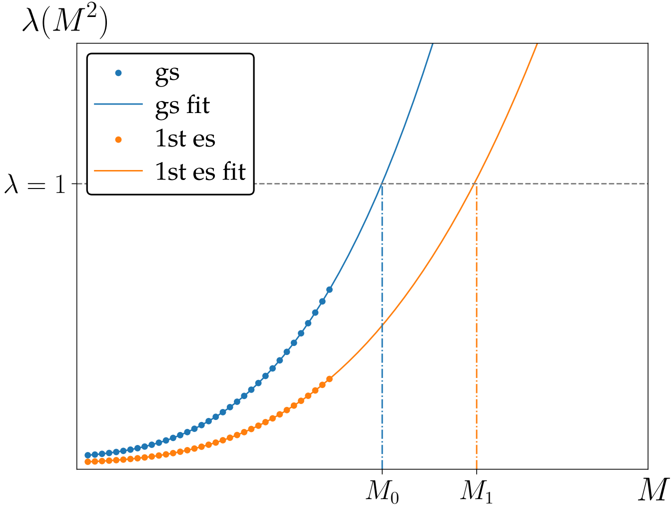
with $\lambda_i(P^2 = -M_i^2) = 1$

$i = 0$: ground state
$i = 1$: 1st radial excited state

Eigenvalue curve



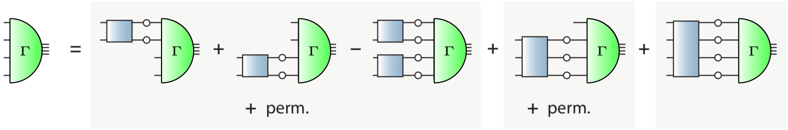
Eigenvalue curve



Four-quark BSE

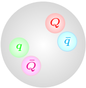
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, Fischer, PLB 718 (2012)
Eichmann, Fischer, Heupel, PLB 753 (2016)

Exact equation:

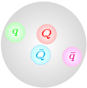


Two-body interactions

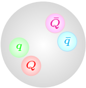
Three- and four-body interactions



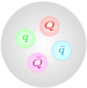
Meson molecule



Hadro quarkonium

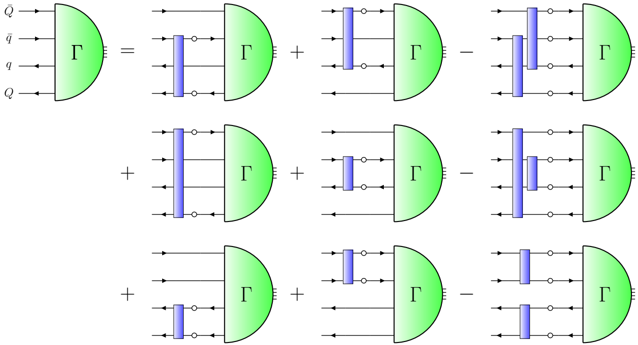


Diquark Antidiquark



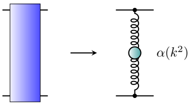
Compact tetraquark

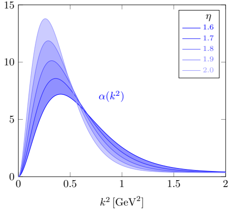
The Four-quark BSE:



$$\Gamma(k,q,p,P) = \sum_i f_i \underbrace{(\ldots)}_{\text{Lorentz-invariants}} \tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Calculations are done in the *Rainbow-Ladder truncation*:



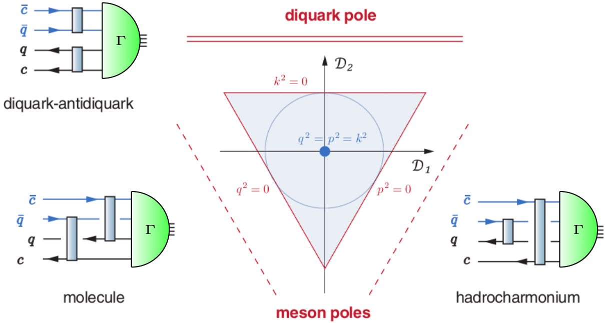


Maris, Tandy, PRC 60 (1999)
Qin et al., PRC 84 (2011)

- Can cast the Lorentz-invariants into multiplets of S_4:

  Eichmann, Fischer, Heupel, PLB 753 (2016)

  - One singlet: S_0
  - One doublet: $D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$
  - Two triples: T_0, T_1 $\rightarrow$ subleading
- Dressing functions: $f_i(S_0, D)$
- Poles dynamically generated in D
- "Physical basis": put poles in externally: $f_i(S_0, D) \rightarrow f_i(S_0) \cdot P_{ab} \cdot P_{cd}$



Eichmann, Fischer, Heupel, Santowsky, Wallbott, Few Body Syst. 61 (2020) 4, 38

Colour structure of the four-quark BSE

$$\Gamma(k,q,p,P) = \sum_i f_i(\dots)\tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Different combinations in Γ_C that form an overall colour singlet:

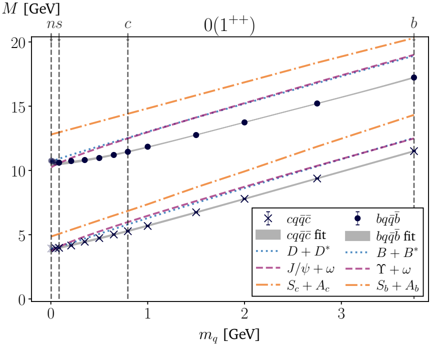
meson-meson	diquark-antidiquark
$\mathbf{1} \otimes \mathbf{1}$	$\bar{\mathbf{3}} \otimes \mathbf{3}$

Physical amplitude

- Structure of the Amplitude Γ is determined according to the quantum numbers of the state in question, the quark content and the physical decay channels.
- Example for the $X(3872) \quad (I(J^{PC}) = 0(1^{++}))$:
  - Meson-molecule: $D\bar{D}^*$
  - Hadro-charmonium: $J/\psi\,\omega$
  - Diquark-Antidiquark: $S_c A_c$

  $\Rightarrow \Gamma = f_{D\bar{D}^*} \cdot \tau_{D\bar{D}^*} + f_{J/\psi\omega} \cdot \tau_{J/\psi\omega} + f_{S_c A_c} \cdot \tau_{S_c A_c}$
- No assumptions of a dominant substructure needed!
- The internal two-body pole structures introduce decay thresholds into the equation.

