Thermal QCD phase transition and its scaling window from lattice simulations with Wilson twisted mass fermions

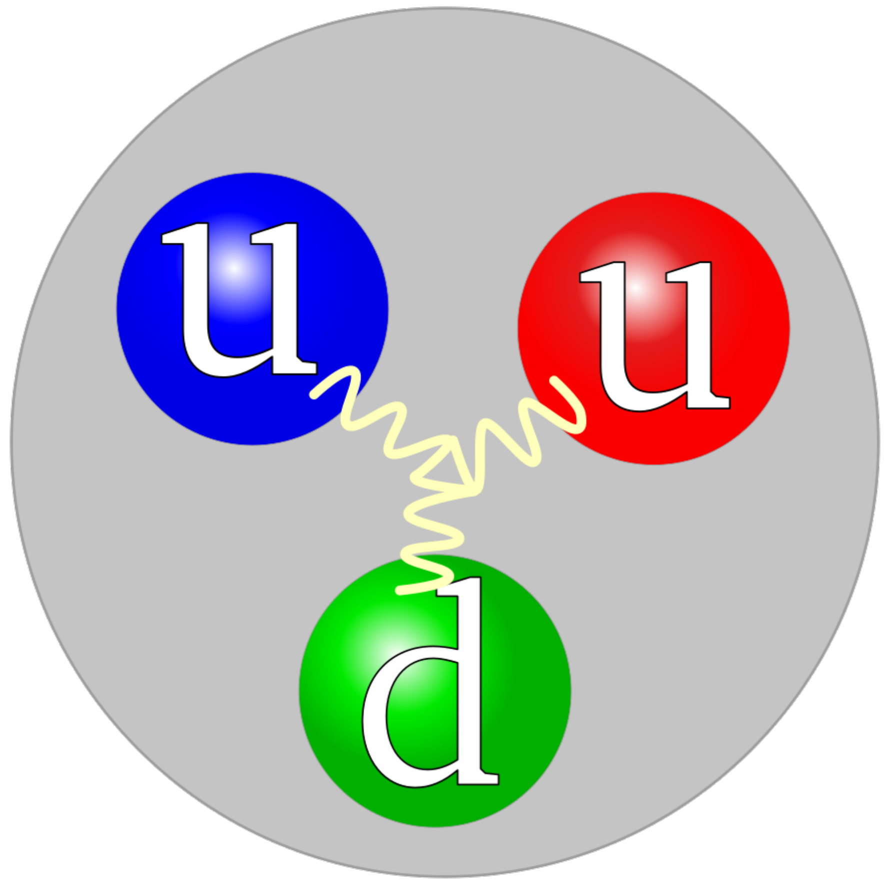
A. Yu. Kotov

JÜLICH Forschungszentrum

[AYuK, M.P. Lombardo, A. Trunin, Phys.Lett.B 823, 2021]
[AYuK, M.P. Lombardo, A. Trunin, Symmetry 13, 2021]
[AYuK, M.P. Lombardo, A. Trunin, PoS Lattice 2021]
[AYuK, M.P. Lombardo, A. Trunin, in progress]

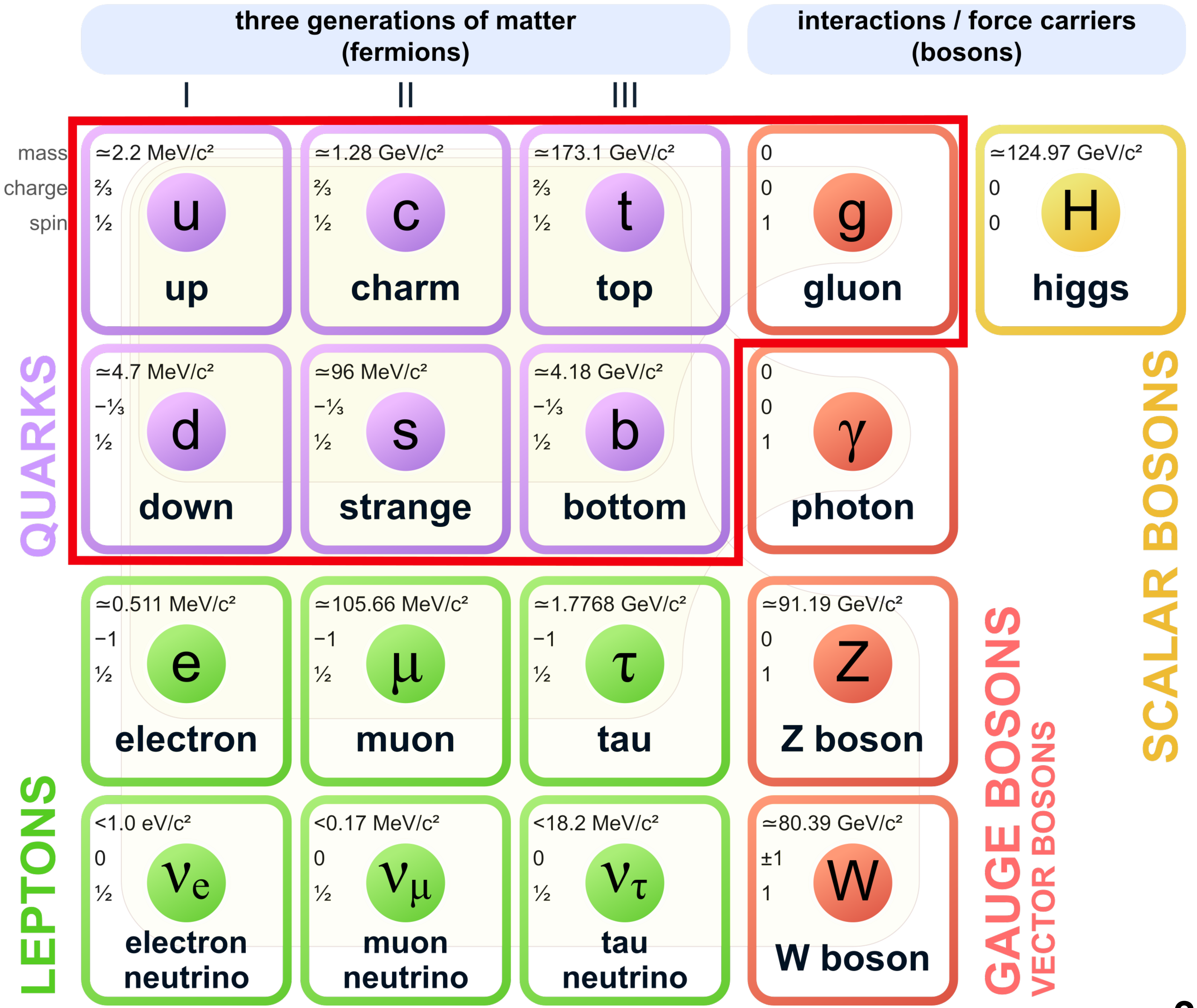
Gießen, 2023

Quantum Chromodynamics: theory of strong interactions



Proton

Standard Model of Elementary Particles



Symmetries of QCD with n quarks

Symmetries of QCD with n quarks

$$L = \sum_a \bar{q}_{La} D q_{La} + \bar{q}_{Ra} D q_{Ra} - m_a(\bar{q}_{La} q_{Ra} + \bar{q}_{Ra} q_{La}) + L_{\text{gauge}}$$

Global symmetry($m_a = 0$):$U_L(n) \times U_R(n) \cong SU_L(n) \times SU_R(n) \times U_B(1) \times U_A(1)$

Symmetries of QCD with n quarks

$$L = \sum_a \bar{q}_{La} D q_{La} + \bar{q}_{Ra} D q_{Ra} - m_a(\bar{q}_{La} q_{Ra} + \bar{q}_{Ra} q_{La}) + L_{\text{gauge}}$$

Global symmetry$(m_a = 0)$: $U_L(n) \times U_R(n) \cong \boxed{SU_L(n) \times SU_R(n)} \times U_B(1) \times U_A(1)$

Spontaneously broken → $SU_V(n)$

Baryon number

Anomalously Broken

Symmetries of QCD with n quarks

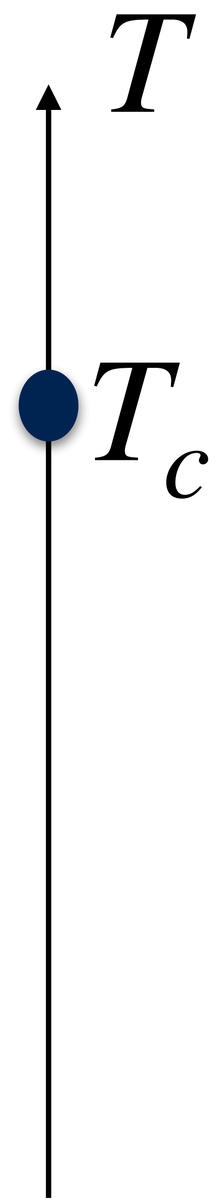
$$L = \sum_a \bar{q}_{La} D q_{La} + \bar{q}_{Ra} D q_{Ra} - m_a(\bar{q}_{La} q_{Ra} + \bar{q}_{Ra} q_{La}) + L_{\text{gauge}}$$

Global symmetry($m_a = 0$): $U_L(n) \times U_R(n) \cong \boxed{SU_L(n) \times SU_R(n)} \times U_B(1) \times U_A(1)$

Spontaneously broken $\downarrow$ $SU_V(n)$

Baryon number

Anomalously Broken



$T > T_c$ ($m = 0$): (which?) symmetry restoration ⇔ <u>order (universality)</u>

Symmetries of QCD with n quarks

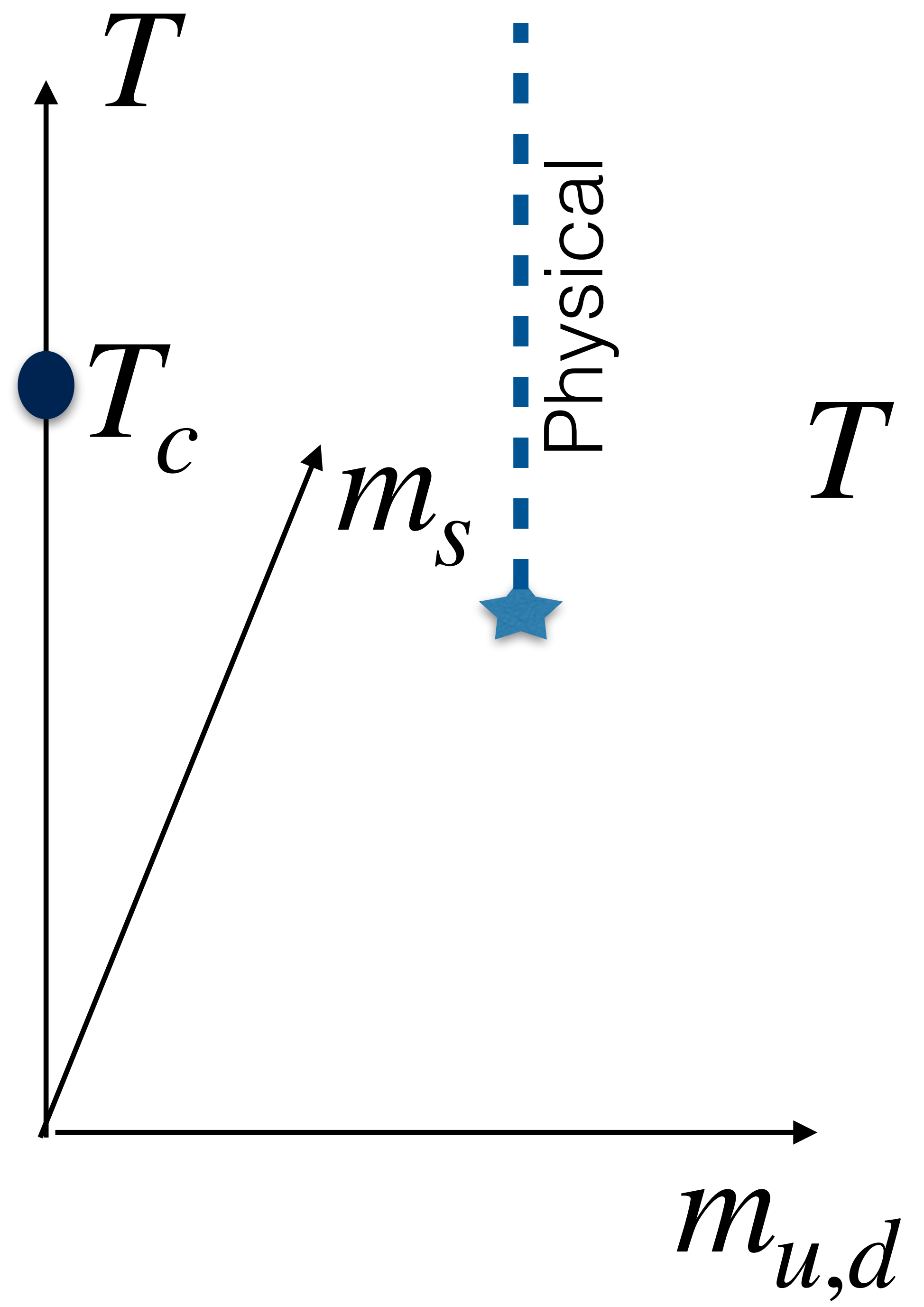
$$L = \sum_a \bar{q}_{La} D q_{La} + \bar{q}_{Ra} D q_{Ra} - m_a(\bar{q}_{La} q_{Ra} + \bar{q}_{Ra} q_{La}) + L_{\text{gauge}}$$

Global symmetry($m_a = 0$): $U_L(n) \times U_R(n) \cong \boxed{SU_L(n) \times SU_R(n)} \times U_B(1) \times U_A(1)$

