QCD mesonic screening masses using Gribov quantization

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Outline

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Introduction

- At finite temperature, Lorentz symmetry is broken $\Rightarrow$ temporal and spatial directions are, in general, unrelated.
- Correlation function in the time direction is used to define spectral functions, which gives information about the plasma's real-time properties, such as particle production rate.
- Correlation functions in spatial direction give info about
  (1) The length scale at which thermal fluctuation are correlated
  (2) The length scale at which the external charges are screened.
- These "static" observables are physical and eminently suited to measurements in lattice experiments.
- Different operator gives different correlation lengths depending on their discrete and continuous global symmetry properties.

Screening mass

- Screening mass can show us how perturbative the medium is.
- Vector-like excitations can reach the perturbative estimate more quickly than pseudo-scalar excitation (HotQCD 19).
- Perturbative estimation: $m/T = 2\pi + \frac{g^2 C_F}{2\pi}(\frac{1}{2} + E_0)$ [Laine & Vepšäläinen, JHEP02(2004)004]

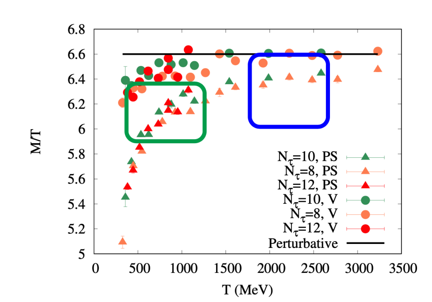


Figure: Credit: Sayantan Sharma's talk in WHEPP 2019, India.

Detailed setup

- Definition: Screening masses are obtained from the exponential decay of the screening correlators

$$C_z\left[O^a, O^b\right] = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^2\mathrm{x}_\perp \left\langle O^a\left(\tau, \mathrm{x}_\perp, z\right) O^b(0, \mathbf{0}, 0)\right\rangle$$

- In the limit of $z \to \infty, C_z\left[O^a, O^b\right] \sim e^{-2\omega_0 z} = e^{-mz} = e^{-z/\zeta}$,

$$\omega_n = 2\pi T\left(n + \frac{1}{2}\right), \quad \zeta^{-1} = 2\pi T = m \to \text{Screening mass}$$

- For the correlation lengths ζ of mesonic observables, $O^a = \bar{\psi}\Gamma F^a \psi$, where

$$\Gamma = \left\{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\right\},$$

- Given $\bar{\psi}, \psi$ we may define, as usual, various bilinears. The $N_f \times N_f$ flavour basis is generated by

$$F^a \equiv \{F^s, F^n\}, \quad F^s \equiv 1_{N_f \times N_f}, \quad n = 1, \ldots, N_f^2 - 1,$$

where $F^s = 1_{N_f \times N_f}$ is an $N_f \times N_f$ identity matrix ("singlet"), and the traceless F^n ("nonsinglet") are assumed normalised such that

$$\mathrm{Tr}\left[F^m F^n\right] = \frac{1}{2}\delta^{mn}$$

- We may then consider scalar, pseudoscalar, vector, and axial vector objects,

$$\begin{aligned} S^a &\equiv \bar{\psi} F^a \psi \\ P^a &\equiv \bar{\psi} \gamma_5 F^a \psi \\ V^a_\mu &\equiv \bar{\psi} \gamma_\mu F^a \psi \\ A^a_\mu &\equiv \bar{\psi} \gamma_\mu \gamma_5 F^a \psi \end{aligned}$$

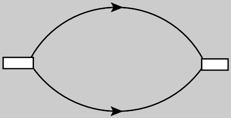
- In the momentum space

$$C_{\mathbf{q}}\left[O^a, O^b\right] \equiv \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3x e^{i\mathbf{q}\cdot\mathbf{x}} \left\langle O^a(\tau, \mathbf{x}) O^b(0, \mathbf{0}) \right\rangle$$

- For scalar and vector operators

$$\begin{aligned} C_{\boldsymbol{q}}\left[S^a, S^b\right], C_{\boldsymbol{q}}\left[V_0^a, V_0^b\right] &= \delta^{ab} f\left(q^2\right), \\ C_{\boldsymbol{q}}\left[V_i^a, V_j^b\right] &= \delta^{ab}\left[\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) t\left(q^2\right) + \frac{q_i q_j}{q^2} l\left(q^2\right)\right] \end{aligned}$$