Example quark mass evolution for the $0(1^{++})$ state



JH, Eichmann, Fischer, PRD 109 (2024)

Thresholds: Hidden-charm Hidden-bottom

$DD^*,\ J/\psi\omega,\ S_cA_c \qquad BB^*,\ \Upsilon\omega,\ S_bA_b$

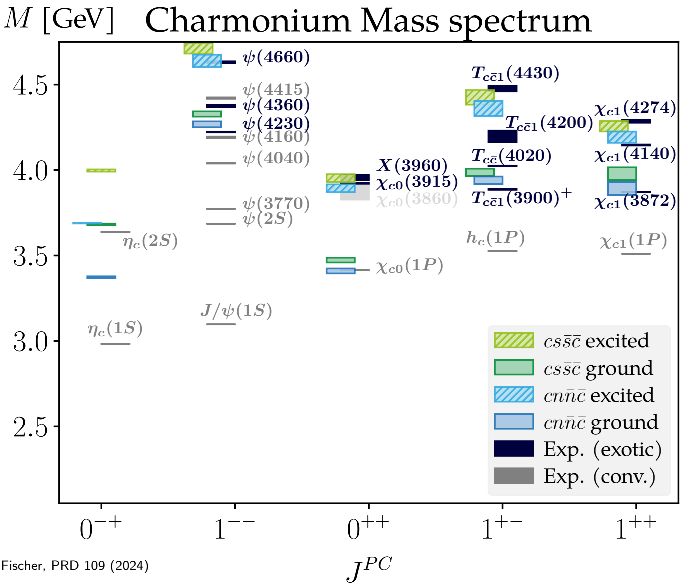
- $M_{1^{++}}^{cq\bar{q}\bar{c}} = 3.89 \pm 0.04$ GeV $\quad\rightarrow\quad X(3872)$
- $M_{1^{++}}^{bq\bar{q}\bar{b}} = 10.52 \pm 0.06$ GeV $\quad\rightarrow\quad (W_{b1}\,?)$

Hidden-flavour physical basis components

	$I(J^{PC})$	Physical components	Exp.
hidden charm	$0(0^{++})$	$D\bar{D}$, $J/\psi\omega$, S_cS_c	$\chi_{c0}(3915)$
$(cq\bar{q}\bar{c})$	$0(1^{++})$	$D\bar{D}^*$, $J/\psi\omega$, S_cA_c	$\chi_{c1}(3872)$
	$1(1^{+-})$	$D\bar{D}^*$, $J/\psi\pi$ S_cA_c	$Z_c(3900)$
	$0(1^{--})$	$D\bar{D}_1$, $\chi_{c0}\omega$, $J/\psi\sigma$	$\psi(4230)$
	$0(0^{-+})$	$D\bar{D}_0$, $\chi_{c0}\eta$, $\eta_c f_0(1370)$	–
hidden bottom	$0(0^{++})$	$B\bar{B}$, $\Upsilon\omega$, S_bS_b	$(W_{b0}?)$
$(bq\bar{q}\bar{b})$	$0(1^{++})$	$B\bar{B}^*$, $\Upsilon\omega$, S_bA_b	$(W_{b1}?)$
	$1(1^{+-})$	$B\bar{B}^*$, $\Upsilon\pi$, S_bA_b	$Z_b(10610)$
	$0(1^{--})$	$B\bar{B}_1$, $\chi_{b0}\omega$, $\Upsilon\sigma$	$\Upsilon(10753)$
	$0(0^{-+})$	$B\bar{B}_0$, $\chi_{b0}\eta$, $\eta_b f_0(1370)$	–

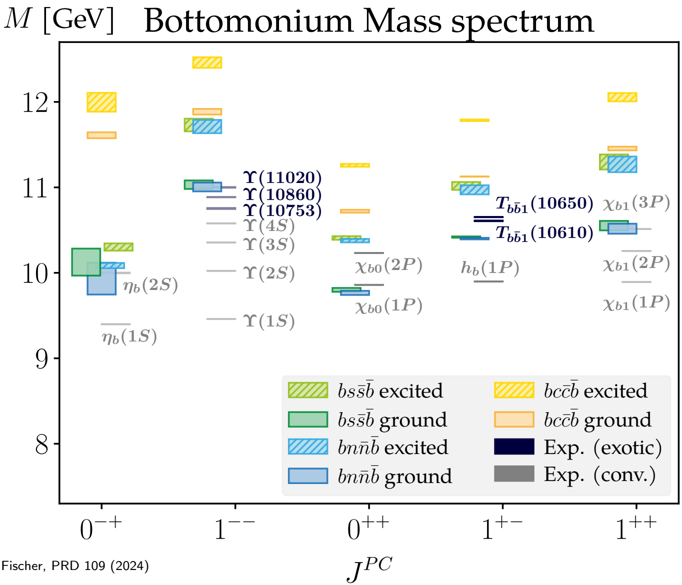
JH, Eichmann, Fischer, PRD 109 (2024)
Wallbott, Eichmann, Fischer, PRD 102 (2020)

Hidden-charm mass spectrum



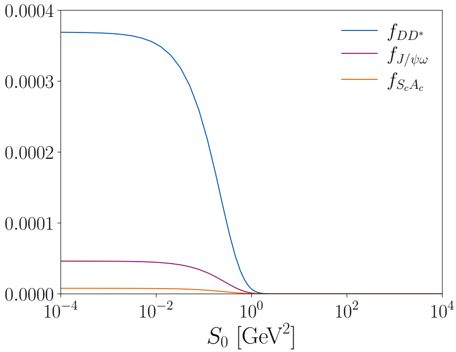
JH, Eichmann, Fischer, PRD 109 (2024)

Hidden-bottom mass spectrum



JH, Eichmann, Fischer, PRD 109 (2024)

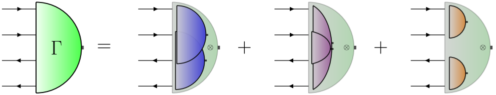
Amplitude for the $0(1^{++})$ state



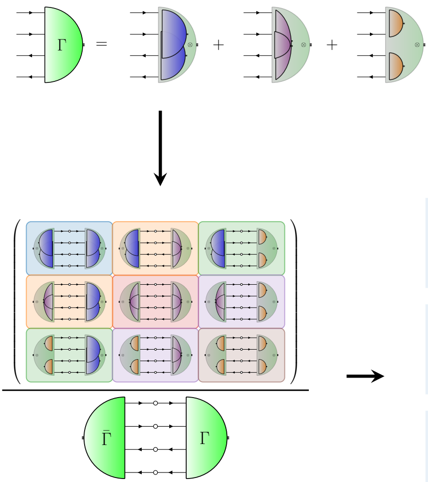
JH, Eichmann, Fischer, PRD 109 (2024)

