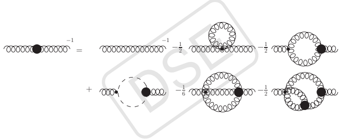
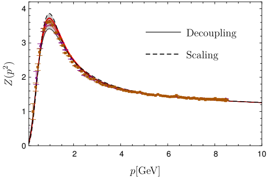
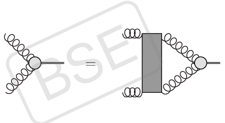
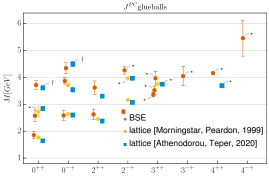
Glueballs in the DSE/BSE framework









Markus Q. Huber

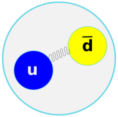
Institute of Theoretical Physics

Giessen University

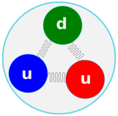
In collaboration with
Christian S. Fischer, Hèlios Sanchis-Alepuz:
Eur.Phys.J.C 80, arXiv:2004.00415
Eur.Phys.J.C 80, arXiv:2110.09180
vConf21, arXiv:2111.10197
HADRON2021, arXiv:2201.05163

Seminar Theoretical Hadron Physics, Giessen, Germany, Jan. 26, 2022

Bound states in QCD



Mesons



Baryons

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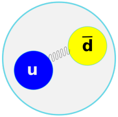

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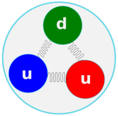

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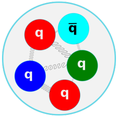
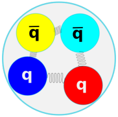
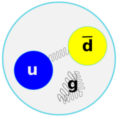


Pentaquarks — First observations 2015 (LHCb)



Tetraquarks — Increasing number of confirmed states. Bound state equations perspective: [Eichmann, Fischer, Heupel, Santowsky, Wallbott ’20]



Hybrids

Glueballs — States of pure ’radiation’

Glueball observations

Experimental candidates, but situation not conclusive.

Scalar glueball candidates (0^{++}): f_0 states

Mixing with scalar isoscalar mesons $\rightarrow$ states have glue and quark content.

Candidate reaction: $J/\psi \rightarrow \gamma + 2g$

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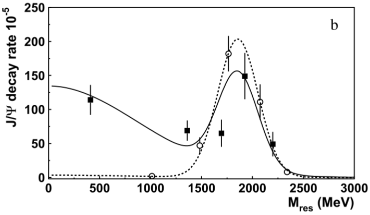
Recent analysis of BESIII data [Sarantsev, Denisenko, Thoma, Klempt '21]:

$$M = 1865 \pm 25^{+10}_{-30}\,\text{MeV},$$
$$\Gamma = 370 \pm 50^{+30}_{-20}\,\text{MeV}$$



Glueball candidates

Lightest glueballs: 0^{++}, 0^{-+}, 2^{++}

- Favored candidates (largest glueball content) for scalar glueball from BESIII data analysis:
  $f_0(1710)$ [Rodas et al. '21], $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt '21]
- Pseudoscalar glueball (0^{-+}) expected around 2.5 GeV:
  η states experimentally not well known
- Tensor glueball (2^{++}) expected around 2.5 GeV:
  f_2-like states experimentally not well known

Glueball calculations

No quarks (no mixing) → Yang-Mills theory

- “Isolated” problem: only gluons
- Clean picture: well-established lattice results

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On the lattice [Gregory et al. ’12]:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360\,\text{MeV}$
- Tiny (e.g., 0^{++}, 2^{++}) to moderate unquenching effects (e.g., 0^{-+}) found

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No quarks (no mixing) $\to$ Yang-Mills theory

- "Isolated" problem: only gluons
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- Functional methods: High quality input available for bound state equations

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Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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$$\boldsymbol{D}(\boldsymbol{x}-\boldsymbol{y}) = \langle O(x)O(y)\rangle$$

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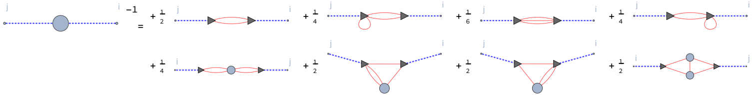
- Functional approach:
  Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawlowski '19]



+ three-loop diagrams

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  Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. → Each can have a pole at the glueball mass.

