Renormalization Group Consistent Treatment of NJL Color-Superconductivity

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Phase structure at large (but not asymptotically large) density and moderate T is relevant for neutron stars and neutron star mergers



- Merger simulations: densities produced in merger remnant might be sufficient to produce quark matter
- Expected implications: Modified post-merger frequency spectrum [Elias R. Most et al.

(2019)] [Bauswein et al. (2019)]

Motivation: signatures of color superconducting phases in neutron star cores or merger remnants?

Method: Use effective models for studying color superconductivity

- This talk: Treatment of unphysical regularization artefacts in NJL color superconductivity (cut-off artefacts)
 - Investigations of some astrophysical aspects of the model

Why (color) Superconductivity

- Noninteracting fermions at T = 0
 - Particles at the Fermi surface can be created at the Fermi surface with no free-energy cost.
- Cooper theorem: With a finite attractive interaction between particles
 - Fermi surface becomes unstable against pair creation → "Cooper pairs"
 - Bose condensation of the Cooper pairs ⇒ Energy gap Δ in exaction spectrum: $\omega = \sqrt{(E - \mu)^2 + \Delta^2}$





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- Solid-state superconductor: Effective attractive interaction between electrons through the interaction with lattice → phonons
- **Color superconductor:** QCD: attractive quark-quark interaction
 - \blacksquare at higher densities via a single gluon exchange in the $\bar{3}$ color channel
 - at intermediate densities also via instanton exchange [Rapp, Schäfer, Shuryak and Velkovsky 1998]



Color Superconductivity



- diquark condensates: $\langle q_i O_{ij} q_j \rangle$
- ▶ Pauli principle: $\mathcal{O} = \mathcal{O}_{spin} \otimes \mathcal{O}_{color} \otimes \mathcal{O}_{flavor} = totally antisymmetric$
- ▶ most attractive channel: color $\overline{3}$ and spin 0 (antisymmetric) \Rightarrow mixing flavors





Nambu Jona-Lasinio (NJL)-type model

$$egin{aligned} \mathcal{L} = & & \ & ar{\psi}(i \partial \!\!\!/ - m) \psi \ & + & G \sum \left[(ar{\psi} au_a \psi)^2 + (ar{\psi} i \gamma_5 au_a \psi)^2
ight] \end{aligned}$$

kinetic term

scalar NJL interaction



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$$\begin{split} \mathcal{L} &= \\ & \bar{\psi}(i\partial \!\!\!/ - m)\psi & \text{kinetic term} \\ &+ G \sum_{i=1}^{n} \left[(\bar{\psi}\tau_a \psi)^2 + (\bar{\psi}i\gamma_5\tau_a \psi)^2 \right] & \text{scalar NJL interaction} \\ &- K \left[\mathsf{det}_f(\bar{\psi}(\mathbbm{1} + \gamma_5)\psi) + \mathsf{det}_f(\bar{\psi}(\mathbbm{1} - \gamma_5)\psi) \right] & \text{'t Hooft (KMT) interaction} \end{split}$$



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kinetic term scalar NJL interaction 't Hooft (KMT) interaction

diquark interaction

with charge conjugated spinor $\psi^c = C \bar{\psi}^T$



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with charge conjugated spinor $\psi^c = C \bar{\psi}^T$ Mean field approximation: Linearise theory around condensates

$$\begin{split} \phi_f = & \langle \bar{\psi}_f \psi_f \rangle & f = u, d, s \\ \Delta_A = & -2G\eta_D \langle \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \rangle & A = 2(ud), 5(us), 7(ds) \end{split}$$

Then minimizing with respect to these condensates

- Λ', G, K, m fitted to vacuum meson spectrum
- Regularization: sharp 3-momentum cutoff $\Lambda^{'} = 602 MeV$



Chemical potential matrix in color-flavor space:

$$\begin{split} \mu_{f,c} = & \mu + Q_f \mu_Q + \lambda_{3,c} \mu_3 + \lambda_{8,c} \mu_8 \\ \text{e.g.} \ \mu_{u,r} = & \mu + \frac{2}{3} \mu_Q + \mu_3 + \frac{1}{\sqrt{3}} \mu_8 \end{split}$$

▶ Neutron star: Enforce charge and color-neutrality locally, i.e. for every phase:

$$\frac{\partial\Omega}{\partial\mu_Q} = \frac{\partial\Omega}{\partial\mu_3} = \frac{\partial\Omega}{\partial\mu_8} = 0$$

• Leptonic contribution: e^- and μ^- in β -equilibrium $\mu_e = \mu_\mu = -\mu_Q$

Optimization problem with nonlinear constraints

- 3 quark mass gap equations
- 3 diquark gap equations
- 3 neutrality constraints
- in total 9 equations to be solved self-consistently

Condensates in Mean Field



 $\blacktriangleright \eta_D = 1$



Condensates in Mean Field



 $\blacktriangleright \eta_D = 1$



 \blacktriangleright Cutoff artefact: Δ values descend at high chemical potentials/number densities