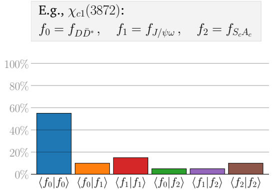
Physical basis:

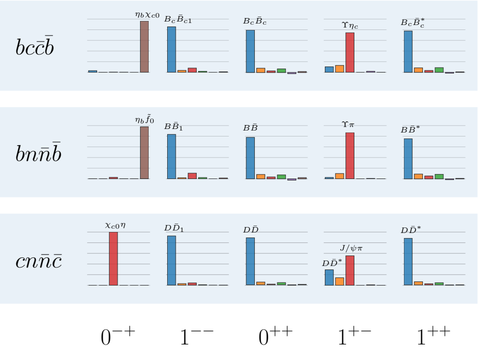
$$\Gamma = f_{D\bar{D}^*} \cdot \tau_{D\bar{D}^*} + f_{J/\psi\omega} \cdot \tau_{J/\psi\omega} + f_{S_cA_c} \cdot \tau_{S_cA_c}$$



Dominant subcluster







$(\tilde{f}_0 = f_0(1370))$

JH, Eichmann, Fischer, PRD 109 (2024)

Colour structure of the four-quark BSE

$$\Gamma(k,q,p,P) = \sum_i f_i(\ldots)\tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Different combinations in Γ_C that form an overall colour singlet:

	meson-meson	diquark-antidiquark
attractive	$\mathbf{1} \otimes \mathbf{1}$	$\bar{\mathbf{3}} \otimes \mathbf{3}$
repulsive	$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{6} \otimes \bar{\mathbf{6}}$

- Repulsive colour components are important (for some channels)!

Open-flavour physical basis components

	$I(J^P)$	Physical components att+rep	att	rep	Exp.
open charm	$1(0^+)$	DD, D^*D^*	$A_{cc}A$	$S_{cc}S$	–
$(cc\bar{q}\bar{q})$	$0(1^+)$	DD^*, D^*D^*	$A_{cc}S$	$S_{cc}A$	T_{cc}^+
	$1(1^+)$	DD^*	$A_{cc}A$	–	–
open bottom	$1(0^+)$	BB, B^*B^*	$A_{bb}A$	$S_{bb}S$	–
$(bb\bar{q}\bar{q})$	$0(1^+)$	BB^*, B^*B^*	$A_{bb}S$	$S_{bb}A$	$(T_{bb}^+?)$
	$1(1^+)$	BB^*	$A_{bb}A$	–	–
open charm-bottom	$1(0^+)$	BD, B^*D^*	$S_{bc}S$	$A_{bc}A$	–
$(bc\bar{q}\bar{q})$	$0(1^+)$	BD^*, DB^*	$A_{bc}S$	$S_{bc}A$	$(T_{bc}?)$

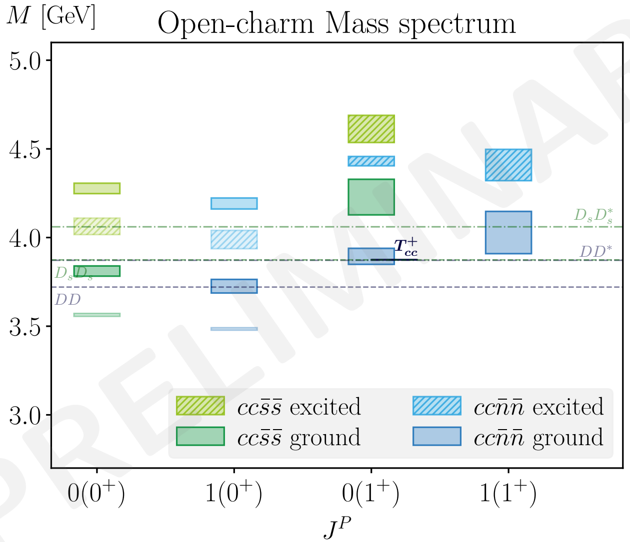
JH, Eichmann, Fischer, in preparation
Wallbott, Eichmann, Fischer, PRD 102 (2020)

Open-flavour physical basis components

	$I(J^P)$	Physical components att+rep	att	rep	Exp.
open charm	$1(0^+)$	DD, ~~D^*D^*~~	~~$A_{cc}A$~~	$S_{cc}S$	–
$(cc\bar{q}\bar{q})$	$0(1^+)$	DD^*, D^*D^*	$A_{cc}S$	$S_{cc}A$	T_{cc}^+
	$1(1^+)$	DD^*	$A_{cc}A$	–	–
open bottom	$1(0^+)$	BB, ~~B^*B^*~~	~~$A_{bb}A$~~	$S_{bb}S$	–
$(bb\bar{q}\bar{q})$	$0(1^+)$	BB^*, B^*B^*	$A_{bb}S$	$S_{bb}A$	$(T_{bb}^+?)$
	$1(1^+)$	BB^*	$A_{bb}A$	–	–
open charm-bottom	$1(0^+)$	BD, ~~B^*D^*~~	$S_{bc}S$	~~$A_{bc}A$~~	–
$(bc\bar{q}\bar{q})$	$0(1^+)$	BD^*, DB^*	$A_{bc}S$	$S_{bc}A$	$(T_{bc}?)$

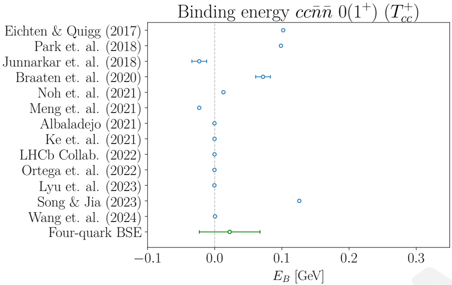
JH, Eichmann, Fischer, in preparation
Wallbott, Eichmann, Fischer, PRD 102 (2020)