A^4-part of $D(x-y)$, total momentum on-shell:

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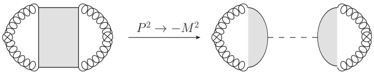
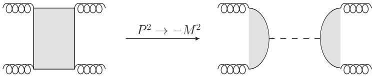
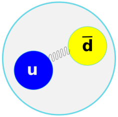
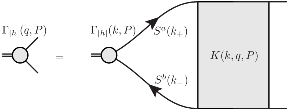




Hadrons from bound state equations

Example: Meson



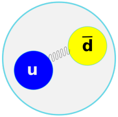


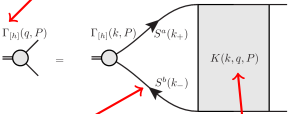
Integral equation: $\Gamma(q,P) = \int dk\, \Gamma(k,P)\, S(k_+)\, S(k_-)\, K(k,q,P)$

Hadrons from bound state equations

Bethe-Salpeter amplitude

Example: Meson





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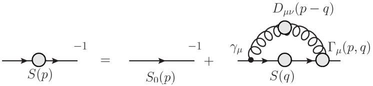
Ingredients:

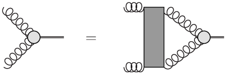
- Quark propagator S



Nonperturbative diagram: full momentum dependent dressings $\to$ numerical solution

- Interaction kernel K
- Constrained by symmetries

Glueball BSE



Need  and , solve for . → Mass

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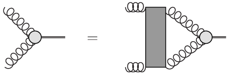
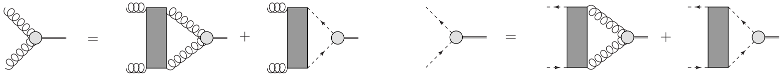


Need  and , solve for . $\rightarrow$ Mass

Not quite. . .

Glueball BSE



Gluons couple to ghosts → Include 'ghostball'-part.

(First step: no quarks
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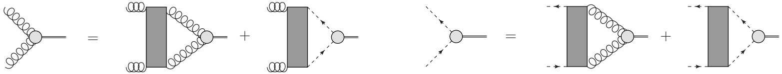

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Need ,  and 4× , solve for  and . → Mass

Construction of kernel

Consistency with input: Apply same construction principle.

Previous BSE calculations of glueballs

Partially using fits/models as input:

- [Meyers, Swanson '13]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal '15]
- [Souza et al. '20]
- [Kaptari, Kämpfer '20]

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→ Either overconstrained or quantitative problems.

→ Qualitative sensitivity to input in contrast to quark sector.

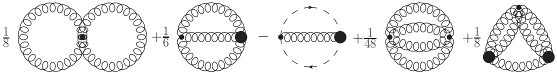
Kernel construction

From 3PI effective action truncated to three-loops:

[Fukuda '87; McKay, Munczek '89; Sanchis-Alepuz, Williams '15; MQH, Fischer, Sanchis-Alepuz '20]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{int,3l}[\Phi, D, \Gamma^{(3)}]$$

$\Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] =$



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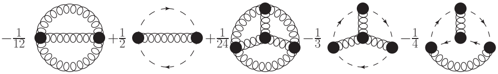

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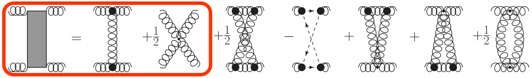
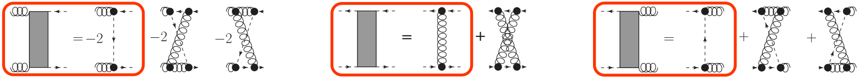
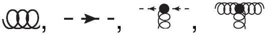


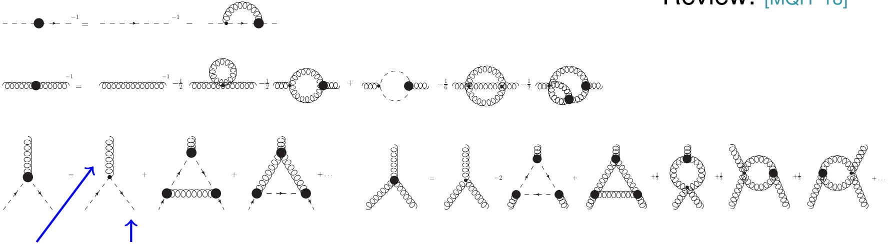


$\rightarrow$ Need .