Spontaneously broken → $SU_V(n)$

Baryon number

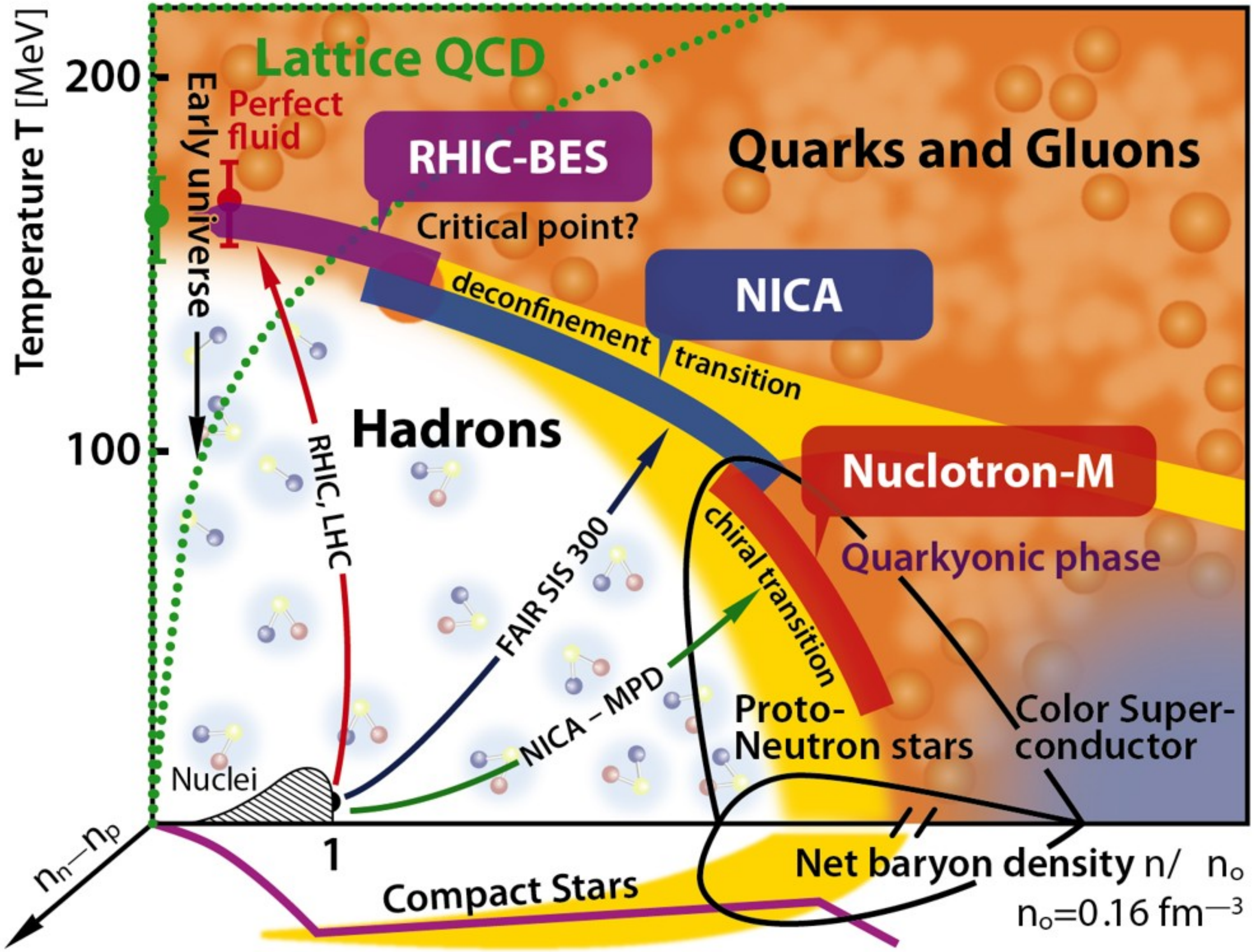
Anomalously Broken



$T > T_c$ ($m = 0$): (which?) symmetry restoration ⇔ order (universality)

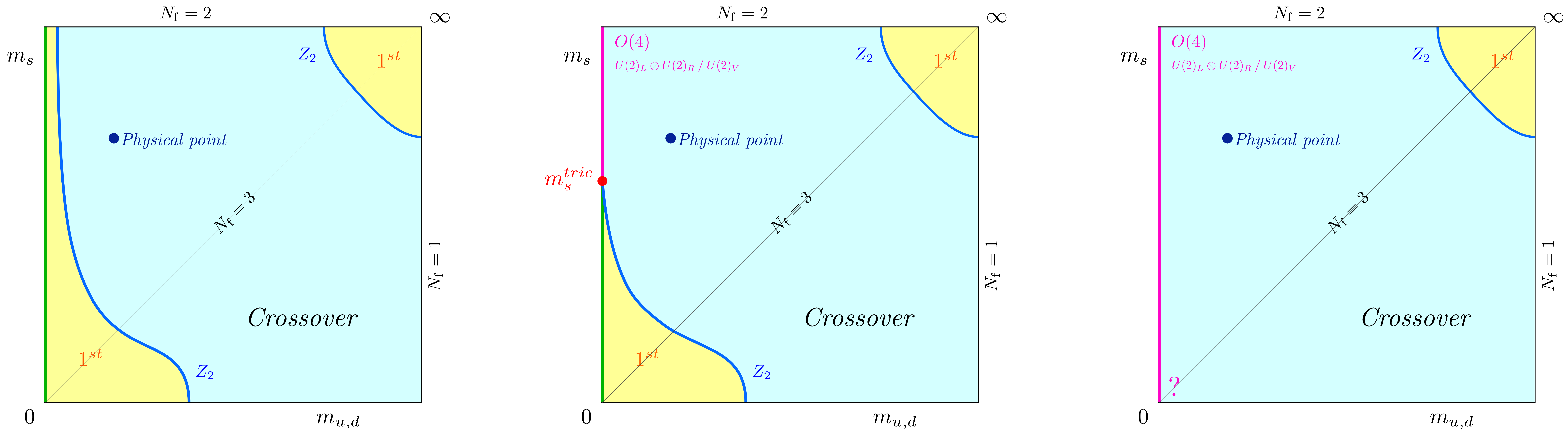
$m_a \neq 0$: explicit symmetry breaking

QCD phase transition



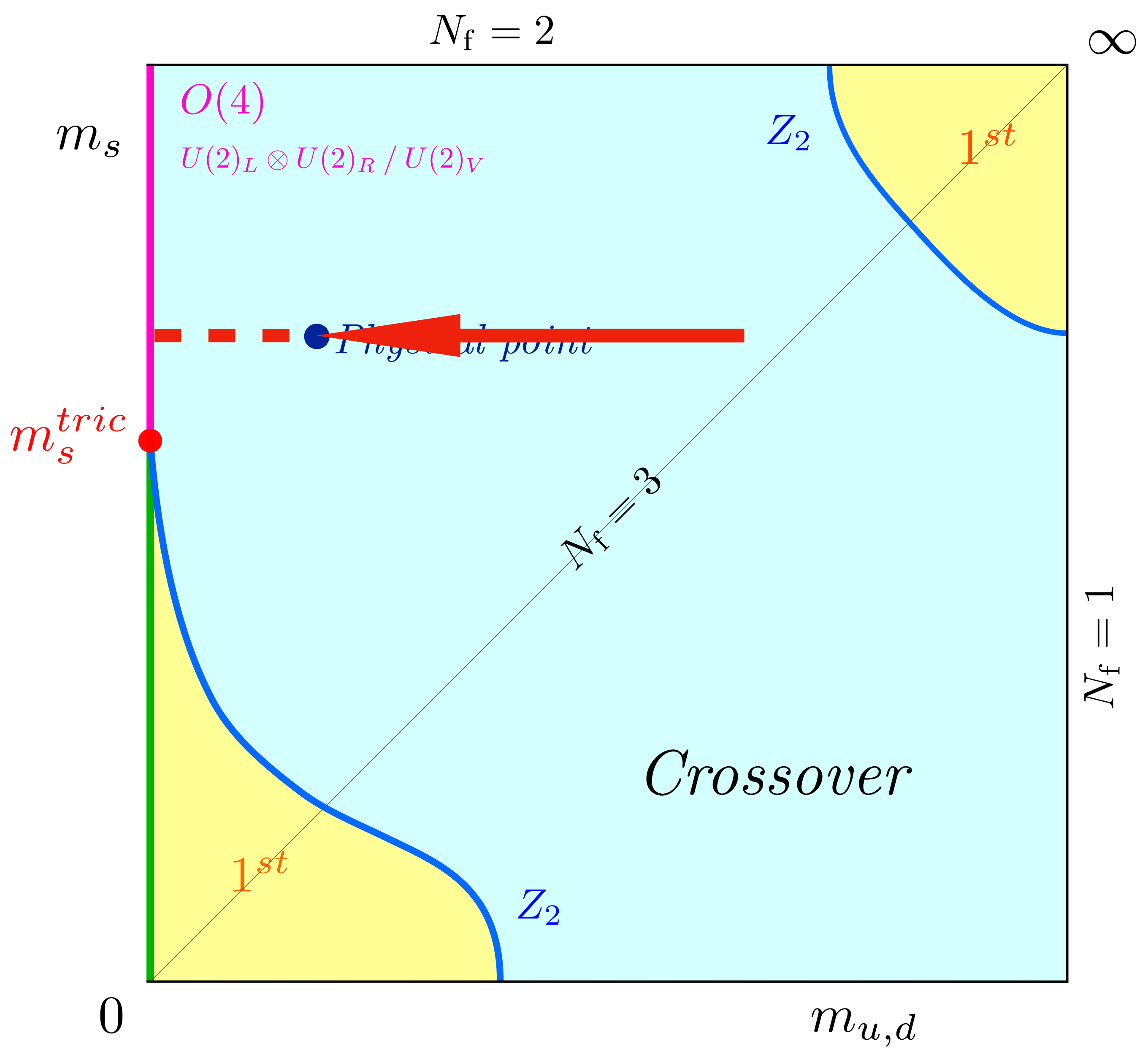
Columbia plot, possible scenarios

[F. Cuteri et al, 2021]



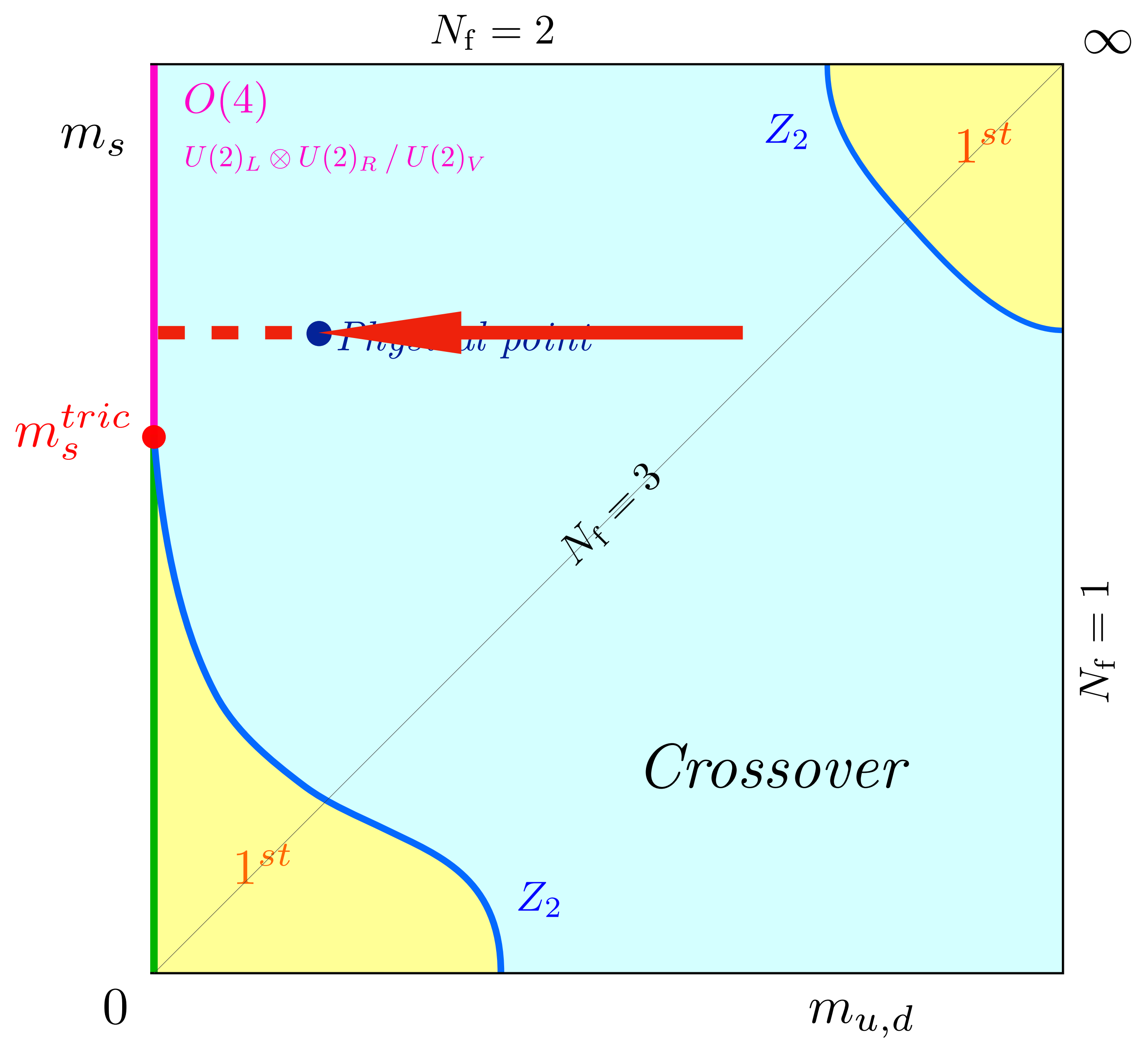
Phase transition in the Nf=2 chiral limit