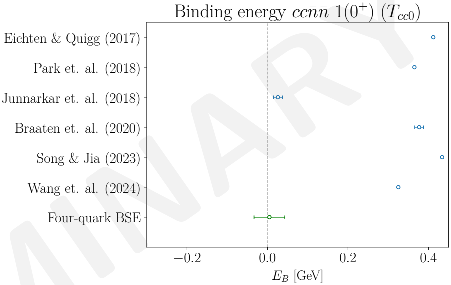
Open-charm spectrum

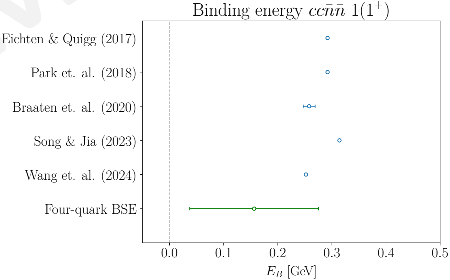


JH, Eichmann, Fischer, in preparation

Open-charm $cc\bar{n}\bar{n}$ binding energy comparison

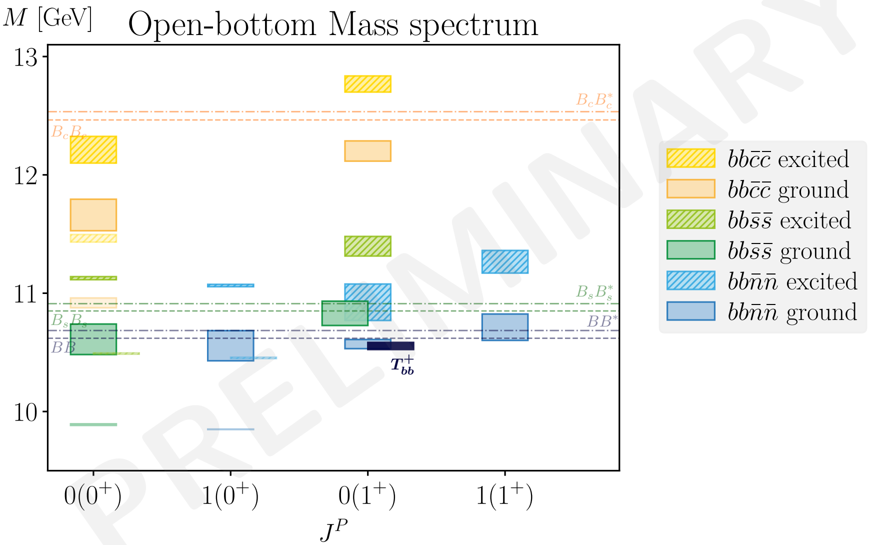






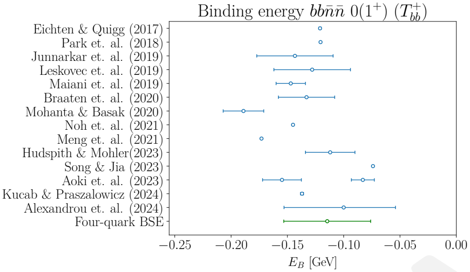
JH, Eichmann, Fischer, in preparation

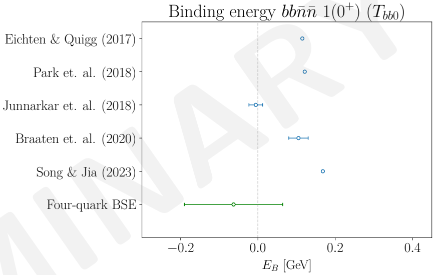
Open-bottom spectrum

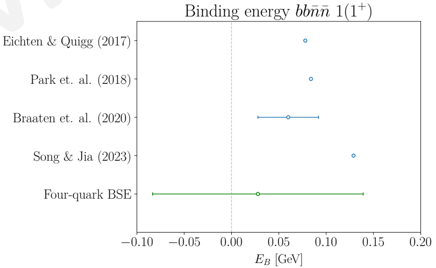


JH, Eichmann, Fischer, in preparation

Open-charm $bb\bar{n}\bar{n}$ binding energy comparison

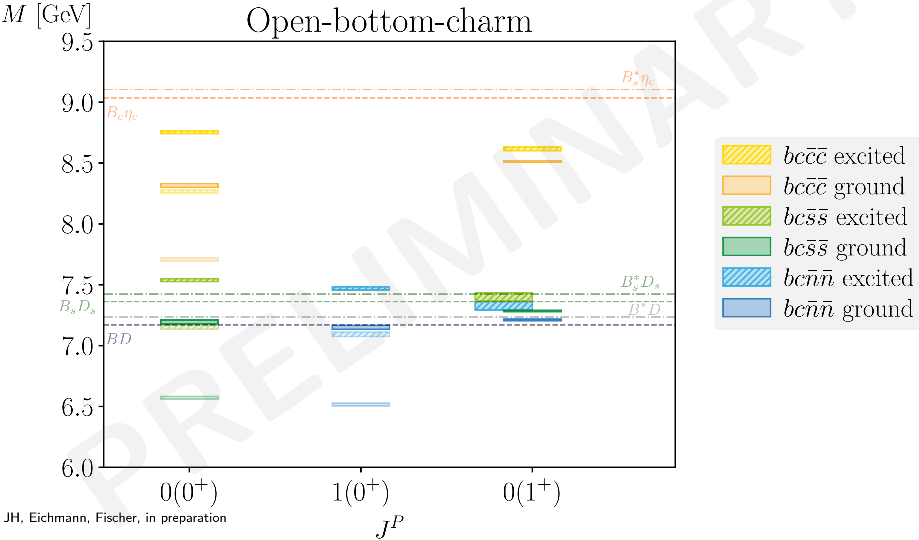






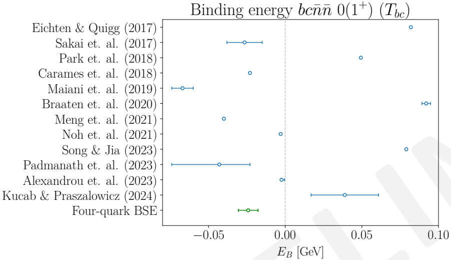
JH, Eichmann, Fischer, in preparation

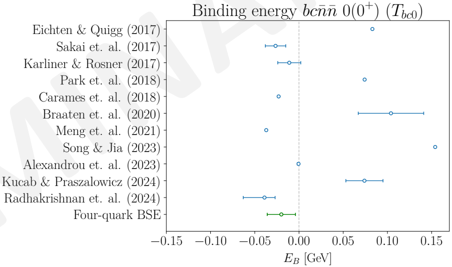
Open-bottom-charm spectrum



JH, Eichmann, Fischer, in preparation

Open-charm $bc\bar{n}\bar{n}$ binding energy comparison