Leading order correlator



- $$C_{\boldsymbol{q}}\left[O^a, O^b\right] = \mathrm{Tr}\left[F^a F^b\right] N_c T \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}} \times \frac{1}{\left[p_n^2+p^2\right]\left[p_n^2+(p+q)^2\right]} \mathrm{Tr}\left[(\not{p}+\not{q})\Gamma^a \not{p}\Gamma^b\right]$$

- The above correlator contains the function

$$B_{3\mathrm{d}}(2p_n) \equiv \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{1}{\left[p_n^2+p^2\right]\left[p_n^2+(p+q)^2\right]} = \frac{i}{8\pi q} \ln \frac{2p_n - iq}{2p_n + iq}.$$

Operator	Particle	Behaviour in free theory	Physical interpretation
S^s	f_0/σ	$q^2 B_{3\mathrm{d}}(2p_0)$	
S^n	a_0/δ	$q^2 B_{3\mathrm{d}}(2p_0)$	
P^s	η'	$q^2 B_{3\mathrm{d}}(2p_0)$	
P^n	π, η	$q^2 B_{3\mathrm{d}}(2p_0)$	
V_0^s	ω	$(q^2 + 4p_0^2) B_{3\mathrm{d}}(2p_0)$	baryon density
$V_\perp^s$		$(q^2 - 4p_0^2) B_{3\mathrm{d}}(2p_0)$	
V_0^n	ρ, ϕ	$(q^2 + 4p_0^2) B_{3\mathrm{d}}(2p_0)$	charge density
$V_\perp^n$		$(q^2 - 4p_0^2) B_{3\mathrm{d}}(2p_0)$	
A_0^s	f_1	$(q^2 + 4p_0^2) B_{3\mathrm{d}}(2p_0)$	
$A_\perp^s$		$(q^2 - 4p_0^2) B_{3\mathrm{d}}(2p_0)$	
A_0^n	a_1	$(q^2 + 4p_0^2) B_{3\mathrm{d}}(2p_0)$	
$A_\perp^n$		$(q^2 - 4p_0^2) B_{3\mathrm{d}}(2p_0)$	

Table: Different particle assignments and the nature of the singularity structure for the mesonic correlators considered

- The non-trivial structure of the correlator appears at the point $iq = 2p_n$.
- At large distances (small q), the correlator is dominated by the smallest Matsubara frequencies $\pm p_0 = \pm\pi T$.
- Consider the correlator not around the pole at $q = 2ip_0$, but for a very large (real) q. Then the correlator can be computed in the operator product expansion as

$$C_{\vec{q}[S^a,S^b]} \approx \mathrm{Tr}\,[F^a F^b]\, T \left[-\frac{N_c}{4\pi} \left(4p_0 + iq \ln \frac{2p_0 - iq}{2p_0 + iq} \right) + \frac{1}{\pi p_0} \frac{(q^2)^2}{(q^2 + 4p_0^2)^2} g^2 \mathrm{Tr}\, A_0^2 + ... \right] .$$

Next-to-leading order for flavour non-singlet correlators

- Infinitely many higher order graphs that need to be considered.

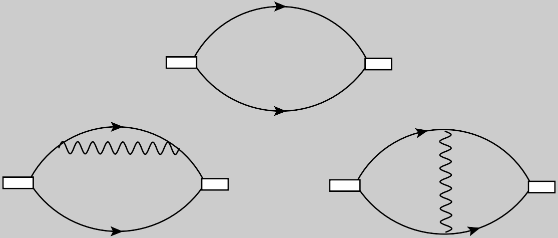


Figure: The diagrams which contribute to meson correlation function: (a) free theory correlator (b) quark self-energy graph (c) interaction of quark and antiquark through gluon exchange.

Continued....