Phase transition in the Nf=2 chiral limit



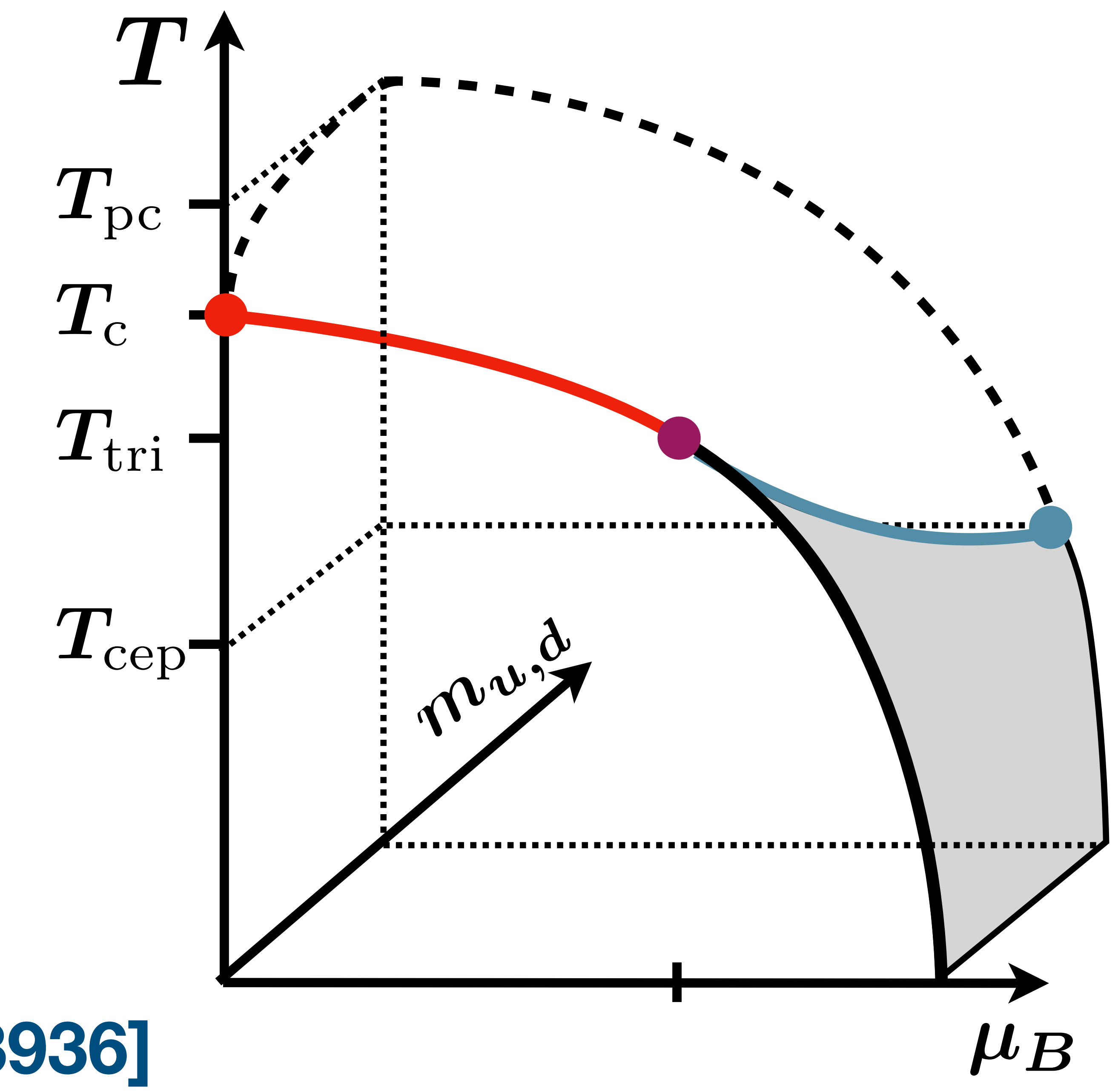
- Favoured scenario: second order, belonging to $SU(2) \times SU(2) \simeq O(4)$ universality class

Phase transition in the Nf=2 chiral limit



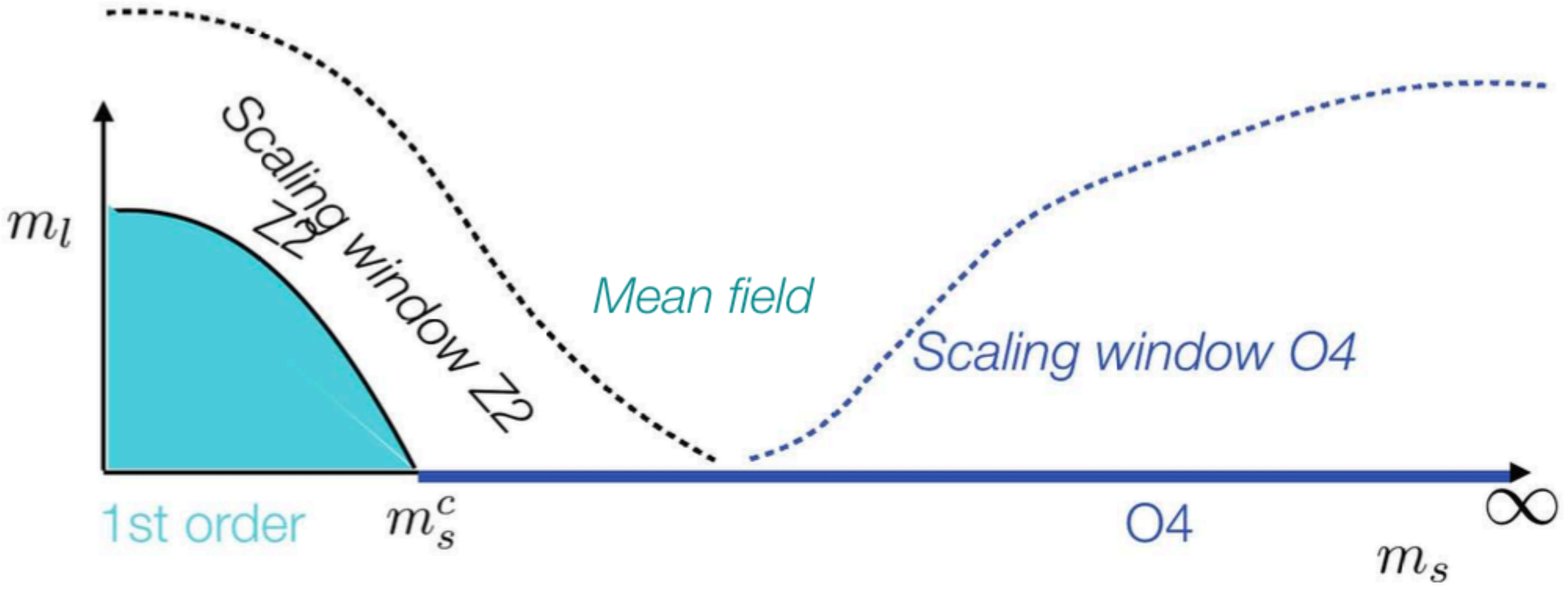
- Favoured scenario: second order, belonging to $SU(2) \times SU(2) \simeq O(4)$ universality class

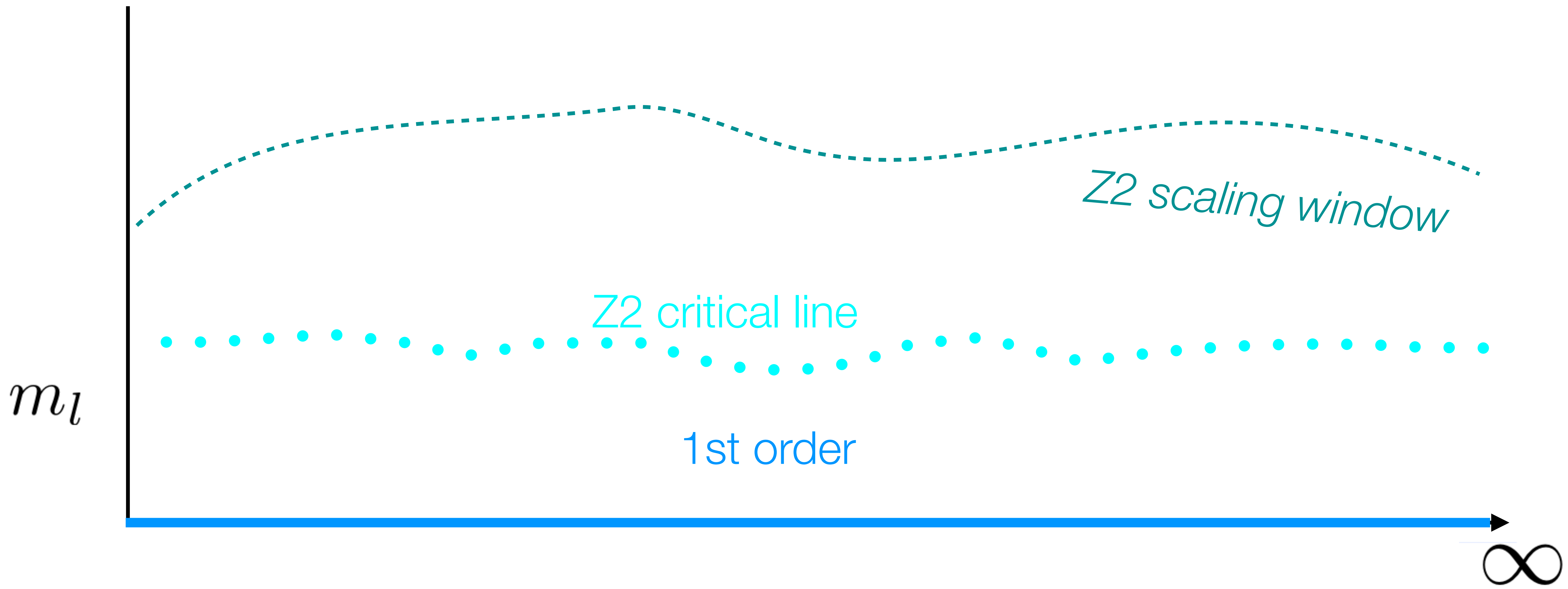
- $T_{\text{CEP}} < T_{\text{Tri}} < T_c \Rightarrow T_c$ puts upper bound on T_{CEP}



[F. Karsch, 1905.03936]

$m_l \neq 0$, possible scenarios





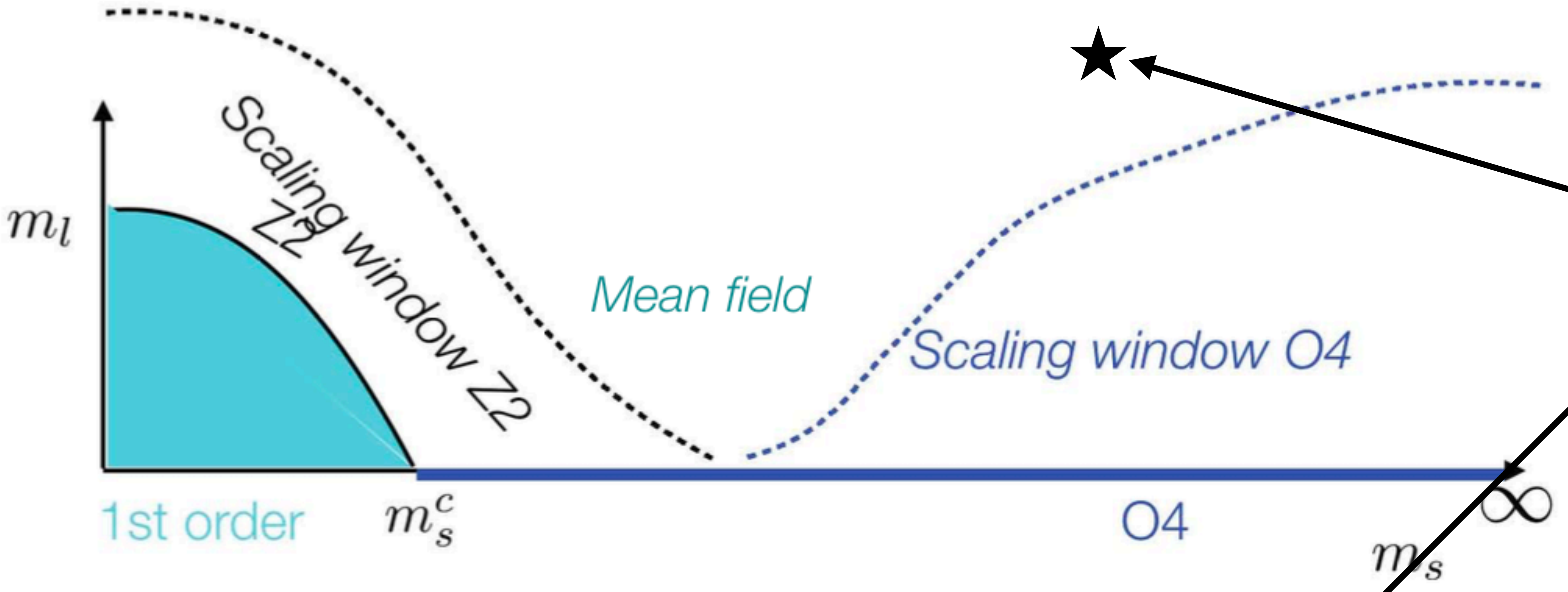
Scaling window:

universal behaviour

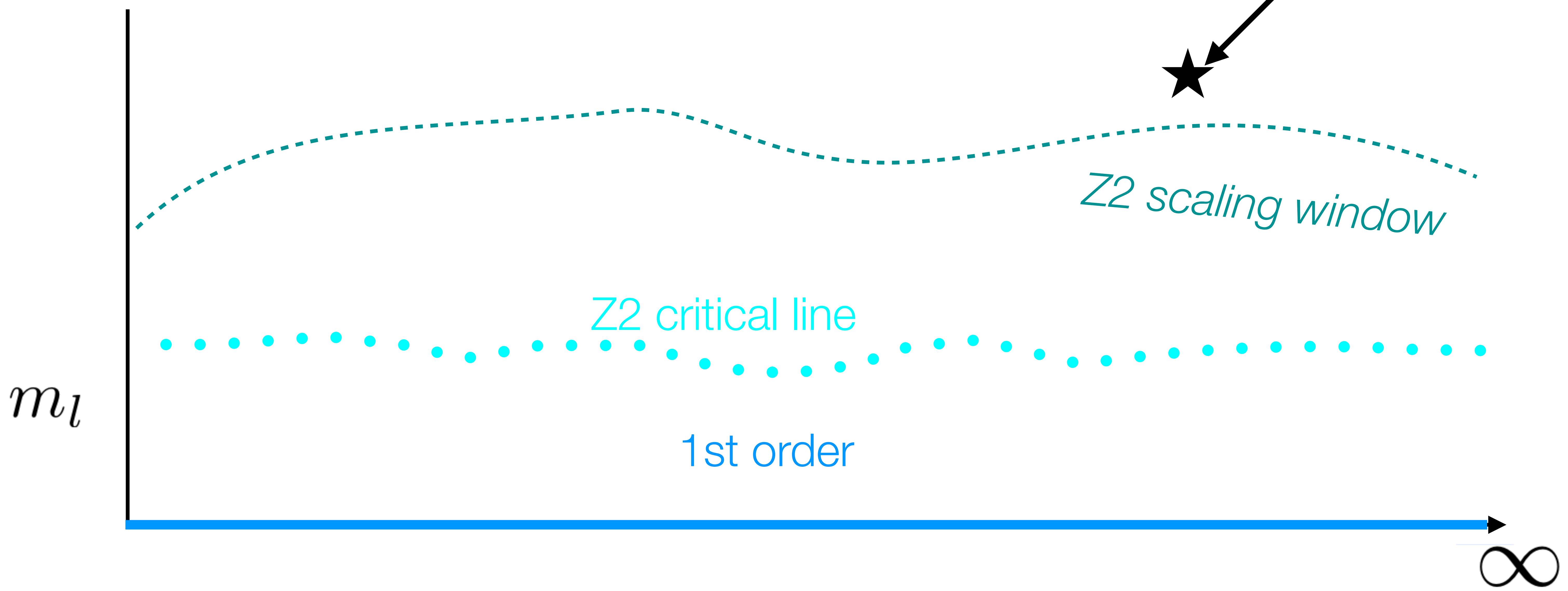
given by EoS

$M = m^{1/\delta} f(t/m^{1/\beta\delta})$ + regular terms

$m_l \neq 0$, possible scenarios



Physical point ?