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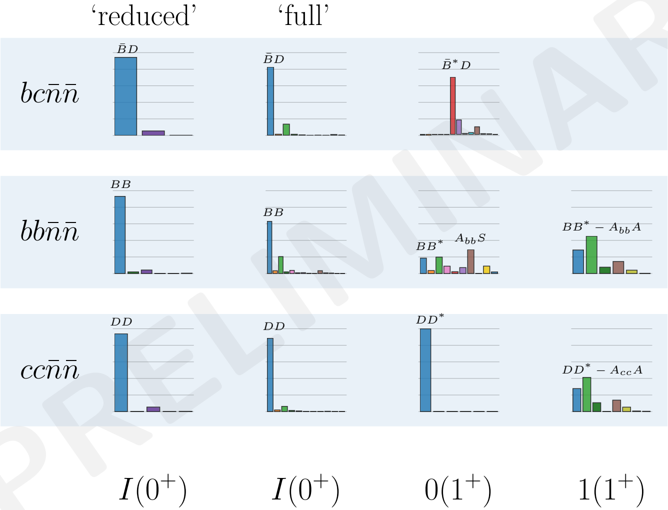
Open-flavour norm contributions



JH, Eichmann, Fischer, in preparation

Summary:

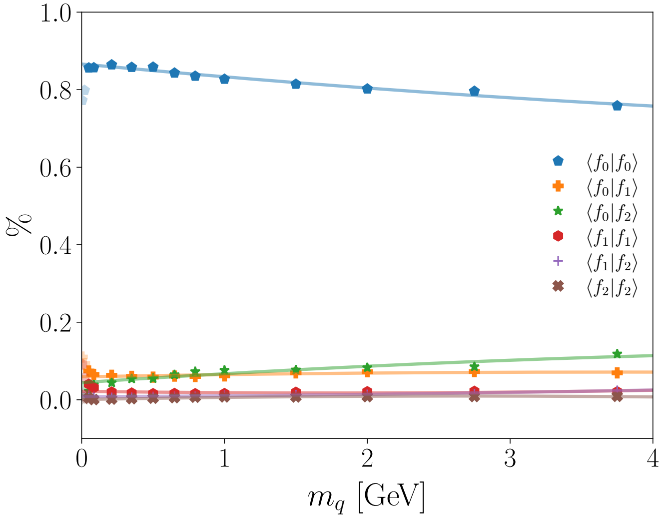
- DSE/BSE framework is a good tool to qualitatively analyse the charm and bottom four-quark state region.
- New results for the the open-flavour four-quark states.
- Analysed the norm contributions as a means to investigate the internal structure.

Outlook:

- Open-flavour states like
  - $\frac{1}{2}(1^{+})$ with $bb\bar{q}\bar{s}$
- Include the two-body quarkonium mixing.
- Investigate the large N_c behaviour.

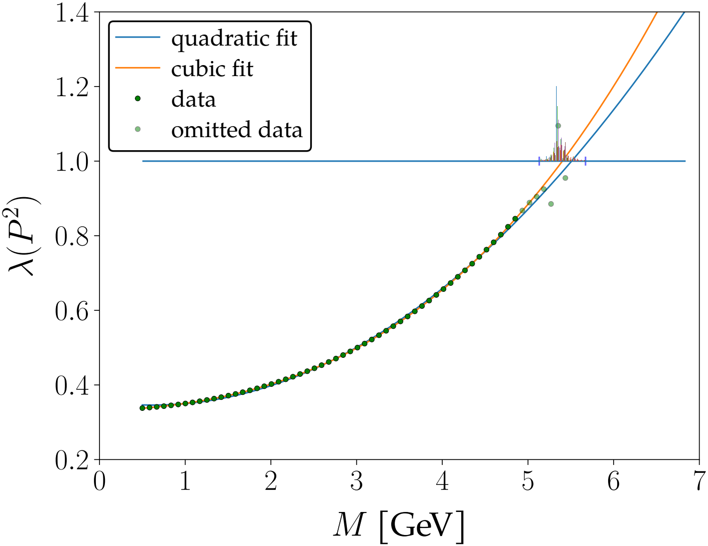
Backup slides

Quark mass evolution of the norm contributions

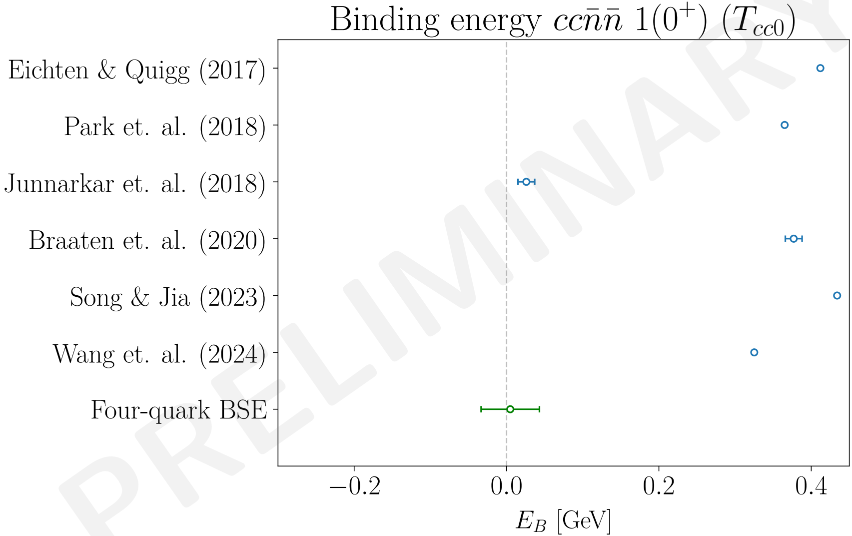


JH, Eichmann, Fischer, PRD 109 (2024)