- A convenient way of resummation of all the diagrams is offered by the effective field theory, namely NRQCD.
- Correlation lengths can be seen as (2+1)-dimensional bound states of heavy particles of mass "p_0", which is much larger than infrared scale gT, g^2T.
- Correlation function in leading order dominates only at zero Matsubara mode and quark Lagrangian for this mode

$$\mathcal{L}_E^{\psi} = \bar{\psi}\left[i\gamma_0 p_0 - ig\gamma_0 A_0 + \gamma_k D_k + \gamma_3 D_3\right]\psi, \qquad [k = 1, 2]$$

- Let us make a basis transformation , $\gamma_\mu^{\rm new} = U\gamma_\mu^{\rm standard}U^{-1}$, with

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

- Denoting $\psi \equiv \begin{pmatrix} \chi \\ \phi \end{pmatrix}$, where χ, ϕ are two-component spinors, the Lagrangian becomes

$$\mathcal{L}_E^{\psi} = i\chi^{\dagger}\Big[p_0 - gA_0 + D_3\Big]\chi + i\phi^{\dagger}\Big[p_0 - gA_0 - D_3\Big]\phi + \phi^{\dagger}\epsilon_{kl}D_k\sigma_l\chi - \chi^{\dagger}\epsilon_{kl}D_k\sigma_l\phi \, .$$

  [χ represents the light mode, while ϕ is heavy].

- We can solve for χ and ϕ from the equations of motion and substitute the solution back to the Lagrangian, we get the “diagonalized” on-shell effective lagrangian for two independent modes with a non-relativistic structure:

$$\begin{aligned}\mathcal{L}_E^{\psi} \approx & i\chi^{\dagger}\left[p_0 - gA_0 + D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\chi + \\ & + i\phi^{\dagger}\left[p_0 - gA_0 - D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\phi + \mathcal{O}\left(\frac{1}{p_0^2}\right).\end{aligned}$$

- The correlators we are interested in, the quarks are almost "on-shell". In the free case, the on-shell point is at $p_0^2 + p^2 = 0$, i.e., $p_3 = \pm i[p_0 + \vec{p}_\perp^2/(2p_0) + ...]$.
- Free propagators become

$$
\begin{aligned}
\langle \chi_u(p)\chi_v^*(q)\rangle &= \delta_{uv}(2\pi)^3\delta^{(3)}(p-q)\frac{-i}{M + ip_3 + \vec{p}_\perp^2/(2p_0)} , \\
\langle \phi_u(p)\phi_v^*(q)\rangle &= \delta_{uv}(2\pi)^3\delta^{(3)}(p-q)\frac{-i}{M - ip_3 + \vec{p}_\perp^2/(2p_0)} ,
\end{aligned}
$$

where $M \equiv p_0$, and u, v contain the spinor, flavour and colour indices.

Power counting

- The parametric scales in the problem are $\sim \pi T, gT, g^2 T$. Let us write $m \equiv \pi T, v \equiv g$, and denote by Δp_3 the off-shellness of the "energy", $\Delta p_3 = p_3 \pm i p_0$. Following the terminology of NRQCD, one can define four different regions of phase space:

$$
\begin{aligned}
\text{hard (h)} &: \quad \Delta p_3 \sim |\vec{p}_\perp| \sim m \,, \\
\text{soft (s)} &: \quad \Delta p_3 \sim |\vec{p}_\perp| \sim mv \,, \\
\text{potential (p)} &: \quad \Delta p_3 \sim mv^2 \,, |\vec{p}_\perp| \sim mv \,, \\
\text{ultrasoft (us)} &: \quad \Delta p_3 \sim |\vec{p}_\perp| \sim mv^2 \,.
\end{aligned}
$$

- Close to on-shell, the quarks of NRQCD_3 are potential, since $\Delta p_3 \sim |\vec{p}_\perp^{\,2}|/m \sim mv^2$.
- For soft and potential quarks, $\chi_\text{s}, \chi_\text{p}$, it follows from $S \sim \int \mathrm{d}z\, \mathrm{d}^2\vec{x}_\perp\, \chi^\dagger i\partial_z \chi \sim 1$ that $\chi \sim 1/|\vec{x}_\perp| \sim mv$.