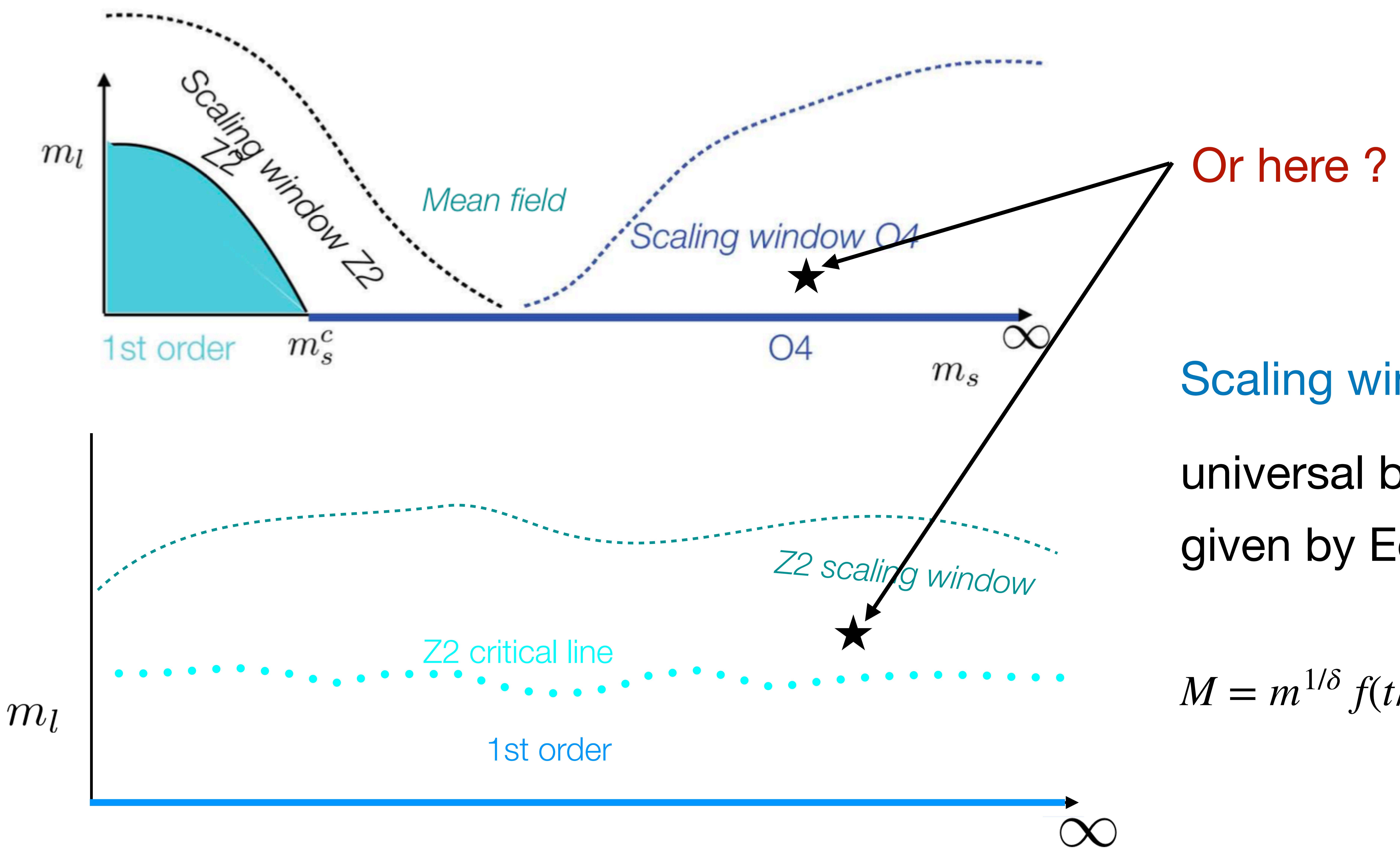
Scaling window:

universal behaviour

given by EoS

$M = m^{1/\delta} f(t/m^{1/\beta\delta})$ + regular terms

$m_l \neq 0$, possible scenarios



Scaling window:

universal behaviour

given by EoS

$M = m^{1/\delta} f(t/m^{1/\beta\delta})$ + regular terms

TWEXT (Twisted Wilson @ EXTreme) Collaboration

Lattice details

- 2+1+1 Wilson twisted mass fermions at maximal twist

  automatically $O(a)$ improved

  [R. Frezzotti, G. Rossi, 2004]

- Heavy quarks (c, s): close to the physical values

- $m_\pi \in [135, 370]$ MeV

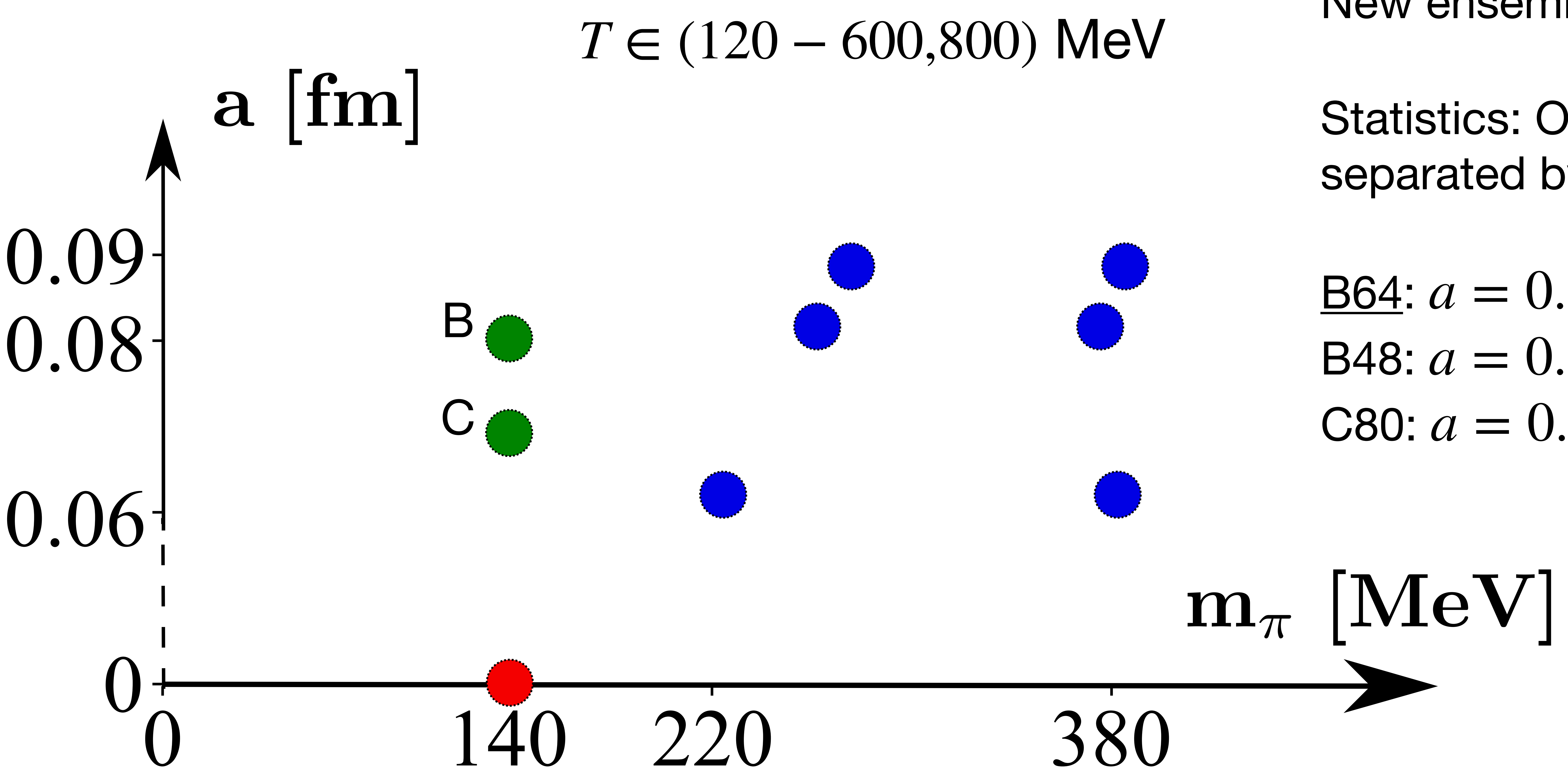
- Fixed scale approach: $a =$ fixed, $T \leftrightarrow N_t$

- Based on ETMC $T = 0$ parameters & tmLQCD code

  [C. Alexandrou et al.,2018][C. Alexandrou et al.,2021]

TWEXT (Twisted Wilson @ EXTreme) Collaboration

Ensemble summary



New ensembles @ $m_\pi = m_\pi^{\text{phys}}$:

Statistics: O(300) config's, separated by several MC traj's

<u>B64</u>: $a = 0.080$ fm, $L = 64a \approx 5.1$ fm
B48: $a = 0.080$ fm, $L = 48a \approx 3.8$ fm
C80: $a = 0.069$ fm, $L = 80a \approx 5.5$ fm

Chiral phase transition & novel order parameter

Novel order parameter

Novel order parameter

- Chiral condensate $\langle\bar{\psi}\psi\rangle$, to be renormalised ($m/a^2$ divergences)

Novel order parameter

- Chiral condensate $\langle\bar{\psi}\psi\rangle$, to be renormalised ($m/a^2$ divergences)
  - Although not important in the fixed scale approach

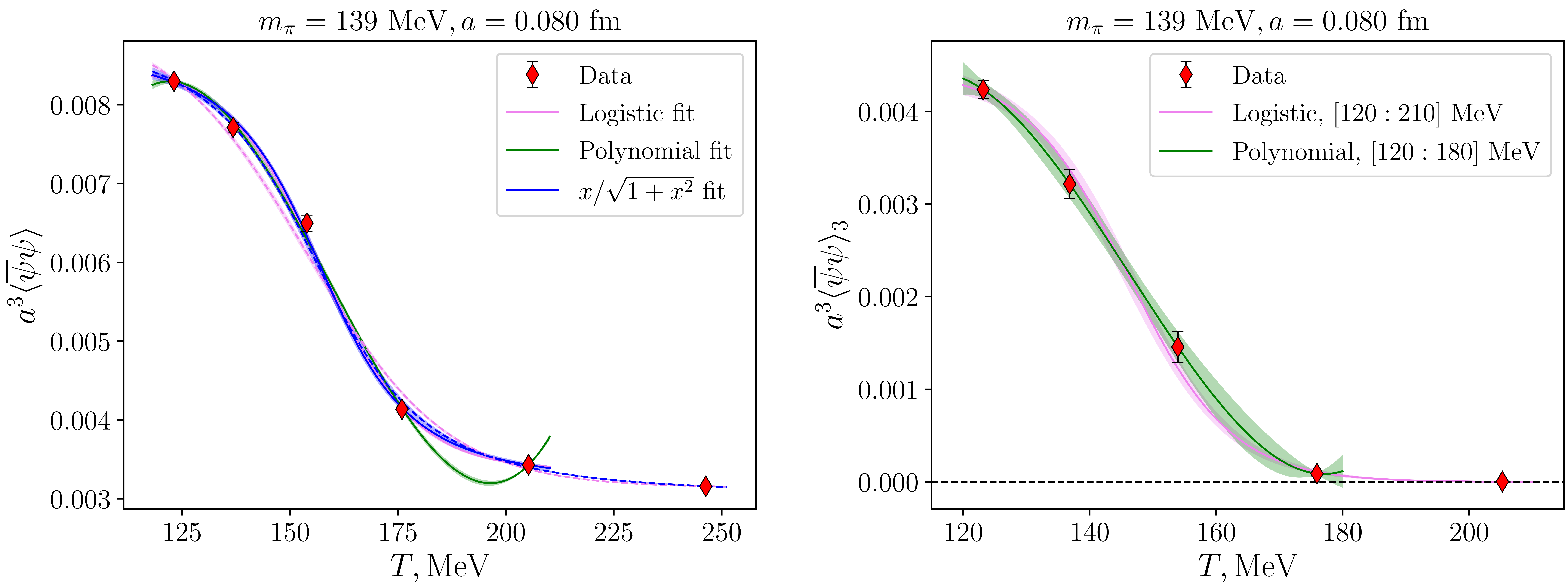
Novel order parameter

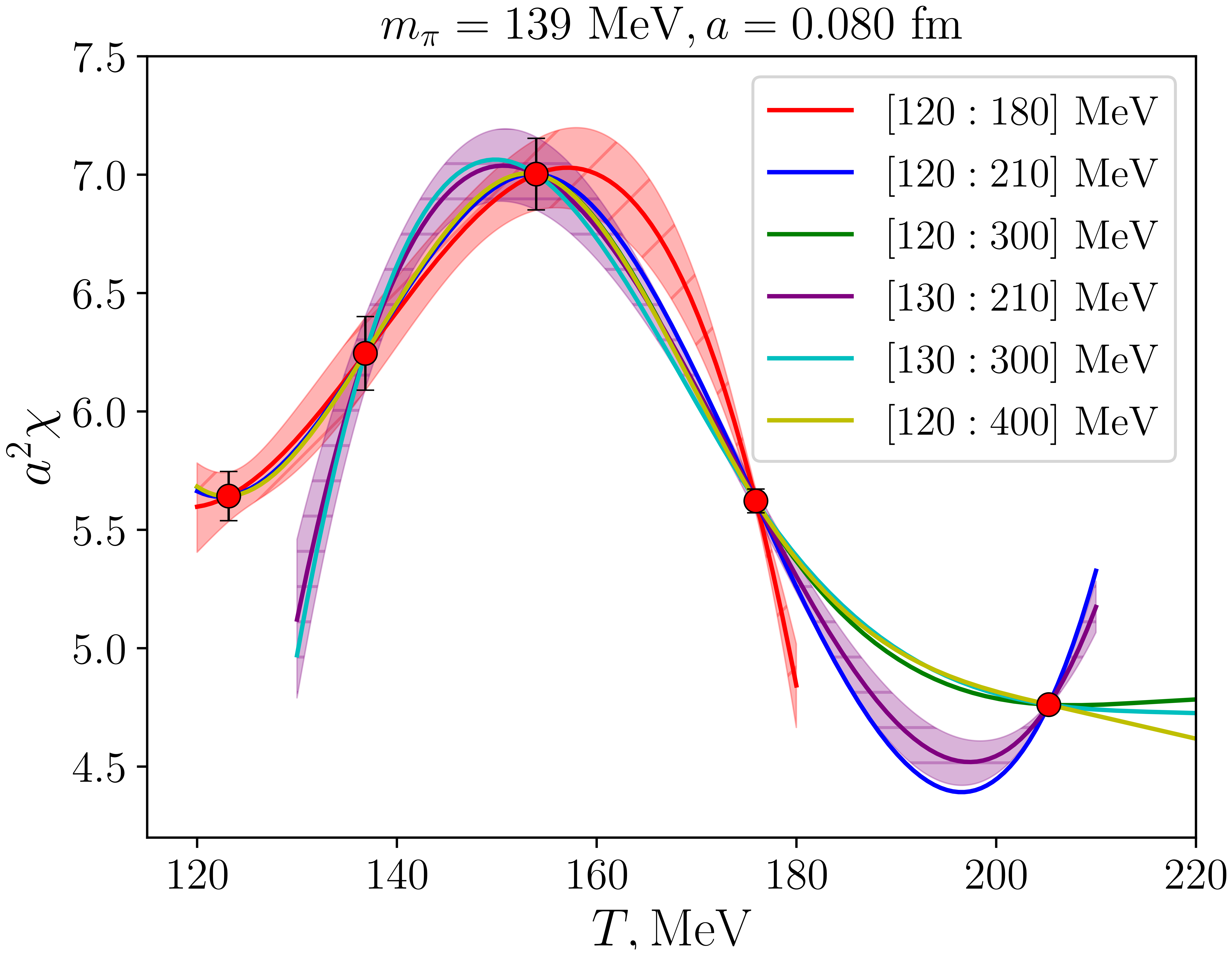
- Chiral condensate $\langle\bar{\psi}\psi\rangle$, to be renormalised ($m/a^2$ divergences)
  - Although not important in the fixed scale approach
- Chiral susceptibility $\chi = \partial\langle\bar{\psi}\psi\rangle/\partial m$