Example extrapolation and error determination

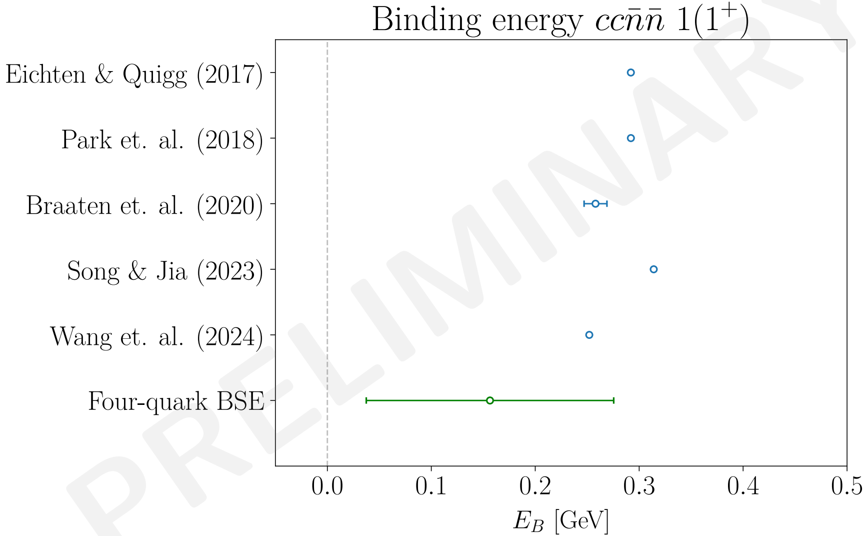


Open-charm $1(0^+)$ (T_{cc0}) binding energy comparison



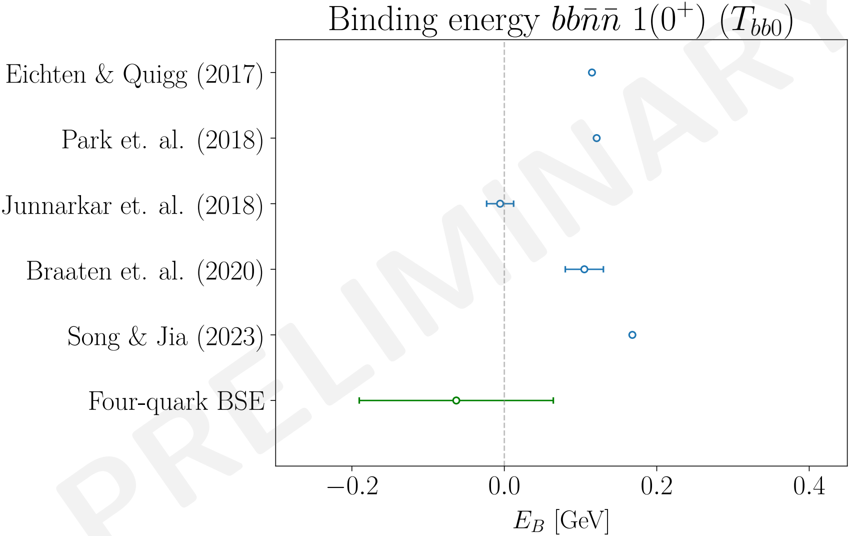
JH, Eichmann, Fischer, in preparation

Open-charm $1(1^+)$ $cc\bar{n}\bar{n}$ binding energy comparison



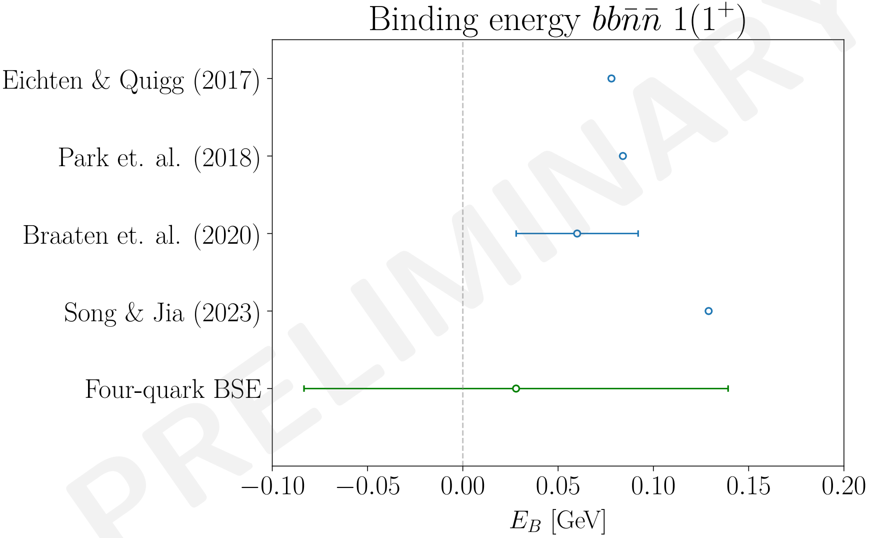
JH, Eichmann, Fischer, in preparation

Open-charm $1(0^+)$ (T_{bb0}) binding energy comparison



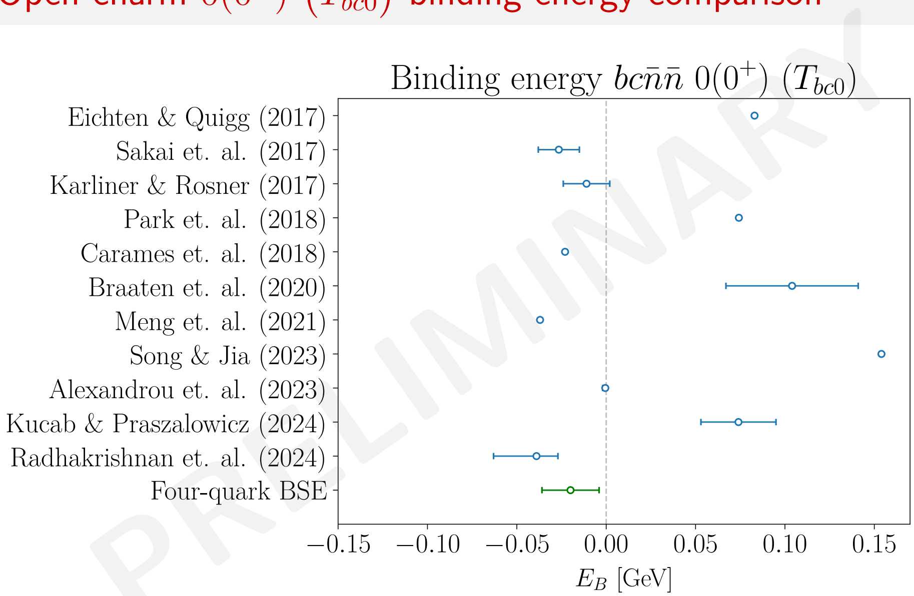
JH, Eichmann, Fischer, in preparation