- Some diagrams vanish for certain quantum numbers.
- Full QCD: Same for quarks $\rightarrow$ Mixing with mesons.

Equations of motion from 3-loop 3PI effective action

Review: [MQH '18]



Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

Self-contained system of equations with the scale as the only input.

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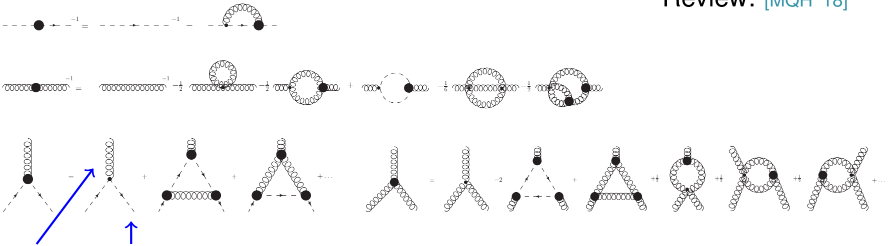

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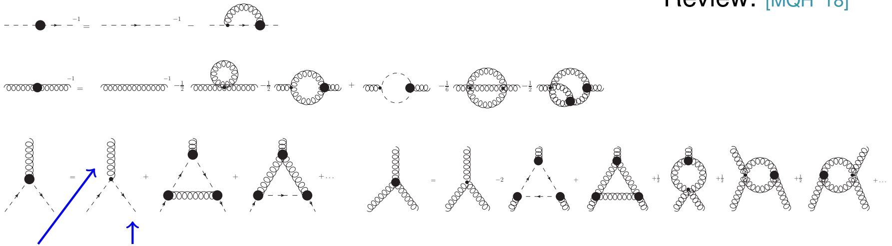


Gluon and ghost fields: Elementary fields of Yang-Mills theory in the Landau gauge

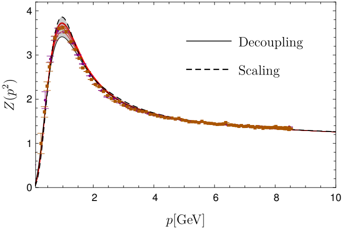
Self-contained system of equations with the scale as the only input.

Truncation→ 3-loop expansion of 3PI effective action [Berges '04]

- 4 coupled integral equations with full kinematic dependence.
- Sufficient numerical accuracy required for renormalization.
- One- and two-loop diagrams [Meyers, Swanson '14; MQH '17; Eichmann, Pawlowski, Silva '21].

Landau gauge propagators

Gluon dressing function:



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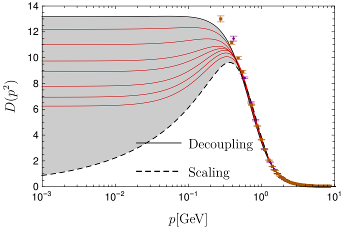
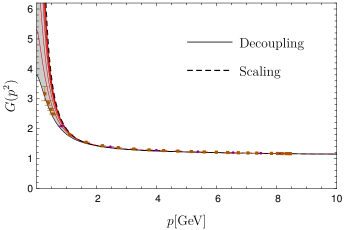


- Family of solutions: Nonperturbative completions of Landau gauge [Maas '10]?
- Realized by condition on $G(0)$ [Fischer, Maas, Pawlowski '08; Alkofer, MQH, Schwenzer '08]
- Results here independent of $G(0)$

Ghost dressing function:

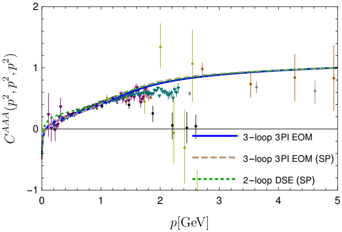


[Sternbeck '06; MQH '20]

Concurrence of functional methods

Exemplified with three-gluon vertex.