Novel order parameter

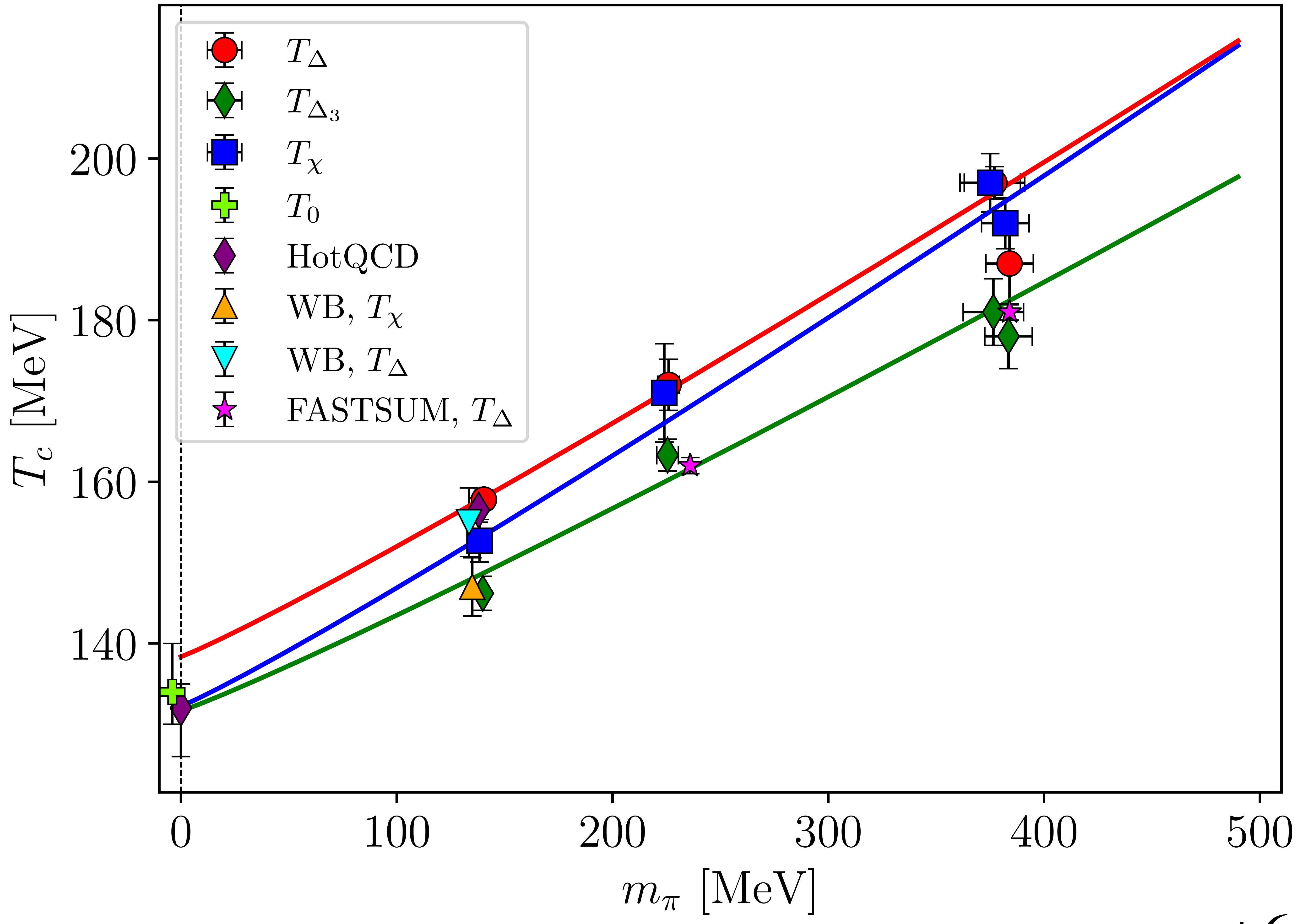
- Chiral condensate $\langle\bar{\psi}\psi\rangle$, to be renormalised ($m/a^2$ divergences)
  - Although not important in the fixed scale approach
- Chiral susceptibility $\chi = \partial\langle\bar{\psi}\psi\rangle/\partial m$
- Novel order parameter: $\langle\bar{\psi}\psi\rangle_3 = \langle\bar{\psi}\psi\rangle - m\chi$
  - $1/a^2$ divergences cancel
  - ~m^3 (symmetric phase, large T)

[W.Unger, 2010]


$m_\pi = 139$ MeV$, a = 0.080$ fm
Data
Logistic fit
Polynomial fit
$x/\sqrt{1+x^2}$ fit
$a^3\langle\overline{\psi}\psi\rangle$
0.008
0.007
0.006
0.005
0.004
0.003
125
150
175
200
225
250
T, MeV
$m_\pi = 139$ MeV$, a = 0.080$ fm
Data
Logistic, [120 : 210] MeV
Polynomial, [120 : 180] MeV
$a^3\langle\overline{\psi}\psi\rangle_3$
0.004
0.003
0.002
0.001
0.000
120
140
160
180
200
T, MeV


$m_\pi = 139$ MeV, $a = 0.080$ fm
[120 : 180] MeV
[120 : 210] MeV
[120 : 300] MeV
[130 : 210] MeV
[130 : 300] MeV
[120 : 400] MeV
$a^2\chi$
T, MeV
7.5
7.0
6.5
6.0
5.5
5.0
4.5
120
140
160
180
200
220

Critical temperature and the chiral limit



	$T(m_\pi = 139$ MeV) [MeV]	$T(m_\pi = 0)$ [MeV]
$\langle\bar\psi\psi\rangle$	157.8(12)	138(2)
χ	153(3)	132(4)
$\langle\bar\psi\psi\rangle_3$	146(2)	132(3)

[AYuK, M.P. Lombardo, A. Trunin, 2021]

$T_0 = 134^{+6}_{-4}$ MeV

$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}, O(4)$

Scaling behaviour

Novel order parameter

Novel order parameter

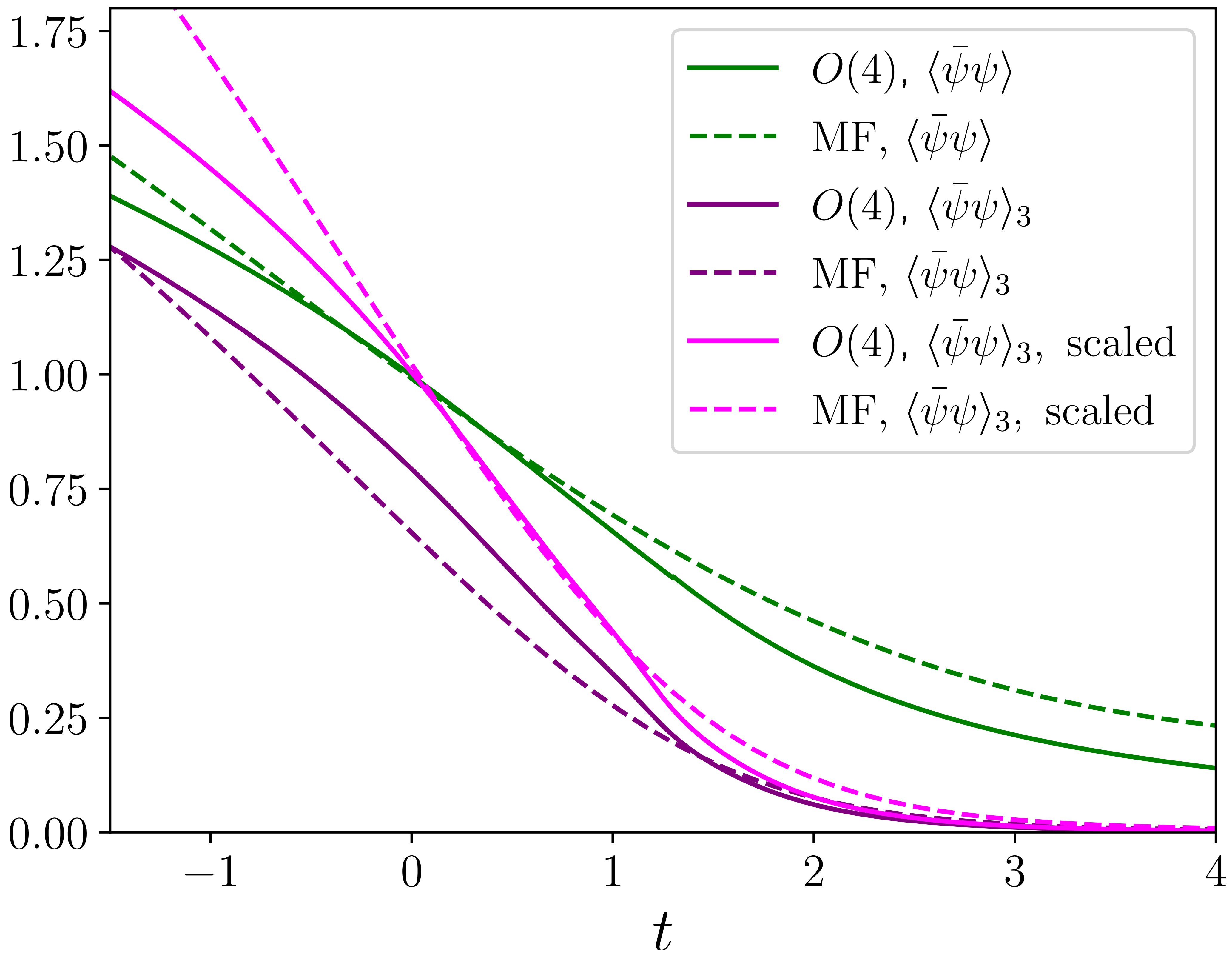
- Novel order parameter: $\langle\bar{\psi}\psi\rangle_3 = \langle\bar{\psi}\psi\rangle - m\chi$
- Suppress non-universal (linear in m) terms

Novel order parameter

- Novel order parameter: $\langle\bar{\psi}\psi\rangle_3 = \langle\bar{\psi}\psi\rangle - m\chi$
- Suppress non-universal (linear in m) terms
- Assume some (e.g., $O(4)$) universality class

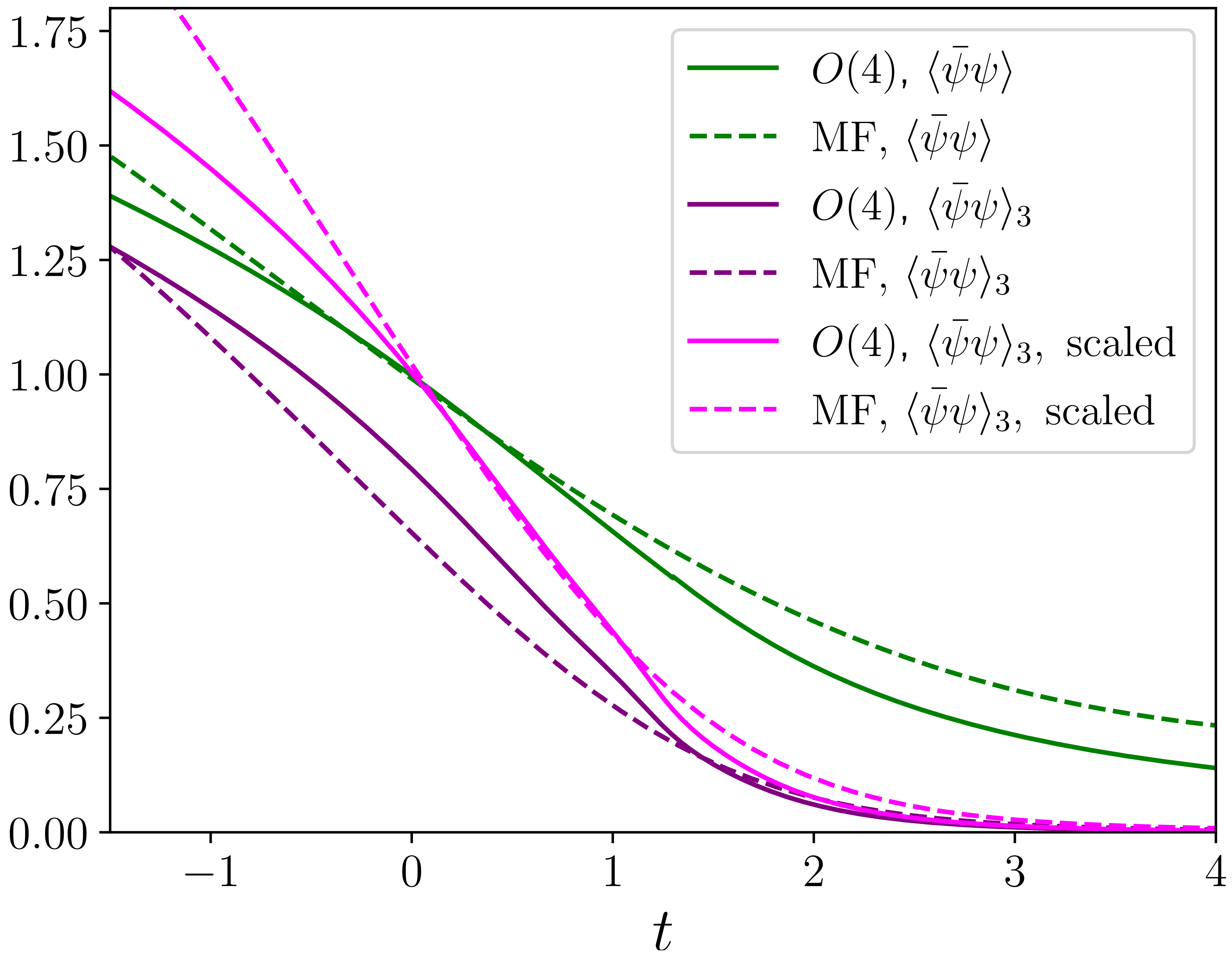
  as $m \to 0$

  - $\langle\bar{\psi}\psi\rangle_3 \sim t^{-\gamma-2\beta\delta}$ as $t \to \infty$
  - $\langle\bar{\psi}\psi\rangle \sim t^{-\gamma}$ as $t \to \infty$