Cutoff artefacts

- Gaps and phase boundary to normal phase decrease in value at $\mu \sim \Lambda'$
- Appearance of uSC phase [Fukushima 2005]
- Previous explanations for uSC: T=0 arguments \rightarrow Not relevant for $T \neq 0$





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- Puzzle: Absence of expected dSC phase in CFL melting pattern [lida et al 2004]
 - Ginzburg-Landau anlaysis around T_c
 - T_c of a pairing is proportional to average Fermi momenta of that pairs: $T_c \propto \bar{p}_F^{ij} = \frac{1}{2}(p_F^i + p_F^j)$





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Renormalization Group-Consistency

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Solution to Regularization artefacts:

 Use renormalization group-consistent regularization presented by Braun et al (2016)



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Renormalization group consistency and low-energy effective theories

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- \blacktriangleright Scale dependent effective action Γ_k includes all physics above momentum scale k
- Full Quantum effective action Γ includes all scales $\Gamma \coloneqq \lim_{k \to 0} \Gamma_k$
- Renormalization group calculation: Ansatz for theory at scale Λ is evolved to k = 0 by solving the RG-flow equation

Renormalization Group Consistency

The full quantum effective action must not depend on the initial scale $k=\Lambda$

$$\Lambda \frac{d\Gamma}{d\Lambda} = 0.$$

Renormalization Group-Consistency



Solution of the flow equation in mean field

$$\Gamma(\mu, T) = \Gamma_{\Lambda}(\mu, T) + \frac{1}{2} \int_{p < \Lambda} \frac{d^3 p}{(2\pi)^3} T \cdot \operatorname{Tr} \log \frac{S^{-1}}{T}$$
(1)

Problem: Λ' isn't big enough with respect to all scales of the system when in medium: $\Lambda' \sim \mu, T, \Delta_i, M_j$.



Renormalization Group-Consistency



Solution of the flow equation in mean field

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(2)

Assumption: Λ bigger than all scales of the system: $\Lambda \gg \mu$, $T, \Delta_i, M_j, \Lambda'$. Then $\Gamma_{\Lambda}(\mu, T) \approx \Gamma_{\Lambda}(\mu = 0, T = 0)$ can be calculated from flow equation in vacuum





In most of the mean field cases: RG consistent treatment is equivalent to only regularizing the vacuum part

$$\Gamma(\mu, T, \chi) = \frac{1}{2\pi^2} \int_0^{\Lambda'} dp \, p^2 \sum_j \left(\epsilon_j(\mu, \chi) + 2T \ln\left(1 + e^{-\frac{\epsilon_j(\mu, \chi)}{T}}\right) \right) - \mathcal{V}$$



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• If
$$\epsilon_j(\mu, \chi) = \tilde{\epsilon}_j(\chi) \pm \mu$$

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RG Consistency and Divergent Medium



- In most of the mean field cases: RG consistent treatment is equivalent to only regularizing the vacuum part→ separation of vacuum and medium
- In the case of color superconductivity: vacuum part can't be separated from the medium part

$$\epsilon_{\pm} = \sqrt{(E \pm \mu)^2 + \Delta^2}$$

$$\Gamma(\mu, T, \chi) \supset \frac{1}{2\pi^2} \int_0^{\Lambda} dp \, p^2 \, (\epsilon_+ + \epsilon_-) \supset \mu^2 \Delta^2 \log \Lambda$$

RG Consistency and Divergent Medium



- In most of the mean field cases: RG consistent treatment is equivalent to only regularizing the vacuum part→ separation of vacuum and medium
- ► In the case of color superconductivity: vacuum part can't be separated from the medium part → issue of medium divergence
- ▶ 3-flavour NJL color-superconductivity suffers from a medium divergence

$$\mu^2 (\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2) \log \Lambda$$

this divergence can be balanced by including a counterterm

$$Y_k(\Delta)\mu^2$$
 with $Y_k(\Delta)=0$ for $k<\Lambda'$

in the Lagrangian: renormalization of the medium part \rightarrow Note that we are not renormalizing a QFT!

This choice of counter-term is equivalent to regularizing the corresponding term raised in the expansion of the medium part



Flowing up to the original cut-off scale $\Lambda=\Lambda'$ (i.e. not RG consistent)





Flowing up to $\Lambda=2\Lambda'$





Flowing up to $\Lambda=10\Lambda'$





Flowing up to $\Lambda=50\Lambda'$







- $\blacktriangleright\,$ At $\Lambda\approx 10\Lambda^{'},$ results become almost independent of $\Lambda\,$
- \blacktriangleright Expected Increasing trend \rightarrow cut-off artefacts removed
- Gap values become enlarged for the same diquark coupling