3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes ’08; Sternbeck et al. ’17; MQH ’20]

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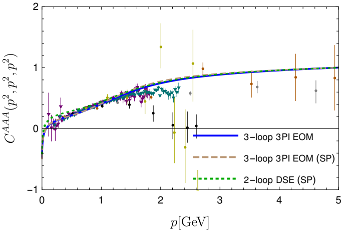

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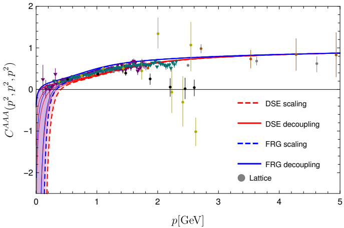
DSE vs. FRG:



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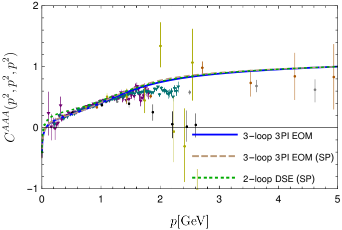

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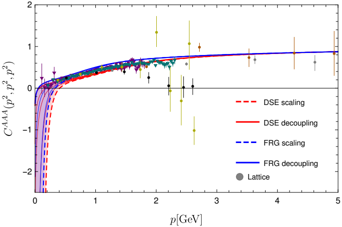


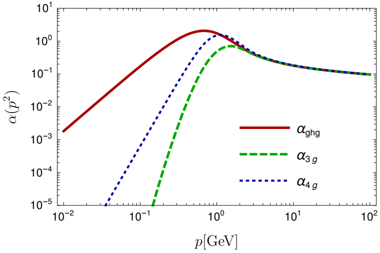
[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; Cyrol et al. '16; MQH '20]

Beyond this truncation

- Further dressings of three-gluon vertex [Eichmann, Williams, Alkofer, Vujinovic '14]
- Effects of four-point functions [MQH '16, MQH '17, Corell et al. '18, MQH '18]

Importance of self-consistency

- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime



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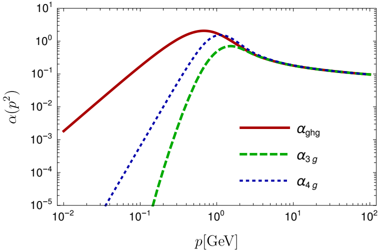
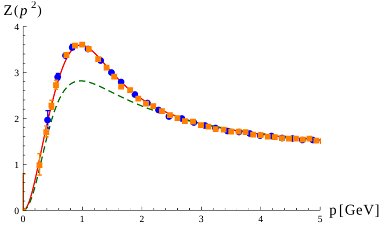
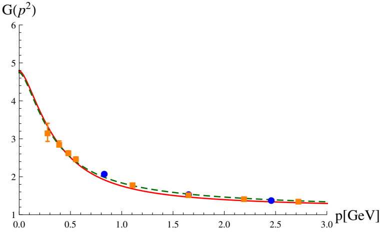


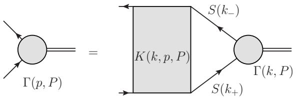
Agreement with lattice is not sufficient!



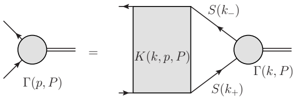


Optimized effective three-gluon vertex [MQH, von Smekal '13]

Solving a BSE



Solving a BSE



Consider the eigenvalue problem (Γ is the BSE amplitude)

$$\mathcal{K}\cdot\Gamma(P)=\boldsymbol{\lambda}(\boldsymbol{P})\,\Gamma(P).$$

$\lambda(P^2)=1$ is a solution to the BSE $\Rightarrow$ Glueball mass $P^2=-M^2$

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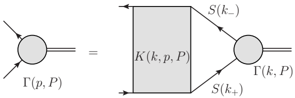

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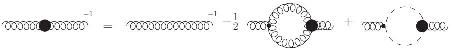
Calculation requires quantities for

$$k_\pm^2 = P^2 + k^2 \pm 2\sqrt{P^2\, k^2}\cos\theta = -M^2 + k^2 \pm 2\, \boldsymbol{i}\, M\sqrt{k^2}\,\cos\theta.$$

$\Rightarrow$ Complex momentum arguments.

Landau gauge propagators in the complex plane

Simpler truncation:



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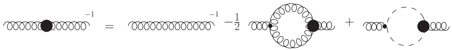

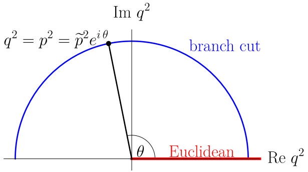

$\rightarrow$ Opening at $q^2 = p^2$.