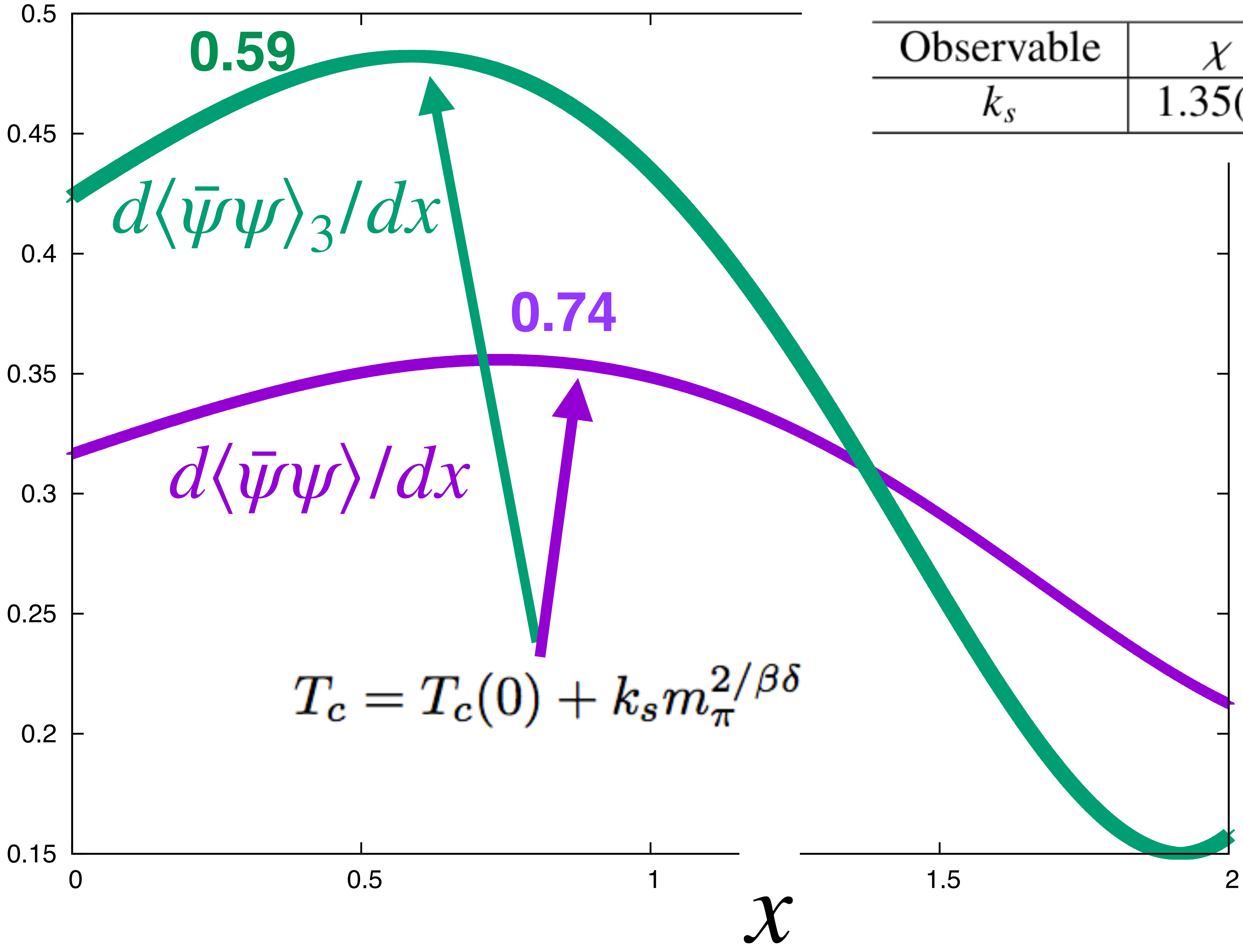
Novel order parameter

- Novel order parameter: $\langle\bar{\psi}\psi\rangle_3 = \langle\bar{\psi}\psi\rangle - m\chi$
- Suppress non-universal (linear in m) terms
- Assume some (e.g., $O(4)$) universality class as $m \to 0$
  - $\langle\bar{\psi}\psi\rangle_3 \sim t^{-\gamma-2\beta\delta}$ as $t \to \infty$
  - $\langle\bar{\psi}\psi\rangle \sim t^{-\gamma}$ as $t \to \infty$
- If $\langle\bar{\psi}\psi\rangle_3 < 0$: possibly first order, or closeness to the first order phase transition (Z_2 scenario ?)

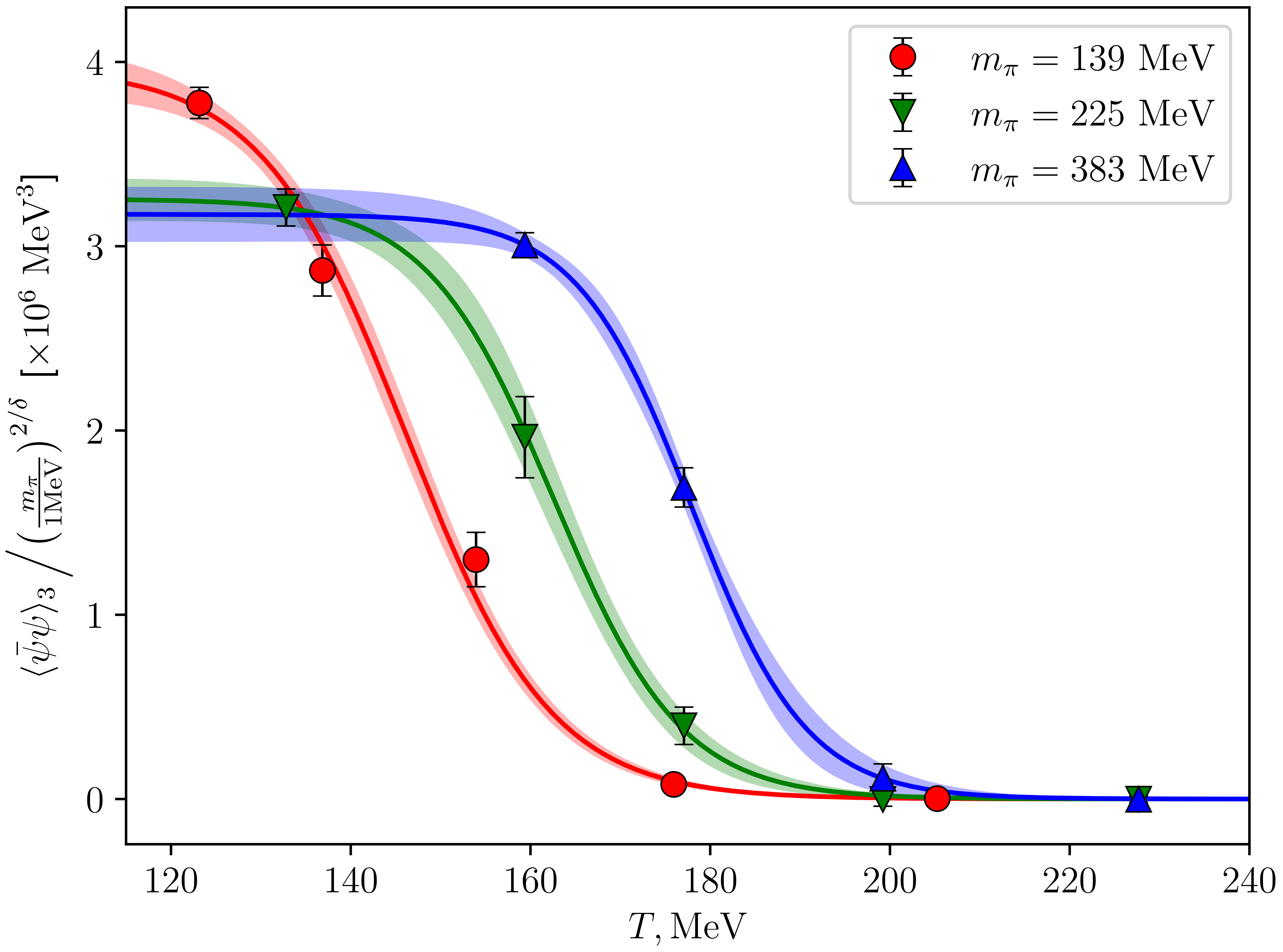


Scaling of T_c with pion mass

Observable	χ	$\bar{\psi}\psi$	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)



Simple estimation of T_0 from EOS



Prediction of EoS:

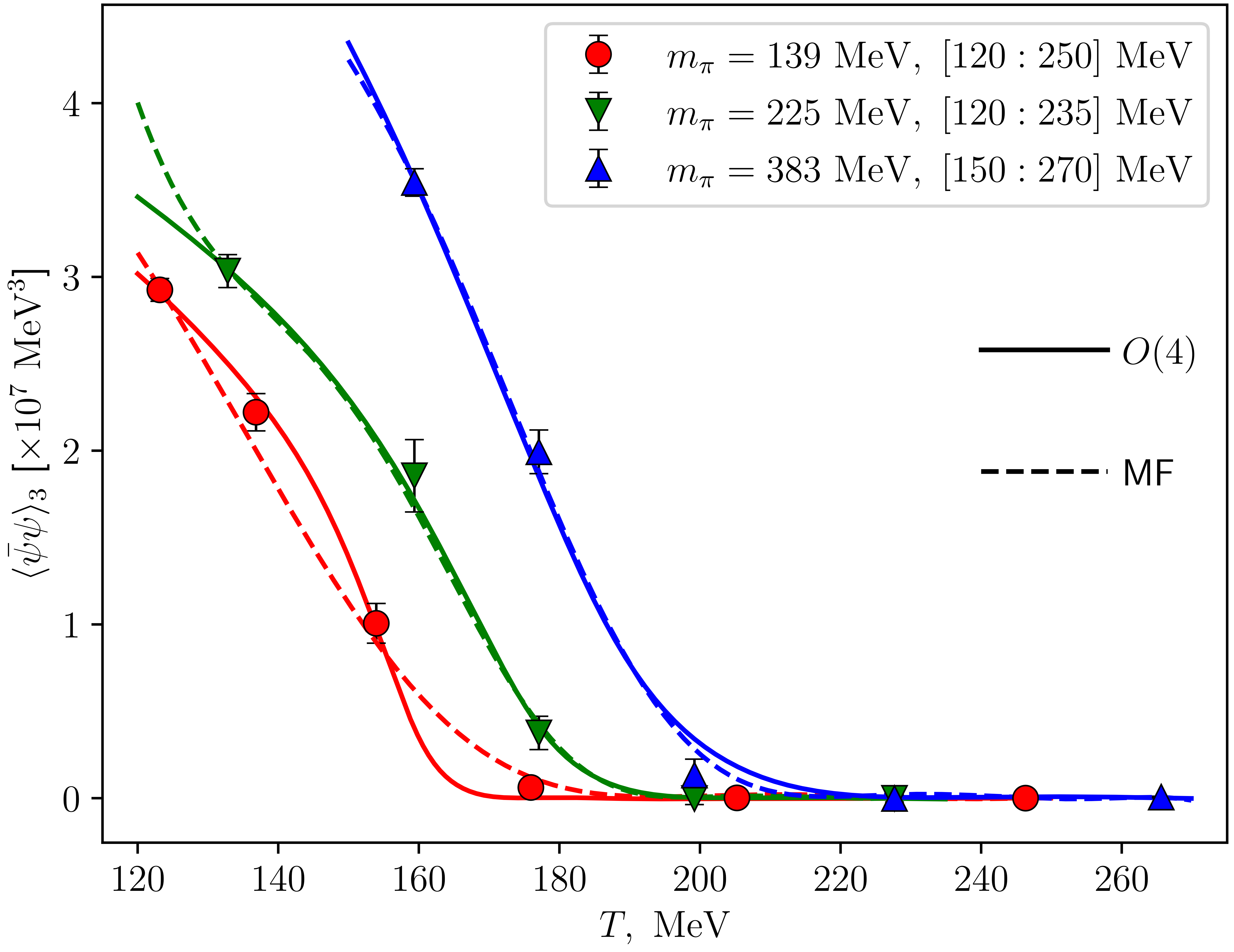
$$\frac{\langle\bar\psi\psi\rangle_3}{m^{1/\delta}} \sim \frac{\langle\bar\psi\psi\rangle_3}{m_\pi^{2/\delta}} = \text{const}$$