- For soft gluons A_s, $S \sim \int \mathrm{d}z\, \mathrm{d}^2\vec{x}_\perp\, A_\mathrm{s}\partial_z^2 A_\mathrm{s} \sim 1$ implies that $A_\mathrm{s} \sim (z/\vec{x}_\perp^2)^{1/2} \sim m^{1/2}v^{1/2}$.
- For ultrasoft gluons A_us, correspondingly, $A_\mathrm{us} \sim (t/\vec{x}_\perp^2)^{1/2} \sim m^{1/2}v$. In addition, when operating on on-shell quarks, the z derivative can be estimated as $\partial_z \sim 1/z \sim mv^2$.
- Given these rules and that $g_\mathrm{E}^2(= g^2T) \sim mv^2$, we note that $g_\mathrm{E}A_\mathrm{s} \sim mv^{3/2}$, $g_\mathrm{E}A_\mathrm{us} \sim mv^2$.
- If we want to keep the Lagrangian up to and including the order $\mathcal{O}(m^3v^4)$, such that we know the on-shell quark self-energy up to order $\mathcal{O}(mv^2)$, Lagrangian becomes

$$\mathcal{L}_E^\psi = i\chi^\dagger\left(M - g_\mathrm{E}A_0 + D_z - \frac{\nabla_\perp^2}{2p_0}\right)\chi + i\phi^\dagger\left(M - g_\mathrm{E}A_0 - D_z - \frac{\nabla_\perp^2}{2p_0}\right)\phi$$

with $D_t \equiv D_3 = \partial_3 - ig_\mathrm{E}A_3$.

Matching conditions from QCD to NRQCD

- On the side of QCD, the inverse propagator evaluated at the position of the tree-level pole, $p^2 = 0$, reads

$$S^{-1} = i\not{p} - ig^2 C_F \sum\!\!\!\!\!\!\int_q \frac{\gamma_\mu(\not{p} - \not{q})\gamma_\mu}{(p-q)_f^2 (q^2+\lambda^2)_b}\Bigg|_{p^2=0} ,$$

-

$$\Big[\gamma_0 S^{-1}(p)\Big]_{11} \approx i\Big\{p_0 + ip_3 + g^2 C_F \frac{T^2}{8p_0}\Big\} ,$$

and the dispersion relation, or pole position, becomes

$$p_3 \approx i\Big[p_0 + g^2 C_F \frac{T^2}{8p_0}\Big] .$$

- In NRQCD side, the inverse quark propagator becomes

$$S^{-1} = i\left\{M + ip_3 - g_{\mathrm{E}}^2 C_F \int \frac{\mathrm{d}^{3-2\epsilon}q}{(2\pi)^{3-2\epsilon}} \frac{1}{M + ip_3 - iq_3}\left[+\frac{1}{q^2+\lambda^2} - \right.\right.$$
$$\left.\left. -\frac{1}{q^2+\lambda^2}\right\} = i\left[M + ip_3\right].$$

- Matching QCD and NRQCD dispersion relation, one gets

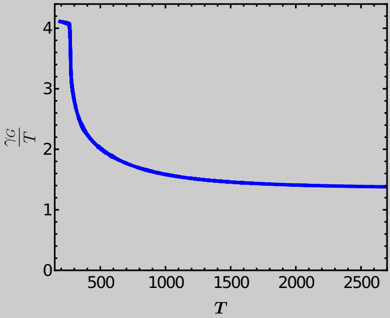
$$M = p_0 + g^2 T \frac{C_F}{8\pi}.$$ [Laine & Vepšaläinen, JHEP02(2004)004]

Matching conditions from QCD to NRQCD, with Gribov

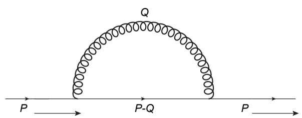
- Gluon Propagator (Gribov modified):

$$D^{\mu\nu}(P) = \left[\delta^{\mu\nu} - (1-\xi)\frac{P^\mu P^\nu}{P^2}\right]\frac{P^2}{P^4+\gamma_G^4}$$

- The Gribov parameter γ_G can be fixed from lattice thermodynamics.



- With Gribov propagator, quark self-energy becomes



$$
\begin{aligned}
\Sigma(P) \;&=\; -ig^2 C_F \sum\!\!\!\!\!\!\!\int_Q \frac{\gamma_\mu(\not{P}-\not{Q})\gamma_\mu}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right) + ig^2 \\
&\times \;\; C_F \sum\!\!\!\!\!\!\!\int_Q \frac{\not{Q}(\not{P}-\not{Q})\not{Q}}{Q^2(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4} - \frac{\xi\, Q^2}{Q^4+\gamma_G^4}\right)
\end{aligned}
$$

- With the above quark self-energy, the Euclidean dispersion relation on the QCD becomes

$$
p_3 \approx i\Big[p_0 - g^2 C_F (I_1 + I_2)\Big]
$$

Continued...