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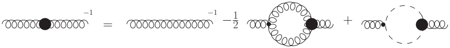
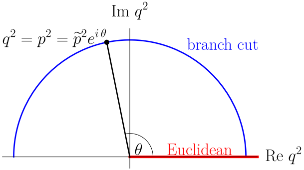
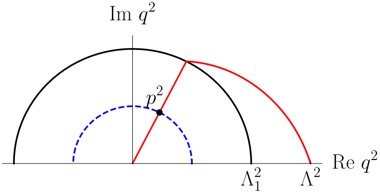






$\rightarrow$ Opening at $q^2 = p^2$.

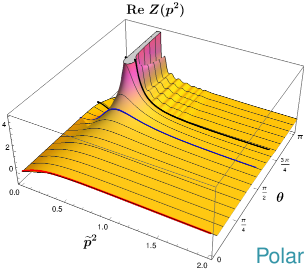
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

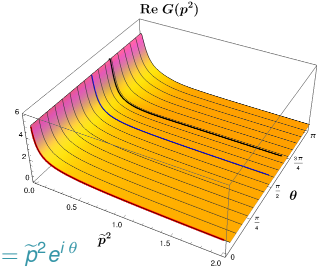
Deformation of integration contour necessary [Maris '95]. Recent resurgence: [Alkofer et al. '04; Windisch, MQH, Alkofer '13; Williams '19; Miramontes, Sanchis-Alepuz '19; Eichmann et al. '19, . . .]

Landau gauge propagators in the complex plane

Ray technique for self-consistent solution of a DSE:

[Strauss, Fischer, Kellermann; Fischer, MQH '20].





Polar coordinates: $p^2 = \widetilde{p}^2 e^{i\,\theta}$

- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)
- No proof of existence of complex conjugate poles due to simple truncation.

[Fischer, MQH '20]

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger '68]
- Average over extrapolations using subsets of points for error estimate

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$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

Coefficients a_i can determined such that $f(x)$ exact at x_i.

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Test extrapolation for solvable system: Heavy meson [MQH, Sanchis-Alepuz, Fischer '20]

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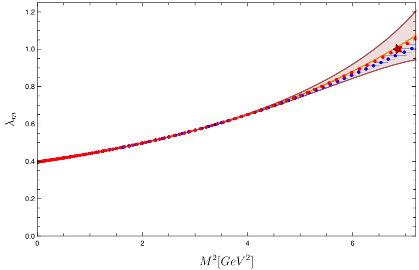

Extrapolation of $\lambda(P^2)$ for glueballs

Higher eigenvalues: Excited states.

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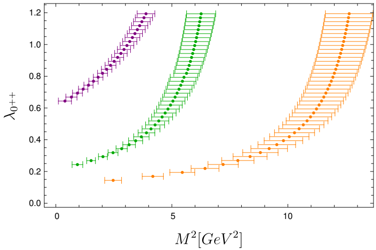

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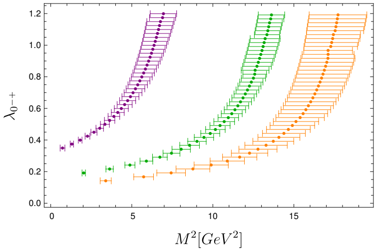
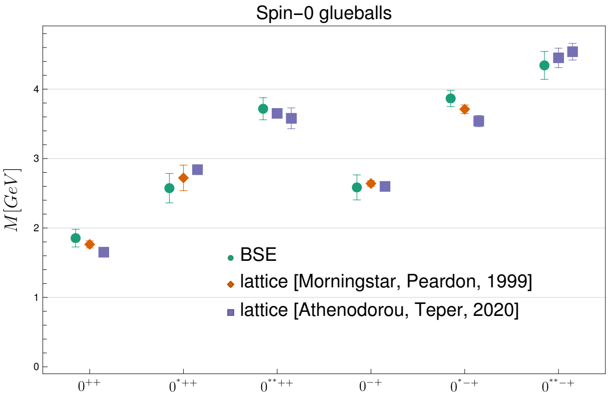


Physical solutions for $\lambda(P^2) = 1$.