Open-charm $1(1^+)$ $bb\bar{n}\bar{n}$ binding energy comparison



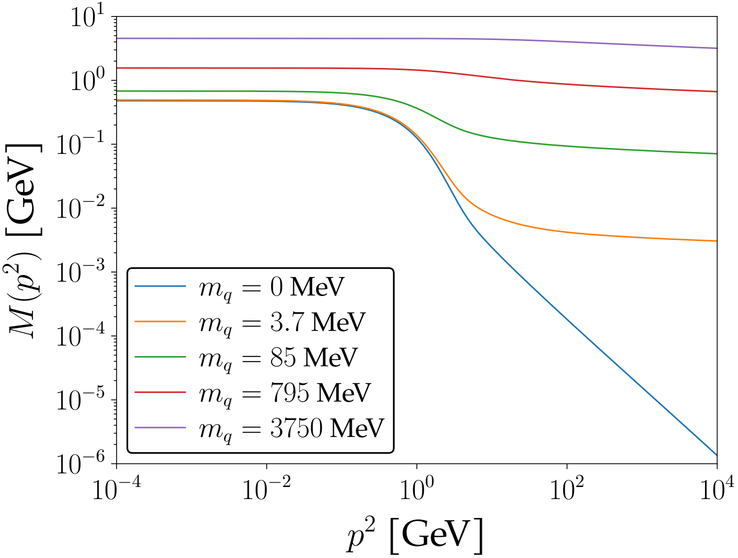
JH, Eichmann, Fischer, in preparation

Open-charm $0(0^+)$ (T_{bc0}) binding energy comparison



JH, Eichmann, Fischer, in preparation

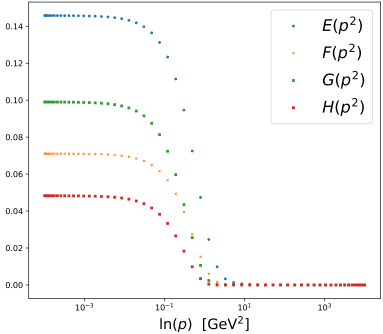
Quark mass generation



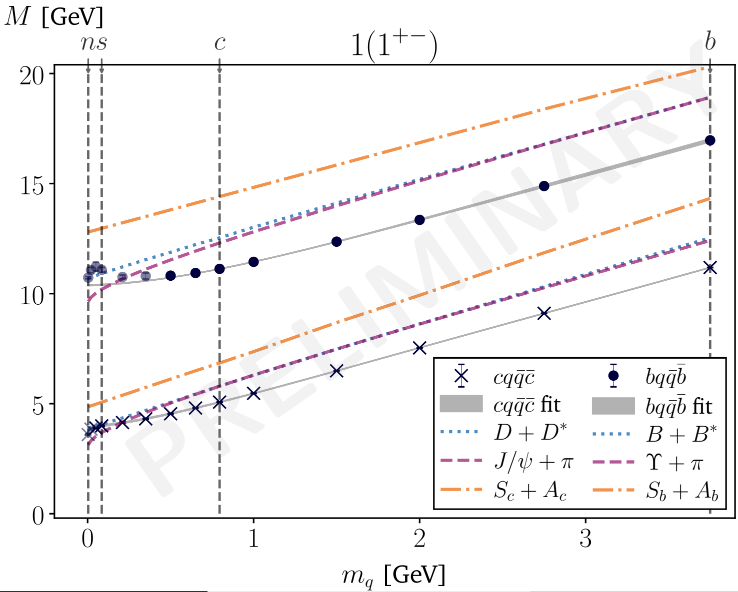
Pion BSE

Pion Bethe-Salpeter amplitude is given by:

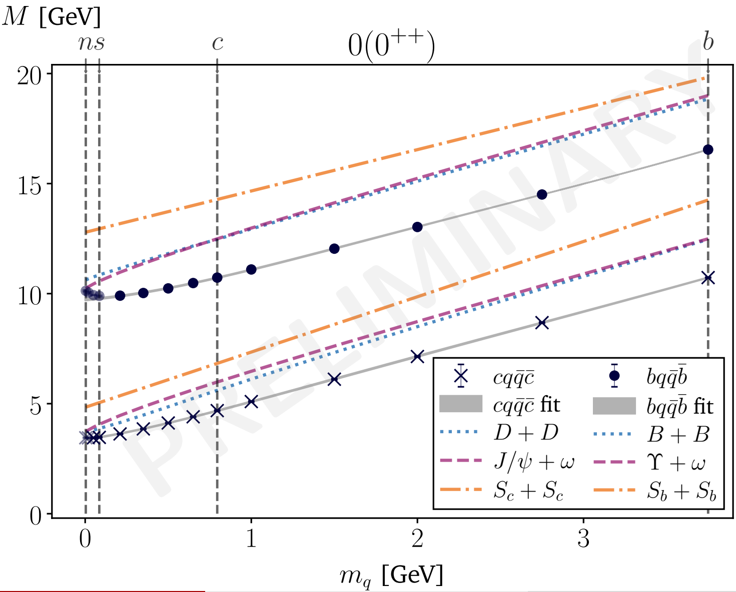
$$\Gamma_{\text{pion}}(p^2) = E(p^2)\cdot\tau_1(p,P) + F(p^2)\cdot\tau_2(p,P) + G(p^2)\cdot\tau_3(p,P) + H(p^2)\cdot\tau_4(p,P)$$