- $$I_1 = \frac{-1}{p_0}\int_0^\infty \frac{q^2 dq}{(2\pi)^2}\left[\frac{n^+}{E_+}+\frac{n^-}{E_-}\right], \quad I_2 = \frac{1}{p_0}\left[\frac{-T^2}{24}+X\right]$$

  with $n^\pm \to$ B.E distribution function, $\tilde{n} \to$ F.D distribution function and $E_\pm = \sqrt{q^2 \pm i\gamma_G^2}$ and

  $$X = \frac{\gamma_G^4}{T^2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{8EE_+E_-}\left[\left\{\frac{\tilde{n}+n^-}{i\pi - E + E_-} - \frac{\tilde{n}+n^-}{i\pi + E - E_-}\right\} - (n^- \to n^+)\right]\frac{1}{E_+ - E_-}$$

- On NRQCD_3 side, the pole location is simply $p_3 = iM$. Now, after doing the matching, we will get

  $$M = p_0 - g^2 C_F (I_1 + I_2)$$

Solution for the screening states

- Correlators we have considered are

$$C_z\left[O^a, O^b\right] \sim \int \mathrm{d}^2 x_\perp \left\langle O^a\left(x_\perp, z\right) O^b\left(\mathbf{0}_\perp, 0\right)\right\rangle,$$

- We focus on the correlators which decay more slowly, that is, which are represented by operators of the type $\phi^\dagger\chi + \chi^\dagger\phi$
- The E.O.M obeyed by the Green's function (at large z), is of the form

$$(\partial_z - H)G(z) = C\,\delta(z)$$

- Exponential decay is determined by the lowest eigenvalue of the H differential operator.
- In order to find H, we now define a point-splitting by introducing a vector $\boldsymbol{r}$ as

$$C(r,z) \equiv \int \mathrm{d}^2\boldsymbol{R}\left\langle \phi^*\left(\boldsymbol{R}+\frac{r}{2}, z\right)\chi\left(\boldsymbol{R}-\frac{r}{2}, z\right)\chi^*(\mathbf{0},0)\phi(\mathbf{0},0)\right\rangle.$$

- The tree-level contribution reads as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla^2_{\boldsymbol{r}}\right] C^{(0)}(r,z) \propto \delta(z)\delta^{(2)}(r).$$

- Similarly, the 1-loop contribution can be written as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla^2_{\boldsymbol{r}}\right] C^{(1)}(\boldsymbol{r},z) = -g_{\mathrm{E}}^2 C_F \mathcal{K}\left(\frac{1}{zp_0}, \frac{\nabla_{\boldsymbol{r}}}{p_0}, \frac{\gamma_G^4}{p_0^4}, rp_0\right) C^{(0)}(r,z).$$

Continued....

- The zeroth-order term (in $1/p_0$) of the kernel $\mathcal{K}$ gives the one-loop static potential as

$$V(r) = g_{\mathrm{E}}^2 \frac{C_F}{2\pi} \left[\ln \frac{\gamma_{\mathrm{G}} r}{2} + \gamma_E - K_0 \left(\gamma_{\mathrm{G}} r \right) \right].$$

- This potential determines the coefficient of the exponential fall-off, $\xi^{-1} \equiv m$, through

$$\left[2M - \frac{\nabla_r^2}{p_0} + V(r) \right] \Psi_0 = m \Psi_0$$

- To find the solution numerically, we rescale $m - 2M \equiv g_{\mathrm{E}}^2 \frac{C_F}{2\pi} E_0$
- EOM becomes $\left[-\frac{\nabla_r^2}{p_0} + V(r) \right] \Psi_0 = g_{\mathrm{E}}^2 \frac{C_F}{2\pi} E_0 \Psi_0$
- After solving for E_0, the Screening mass can be obtained as $m = 2\pi T + g^2 T \frac{C_F}{2\pi} \left[E_0 - \frac{4\pi}{T} \left(I_1 + I_2 \right) \right].$