Glueballs masses for $0^{\pm +}$



Lattice 0^{**++}: Conjectured based on irred. rep. of octahedral group

All results for $r_0 = 1/418(5)$ MeV.

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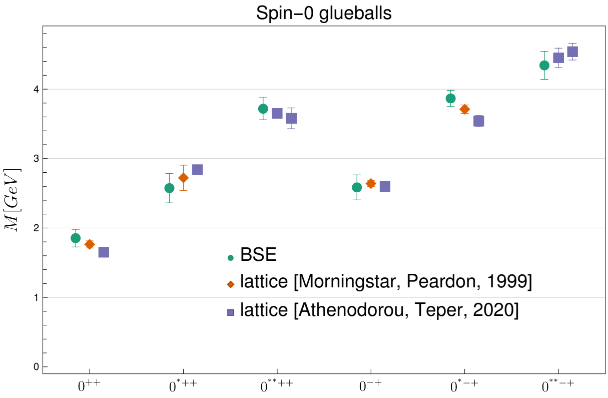

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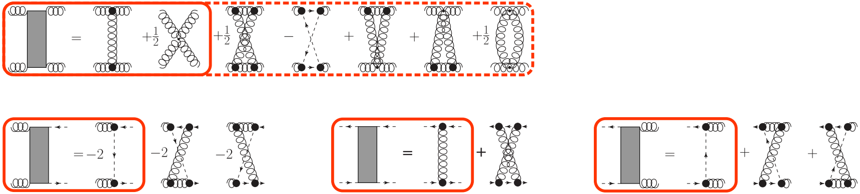
Under conjecture that choice of solution is a gauge choice: Explicit test of gauge independence!

Tested that results are independent of family of solutions.

Two-loop diagrams

Include leading two-loop diagrams:



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(Subleading two-loop terms still missing for scalar glueball.)

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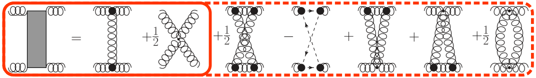

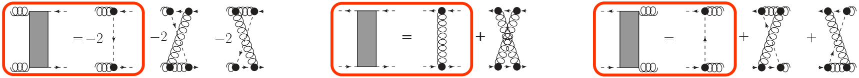

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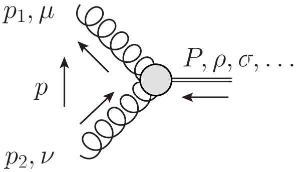
0^{-+}: No effect on mass. [MQH, Fischer, Sanchis-Alepuz , vConf21, arXiv:2111.10197]
0^{++}: Tiny effect on mass ($< 2\%$).
[MQH, Fischer, Sanchis-Alepuz , HADRON2021, arXiv:2201.05163]

Glueball amplitudes

$$\Gamma_{\mu\nu\rho\sigma\ldots}(p_1,p_2) = \sum \tau^i_{\mu\nu\rho\sigma\ldots}(p_1,p_2)h_i(p_1,p_2)$$



Example: 2^{++}

- Four indices → 43 tensors

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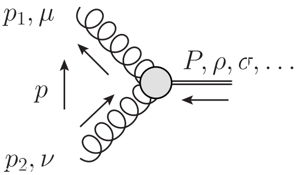

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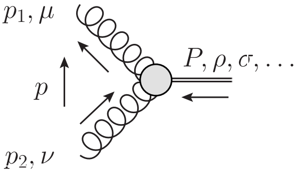

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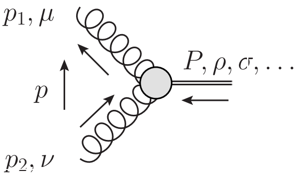

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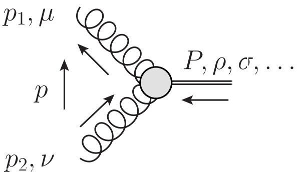

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J	P $=+$	P $=-$
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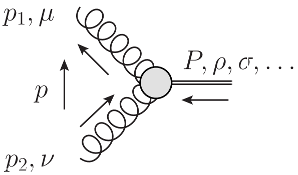

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'Low' number of tensors, but high-dimensional tensors!

$\rightarrow$ Computational cost increases with J.