at

$$T = T_0(m_\pi = 0) = 138(2)\ \text{MeV}$$

$$M = h^{1/\delta} f(t/h^{1/\beta\delta}) + \text{regular terms}$$

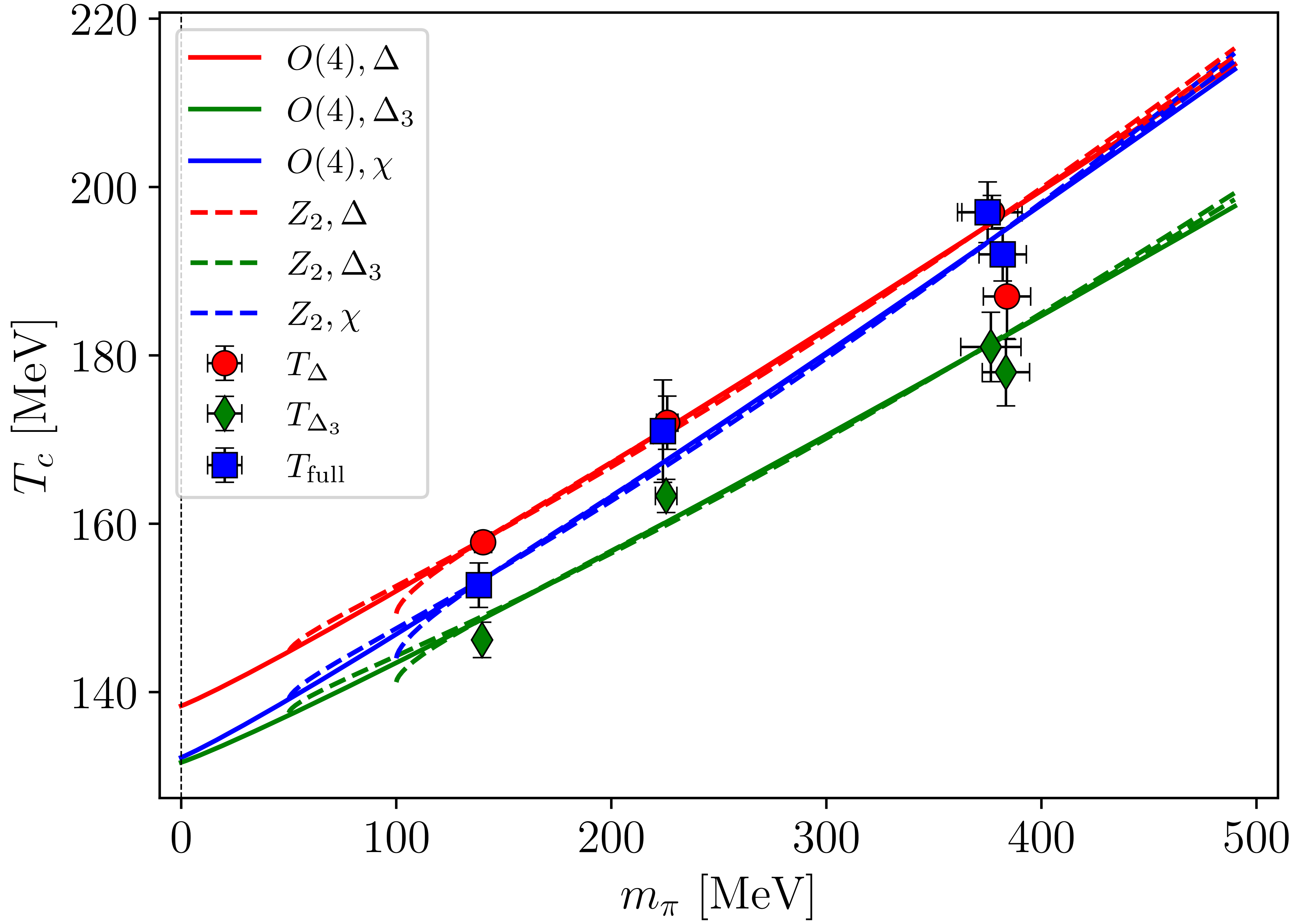
O(4) vs mean field



Mild tension between data and MF for m_π=139 MeV

m_π [MeV]	T_0 [MeV]
139	142(2)
225	159(3)
383	174(2)

Z_2 vs O(4) scaling



O(4) scaling:

Observable	T_0 [MeV]	$z_p/z_{\bar\psi\psi_3}$	$z_p/z_{\bar\psi\psi_3}$ $O(4)$	z_p $O(4)$
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar\psi\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar\psi\psi\rangle_3$	132(3)	1	1	0.55(1)

Z_2 scaling:

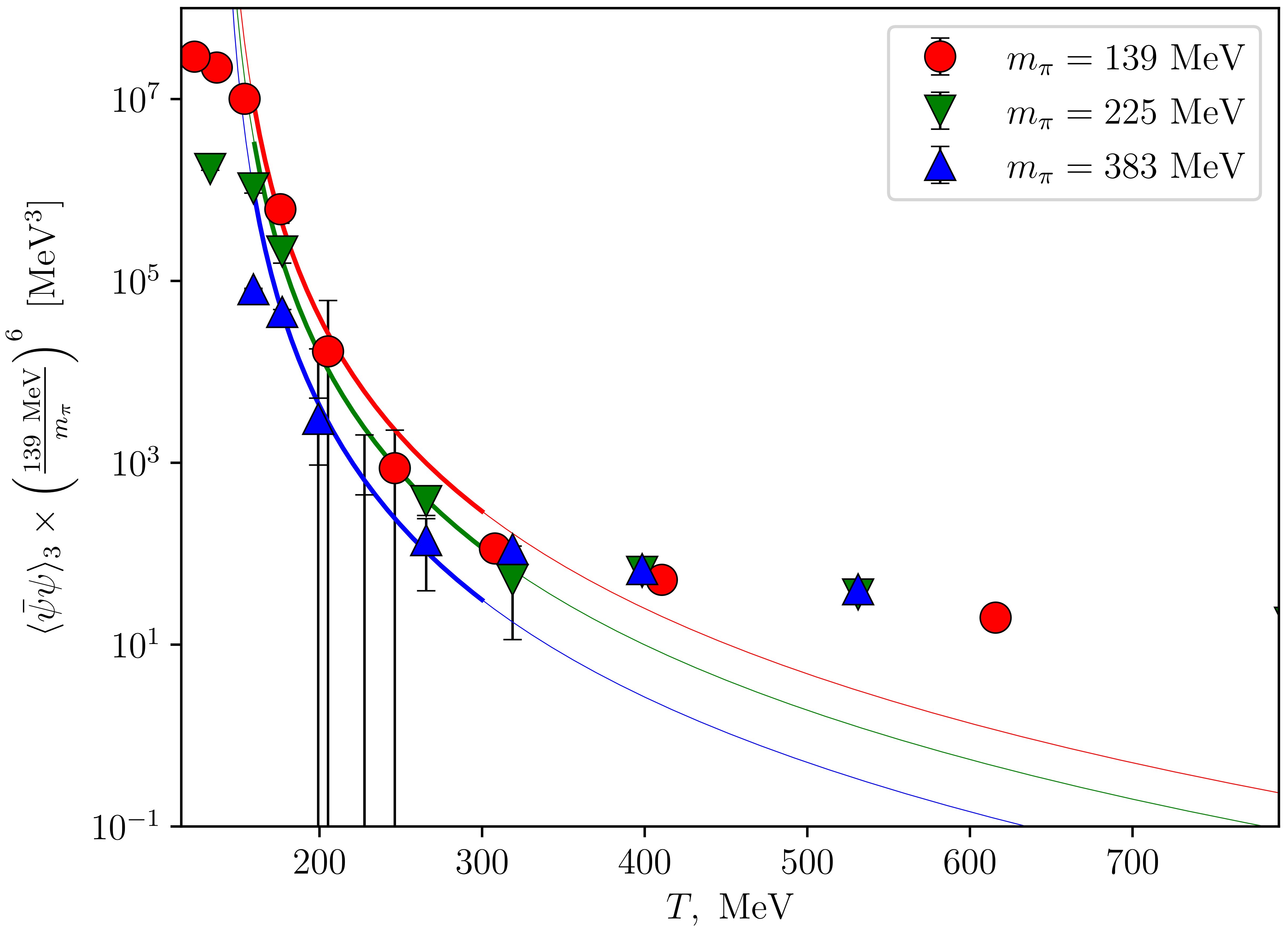
$m_\pi^c = 100$ MeV is still ok

$m_\pi^c = 0$ MeV is indistinguishable from O(4)

$T_0 = T_c(m_\pi \to 0) = 134^{+6}_{-4}$ MeV

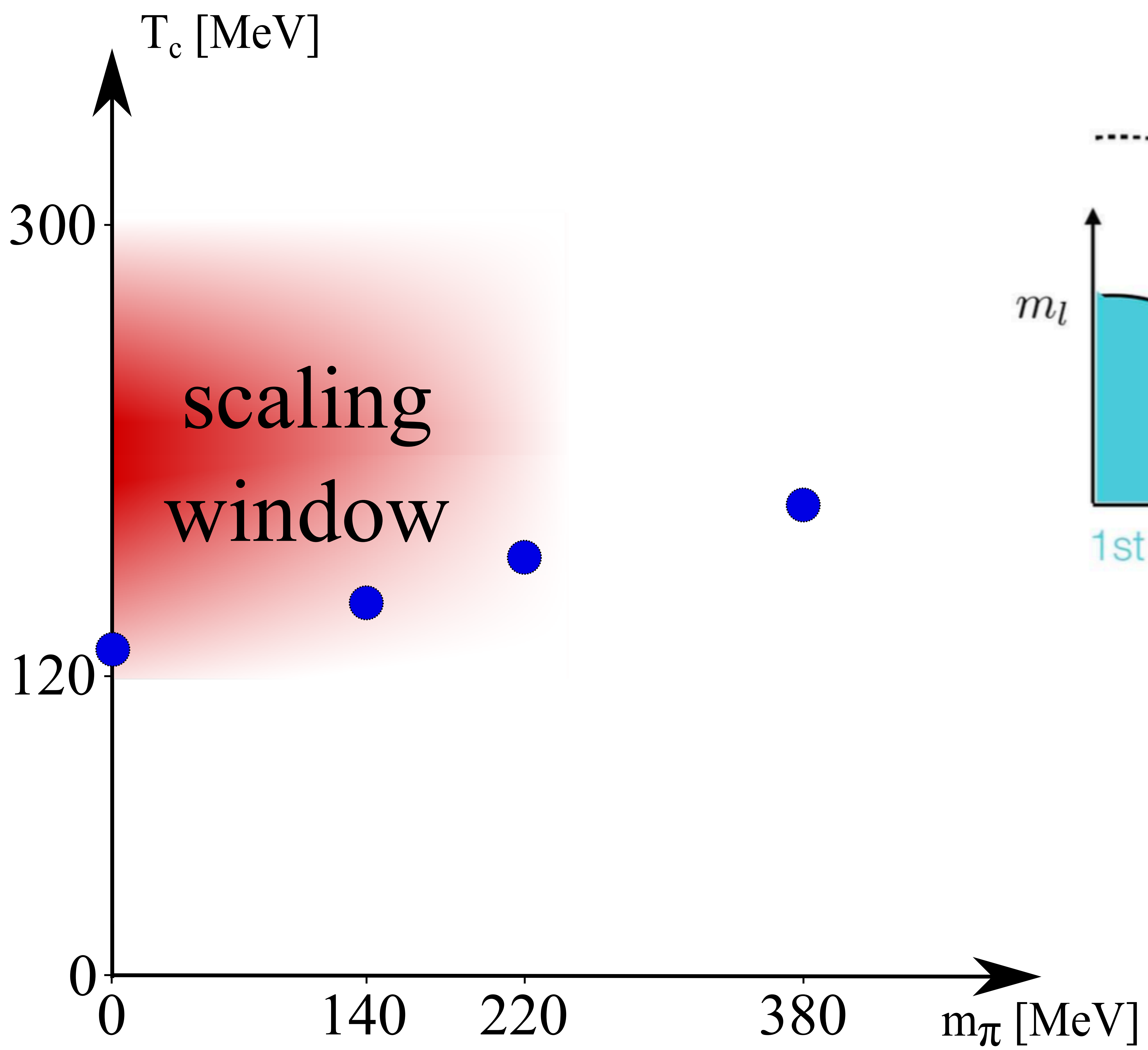
$T_c = T_c(0) + k_s(m - m_c)^{1/\beta\delta}, m \sim m_\pi^2$

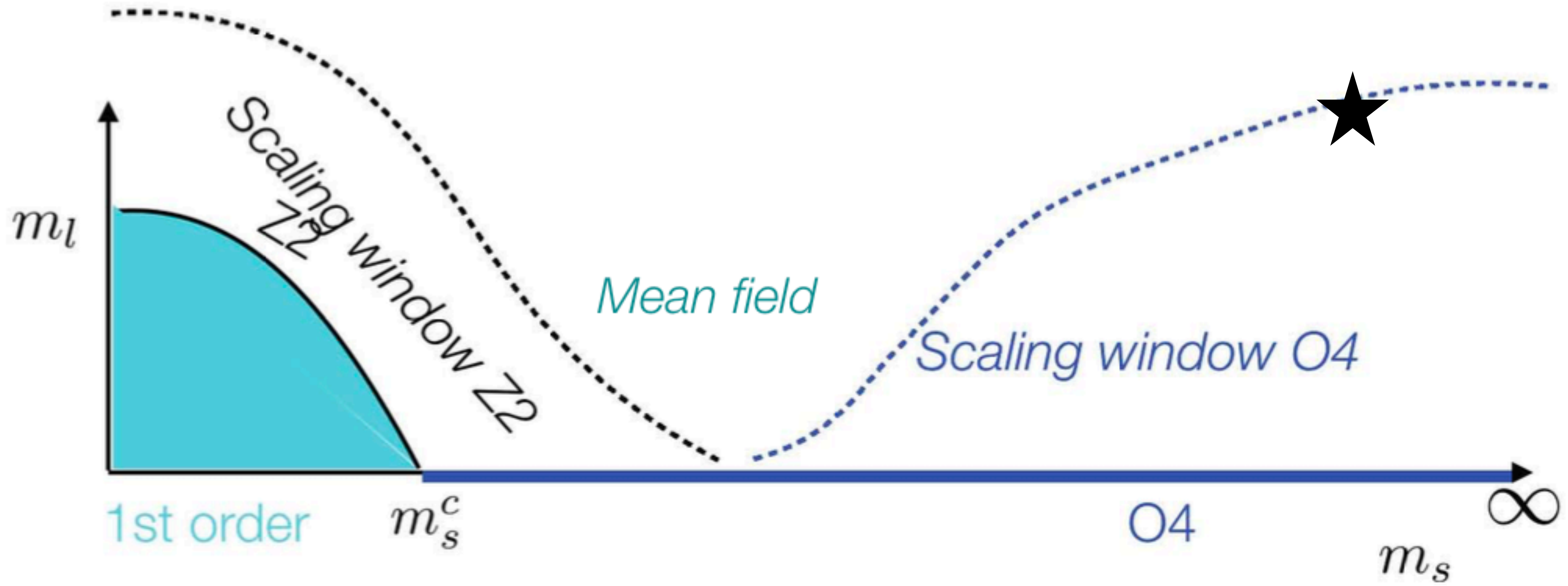
Large temperature behaviour



- O(4): $\langle\bar{\psi}\psi\rangle_3 \sim t^{-\gamma-2\beta\delta}$
- Griffith analyticity: $\langle\bar{\psi}\psi\rangle_3 \sim m^3 \sim m_\pi^6$
- $T \sim 300$ MeV

Scaling window





FRG: Tiny scaling window ($m_\pi < 1$MeV) ?