Results and Discussion

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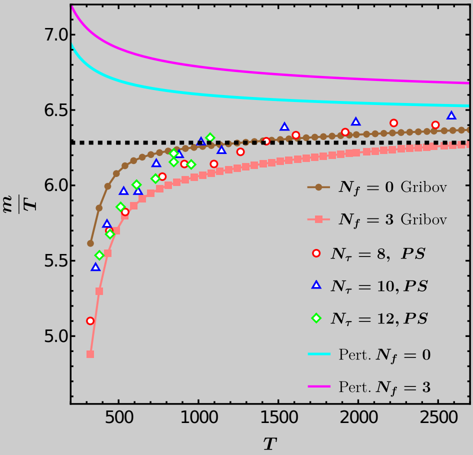
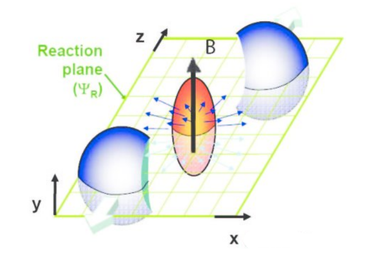


Figure: The temperature dependence of the scaled screening mass. The dashed line represents the free theory result from $m/T = 2\pi$.

Magnetic field in HIC

Magnetic field in HIC

Magnetic field in HIC

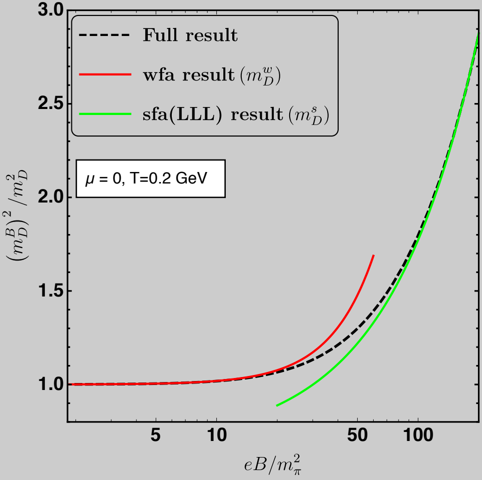


- The non-central heavy ion collision produces a very strong magnetic field normal to the reaction plane.
- At LHC energies, the strength of the magnetic field is estimated to be as high as $eB = 15m_\pi^2 = 1.5 \times 10^{19}$Gauss, the largest magnetic field ever produced in the laboratory.

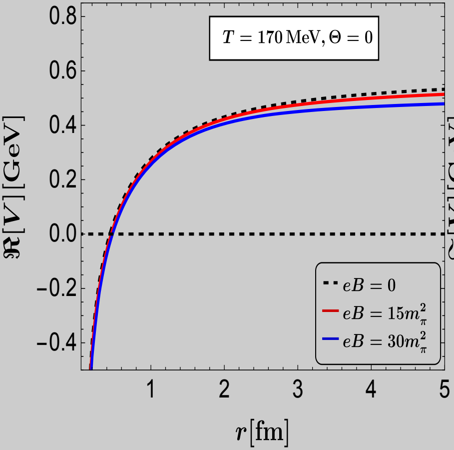
V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009)

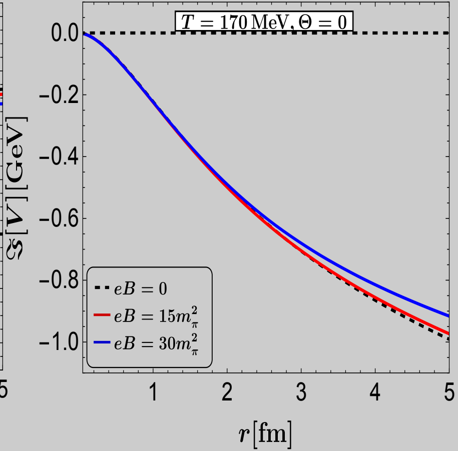
- As the magnetic field is very strong, LLL approximation is used in literature.
- Very recently, we have studied the Debye mass and heavy-quarkonium potential without LLL approximation.

Debye mass


Full result
wfa result (m_D^w)
sfa(LLL) result (m_D^s)
μ = 0, T=0.2 GeV
3.0
2.5
2.0
1.5
1.0
$\left(m_D^B\right)^2/m_D^2$
5
10
50
100
eB/m_π^2

Results: Radial dependence





- The magnetic field dependence is very small and is insignificant even at $eB = 15m_\pi^2$.

Thank you for your attention.