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^{\pm}$ or $(2n+1)^{-}$ [Landau '48; Yang '50].
($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

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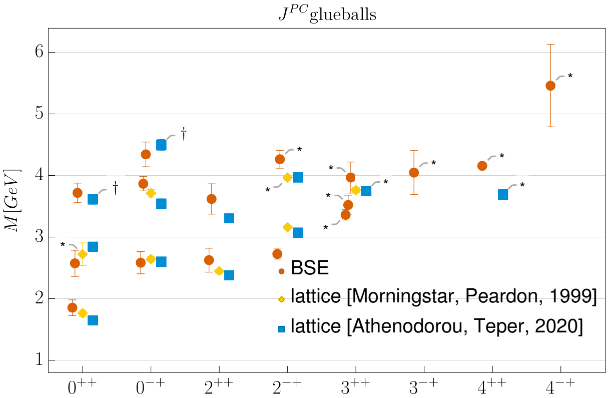
$J = 1$ not found here.

Glueball masses for $J^{\pm +}$



Lattice:
*: identification with some uncertainty
†: conjecture based on irred. rep of octahedral group

[MQH, Fischer, Sanchis-Alepuz '21]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

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Parameter-free determination of glueball masses from functional methods.

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Thank you for your attention.

Charge parity

Transformation of gluon field under charge conjugation:

$$A^a_\mu \to -\eta(a) A^a_\mu$$

where

$$\eta(a) = \begin{cases} +1 & a = 1, 3, 4, 6, 8 \\ -1 & a = 2, 5, 7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A^a_\mu A^a_\nu \to \eta(a)^2 A^a_\mu A^a_\nu = A^a_\mu A^a_\nu.$$

$\Rightarrow C = +1$

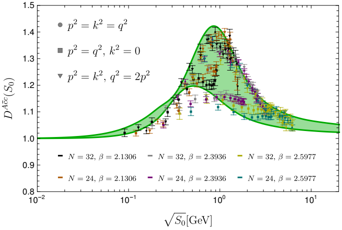
Negative charge parity, e.g.:

$$d^{abc} A^a_\mu A^b_\nu A^c_\rho \to -\, d^{abc} \eta(a)\eta(b)\eta(c) A^a_\mu A^b_\nu A^c_\rho = \\ -\, d^{abc} A^a_\mu A^b_\nu A^c_\rho.$$

Only nonvanishing elements of the symmetric structure constant d^{abc}: zero or two indices equal to 2, 5 or 7.

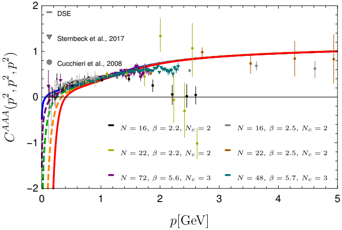
Landau gauge vertices

Ghost-gluon vertex:



[Maas '19; MQH '20]

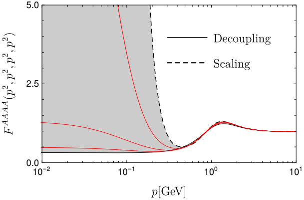
Three-gluon vertex:



[Cucchieri, Maas, Mendes '08; Sternbeck et al. '17; MQH '20]

- Nontrivial kinematic dependence of ghost-gluon vertex
- Simple kinematic dependence of three-gluon vertex
- Four-gluon vertex from solution

Four-gluon vertex:

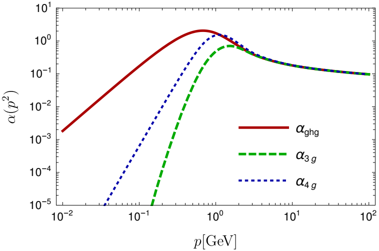


[MQH '20]

Some properties of the Landau gauge solution

[MQH '20]

- Slavnov-Taylor identities (gauge invariance): Vertex couplings agree down to GeV regime



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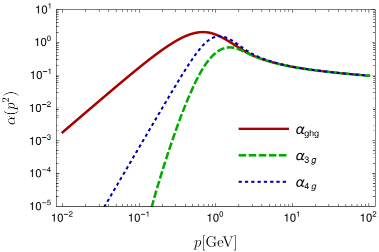

- Renormalization: First parameter-free subtraction of quadratic divergences
  $\Rightarrow$ One unique free parameter (family of solutions)