RG Consistency for Neutral CSC



For a generalized chemical potential analytically derived form of divergence

$$\left(\Delta^2_{ud}\bar{\mu}^2_{ud} + \Delta^2_{us}\bar{\mu}^2_{us} + \Delta^2_{ds}\bar{\mu}^2_{ds}\right)\log\Lambda$$

with $\bar{\mu}_{ij}$ being the average chemical potential of quark flavour species i and j

- ► These divergences should be removed for an RG consistent model
- Behaviour of divergence suggests that

$$\mathcal{L}^R \supset \frac{1}{2} Y_{ud}(\Delta_{ud}) \bar{\mu}_{ud}^2 + \frac{1}{2} Y_{ds}(\Delta_{ds}) \bar{\mu}_{ds}^2 + \frac{1}{2} Y_{us}(\Delta_{us}) \bar{\mu}_{us}^2$$

One can analytically drive the expression for these 3 renormalization factors

Results: Phase Diagram





- Cut-off artefacts are removed
 - Expected increasing trend of phase boundaries
 - No uSC phase
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Thermodynamics



- Pressure: $P(T,\mu) = \frac{T}{V_3}\Gamma(T,\mu)$
- Equation of State: Energy density $\epsilon = -P + \mu n + sT$ as a function of P
- \blacktriangleright Connects microphysics to astrophysics of neutron stars via TOV equation \rightarrow Mass-radius curves
- ▶ **Example:** Speed of sound indicates the stiffness of the EOS: Higher C_s means stiffer EOS \rightarrow higher TOV mass

•
$$C_s^2 = \frac{dP}{d\epsilon}$$

• Causality:
$$C_s^2 < 1$$

• Chiral limit: $C_s^2 = 1/3$

Results: Thermodynamics



A natural outcome of the RG consistent treatment: correct thermodynamic limits

- Correct thermodynamics are mostly important for the astrophysics related calculations
- \blacktriangleright Above the T_c at high densities speed of sound should go to the chiral limit $C_s^2=1/3$
 - can only be achieved if the medium part is not regularized
 - RG consistent treatment automatically satisfies this



Results: Thermodynamics

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A natural outcome of the RG consistent treatment: correct thermodynamic limits

- Correct thermodynamics are mostly important for the astrophysics related calculations
- One expects at higher densities: $P \approx \alpha \mu^4 + \beta \mu^2 \Delta^2$

 \blacksquare if $\Delta < \mu \Rightarrow C_s^2 \rightarrow 1/3$ at high densities



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CRC-TR 2

Procedure:

- Choose EoS for Hadronic Matter (HM) at low densities that satisfies all observational constraints (here: relativistic mean-field model with density-dependent couplings DDME2 [Lalazissis et al. (2005)])
- Calculate Maxwell construction: $\{P, \mu_B, T\}_{HM} = \{P, \mu_B, T\}_{QM}$ in β -equilibrium at the point of the phase transition By construction, this gives a first order phase transition from HM to QM
- Calculate M-R-relation



 $\mathcal{L} =$



Vector interaction provides stiffness of the equation of state at high temperatures to reach $2M_{\odot}$ hybrid stars $_{\rm [Klähn\ et\ al\ (2007,\ 2013),\ Alaverdyan\ (2022)]}$

$$\begin{split} \bar{\psi}(i\partial \!\!\!/ - m)\psi & \text{kinetic term} \\ + G \sum_{i} \left[(\bar{\psi}\tau_a \psi)^2 + (\bar{\psi}i\gamma_5\tau_a \psi)^2 \right] & \text{scalar NJL interaction} \\ - K \left[\det_f(\bar{\psi}(\mathbbm{1} + \gamma_5)\psi) + \det_f(\bar{\psi}(\mathbbm{1} - \gamma_5)\psi) \right] & \text{'t Hooft (KMT) interaction} \\ + G \eta_D \sum_{i} (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) & \text{diquark interaction} \\ - G \eta_V(\bar{\psi}\gamma^\mu\psi)^2 & \text{vector interaction} \end{split}$$

► 2 free parameters η_D , η_V can be constrained by observational constraints on the static EoS of isolated hybrid stars

Results: MR curves



Variation of the vector coupling at constant $\eta_D = 1.49$



Message: Model allows for hybrid EoS containing stable 2SC, 2SC+CFL and CFL-cores consistent with the observation of $2M_{\odot}$ neutron stars

Results: Parameter scan



Work in preparation



Summary

- ► NJL color-superconductivity suffers from cut-off artefacts
- RG-consistent formulation systematically removes the cutoff artefacts and changes the phase diagram in terms of critical temperatures, diquark condensate values and phase transition points
- RG-consistent formulation for neutral CSC matter is in agreement with expected dSC phase in CFL melting patter
- Different type of possible phase transitions to CSC matter in agreement with astrophysical observations
- ▶ We can limit free parameters of the model by the astrophysical constrains

Outlook

- Main interest: Study imprints of color superconductivity in neutron star mergers
- Publishing results...

Thank you for listening!