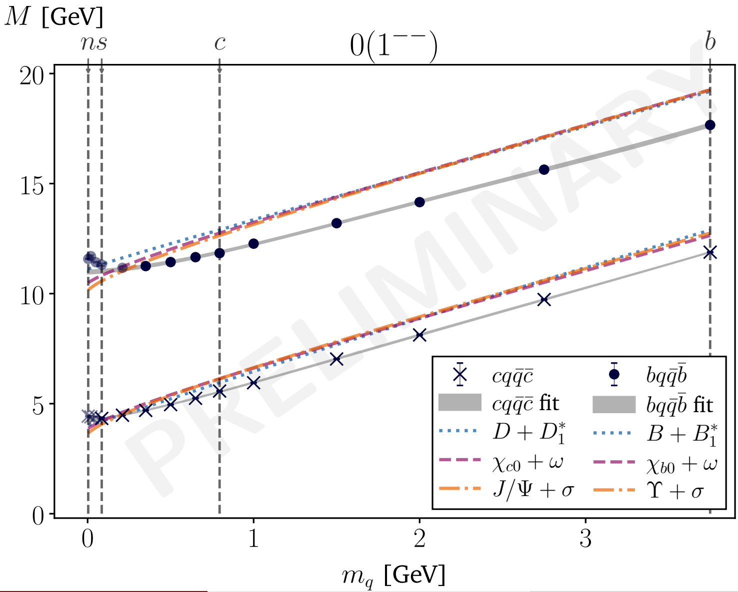
Quark mass evolution 1^{+-}



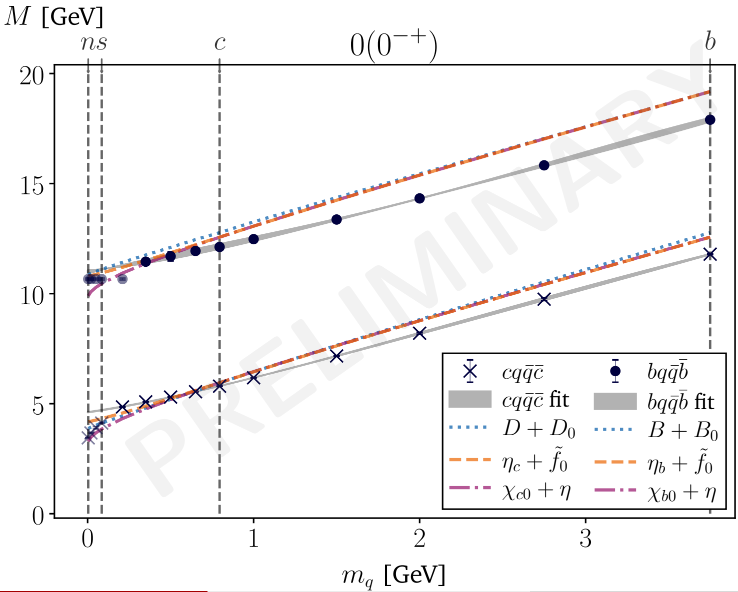
Quark mass evolution 0^{++}



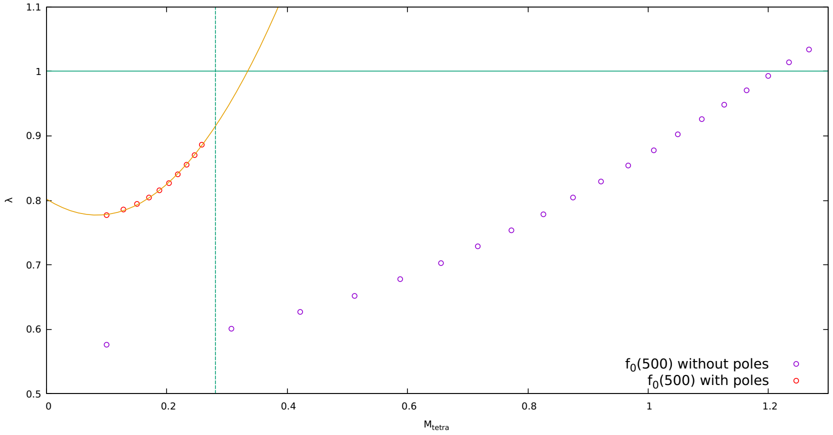
Quark mass evolution 1^{--}



Quark mass evolution 0^{-+}

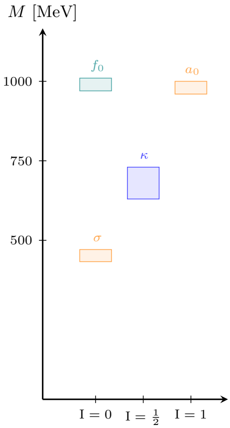


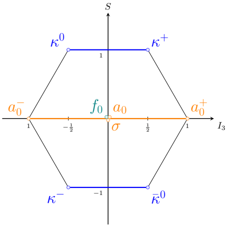
Eigenvalue curve



Light scalar mesons

The light scalar (0^{++}) mesons is an example where the Quark Model yields wrong predictions:





$f_0(980)$ $s\bar{s}$

$\kappa(700)$ $u\bar{s},\ d\bar{s}$

$\left.\begin{array}{l} a_0(980) \\ \sigma(500) \end{array}\right\} u\bar{u},\ d\bar{d},\ u\bar{d}$

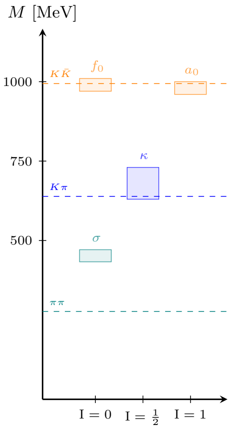
- Why are a_0, f_0 almost mass degenerate?
- Why are the decay widths so different?

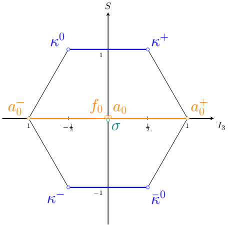
$$\Gamma(\sigma,\kappa) \approx 550\,\mathrm{MeV}$$
$$\Gamma(a_0, f_0) \approx 50 - 100\,\mathrm{MeV}$$

- Why are they so light?

Light scalar mesons

Suppose they were tetraquarks: Jaffe 1977; Close, Tornqvist 2002; Maiani, Polosa, Riquer 2004





$f_0(980)$, $a_0(980)$ } $us\bar{u}\bar{s}, \ldots$

$\kappa(700)$ $us\bar{u}\bar{d}, \ldots$

$\sigma(500)$ $ud\bar{u}\bar{d}$

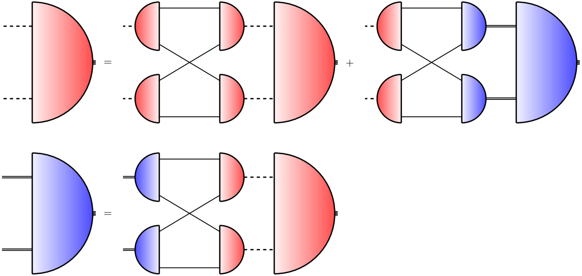
- Explains mass ordering and decay widths:
  a_0, f_0 couple to $K\bar{K}$,
  σ, κ large decay widths
- Non-$q\bar{q}$ nature of σ is supported by dispersive analyses, unitarized ChPT, large N_C, extended linear σ models, quark models

Pelaez, Phys.Rept. 658 (2016)

2-body approach

- Assume dominant 2-body forces → simplify the 4-body BSE to get the **2-body approach**.

Santowsky, Fischer, Eur.Phys.J.C 82 (2022) 4, 313
Santowsky, Eichmann, Fischer, Wallbott, Williams, Phys.Rev.D 102 (2020) 5, 056014



- It is a coupled system of meson-meson and diquark-antidiquark components which interact via quark exchange.
- This approach is close in spirit to an effective field theory description.