[J. Braun et al., 2020]

Threshold at $T = 300$ MeV and topology

Method to measure topological susceptibility

QCD and topology, finite temperature

$$\chi_{top} = \langle Q_{top} \rangle^2 / V = m_l^2 \chi_{5,disc}$$

$$m \int d^4x \bar{\psi}\gamma_5\psi = Q_{top}$$

[Kogut, Lagae, Sinclair, 1998]

$SU_L(2) \times SU_R(2)$

$\chi_{5,con}$ $\quad \pi : \bar{\psi}\gamma_5 \frac{\tau}{2}\psi$ ⟷ $\sigma : \bar{\psi}\psi$ $\quad \chi_{con} + \chi_{disc}$

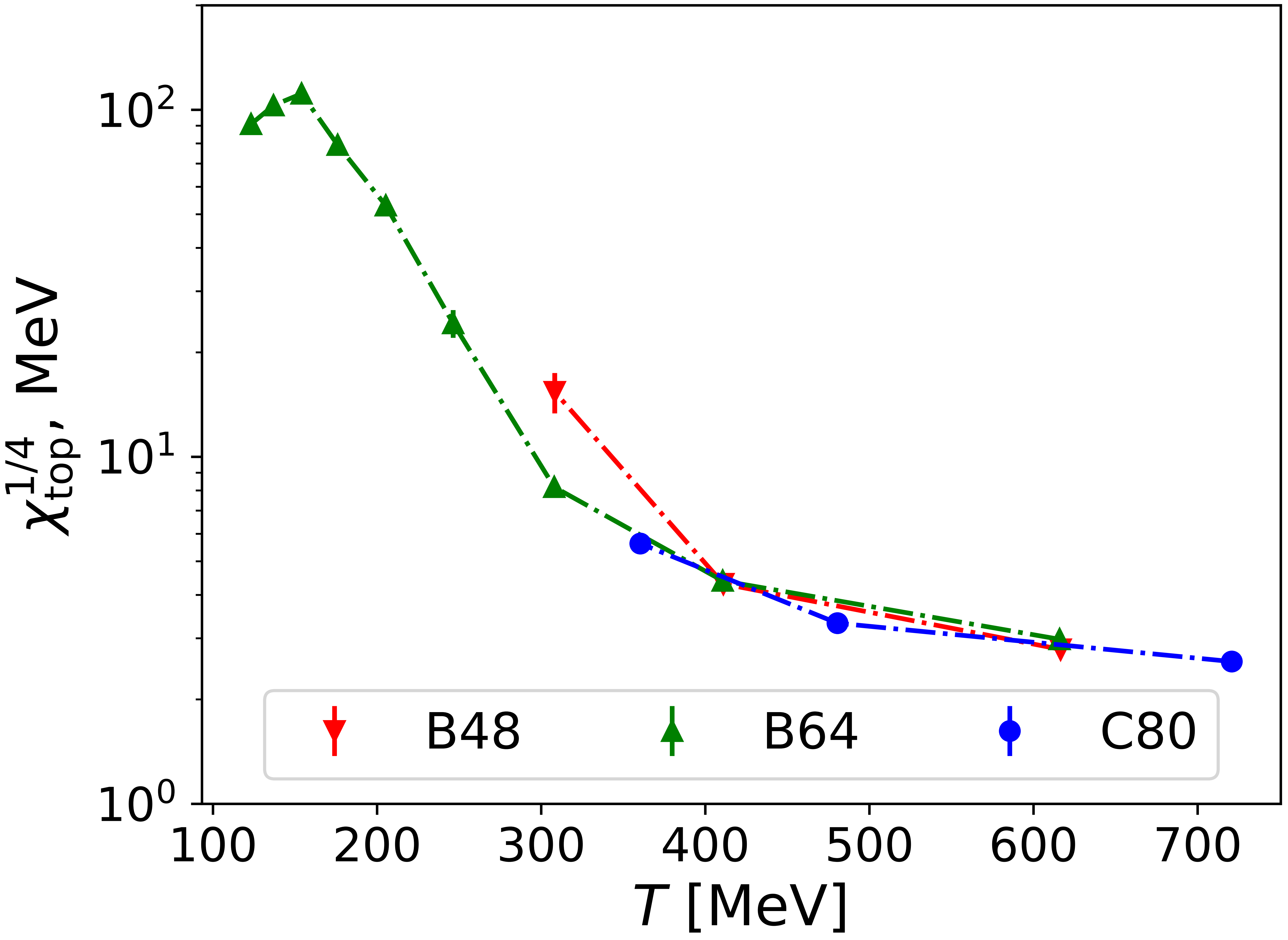
$U_A(1)$ ↕ $\qquad$ ↕ $U_A(1)$

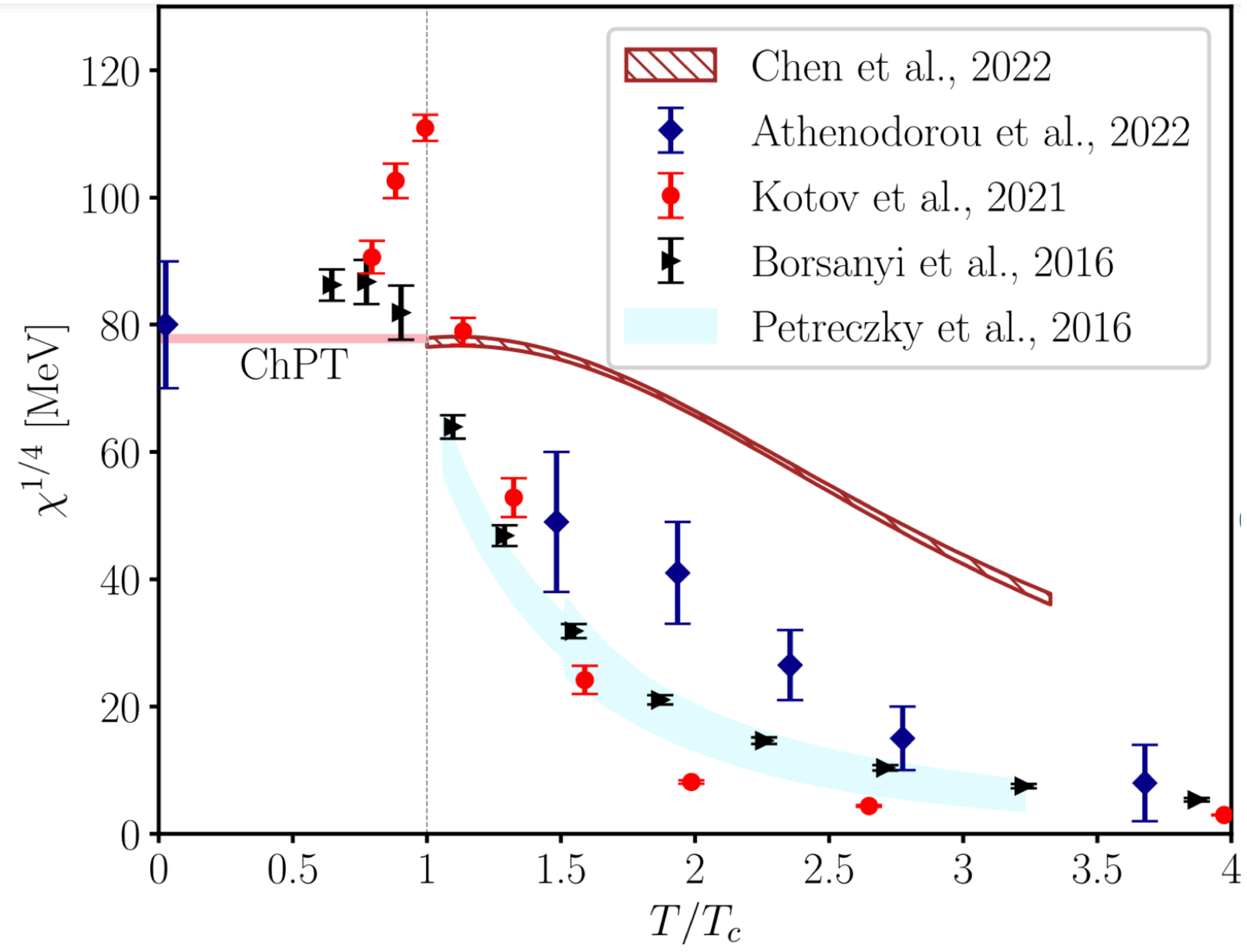
χ_{con} $\quad \delta : \bar{\psi}\frac{\tau}{2}\psi$ ⟷ $\eta : \bar{\psi}\gamma_5\psi$ $\quad \chi_{5,con} - \chi_{5,disc}$

$SU_L(2) \times SU_R(2)$

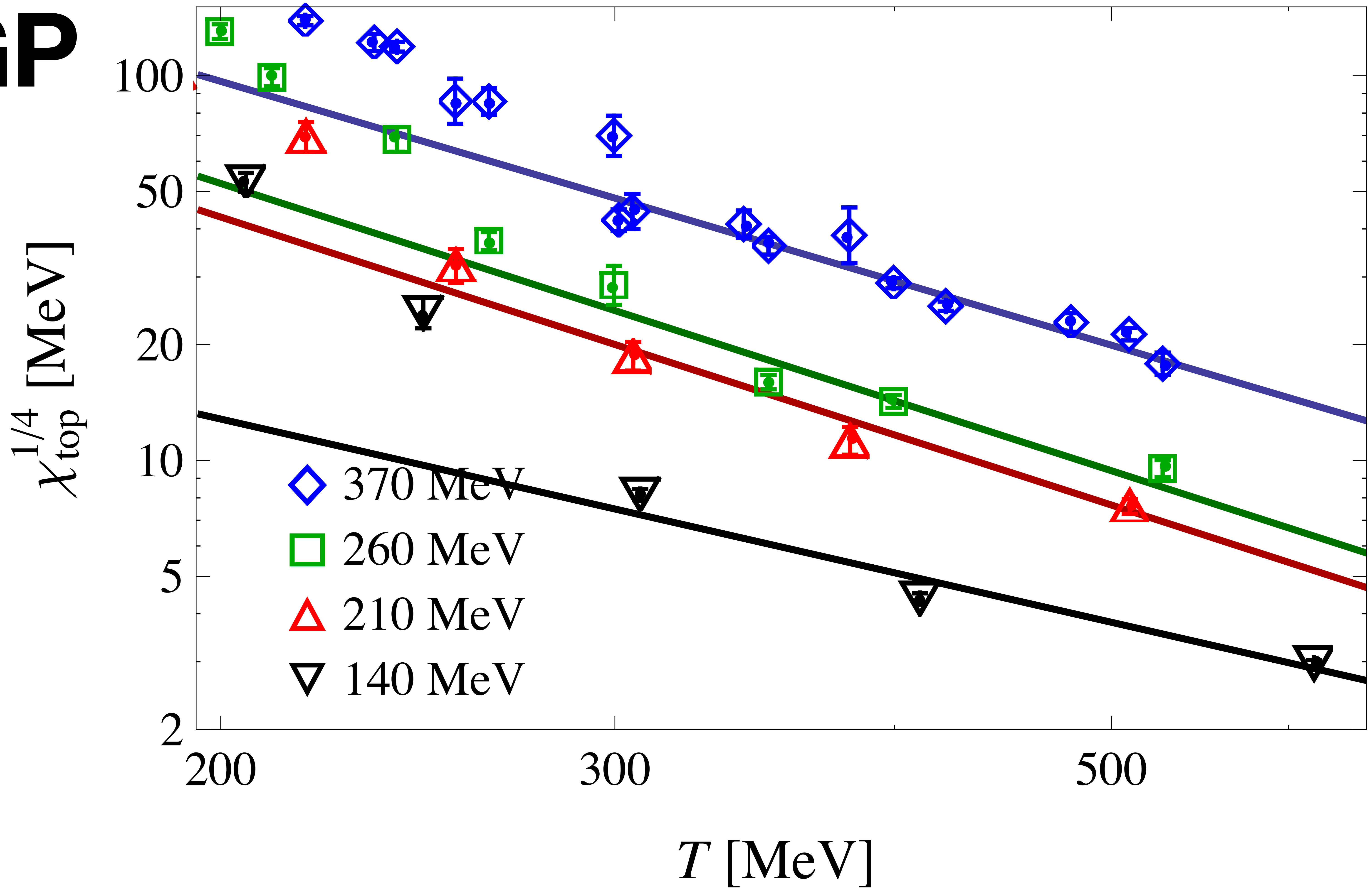
$\chi_\pi - \chi_\delta = \chi_{disc} = \chi_{5,disc}$, for $T \geq T_C, m_l \to 0$ $\quad \Longrightarrow \chi_{top} = \langle Q_{top} \rangle^2 / V = m_l^2 \chi_{disc}$

Plot by C. Bonanno





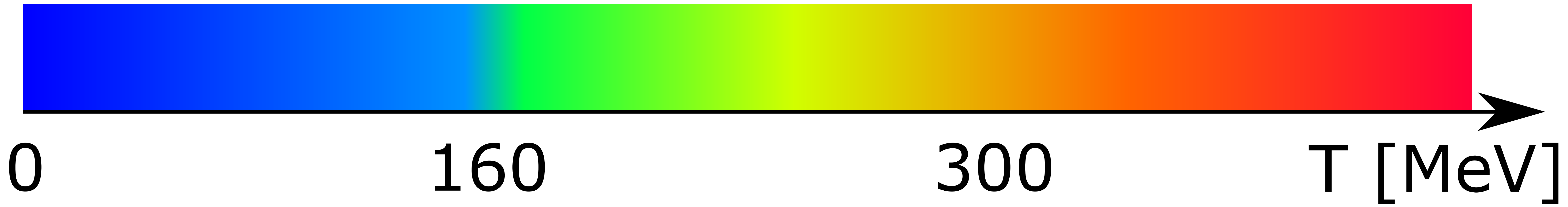
Threshold in QGP



$T \sim 300$ **MeV**

- Onset of DIGA behaviour

$$\chi \sim T^{-d}$$



Conclusions

- Thermal QCD with Wilson twisted mass fermions
- $\langle \bar{\psi}\psi \rangle_3 = \langle \bar{\psi}\psi \rangle - m\chi$ for critical/scaling phenomena
- $T_0 = 134^{+6}_{-4}$ MeV in the chiral limit $m \to 0$
- Consistent with $O(4)$ scaling for $m_\pi \lesssim 140$ MeV, $T \in [120,\ 300]$ MeV
- $T \sim 300$ MeV: indications of threshold in QGP