From quantum kinetic theory to dissipative spin hydrodynamics

Nora Weickgenannt

NW, E. Speranza, X.-l. Sheng, Q. Wang. D. H. Rischke, PRL127 (2021) 5, 052301; PRD104 (2021) 1, 016022
NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)
NW, D. Wagner, E. Speranza, 2204.01797 (2022)

Lunch club seminar in Gießen
June 22, 2022

Polarization through rotation

- Non-central heavy-ion collisions: large orbital angular momentum
- Conversion of orbital angular momentum into spin

  $\Longrightarrow$ Global rotation leads to polarization

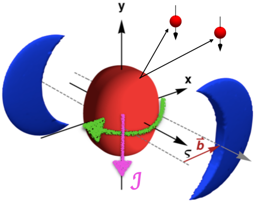


Figure from W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- Estimate vorticity from thermal approach in global equilibrium

  F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, PRC 95 (2017) 5, 054902

$$\omega \approx T(\mathcal{P}_\Lambda + \mathcal{P}_{\bar{\Lambda}})$$

Quark-gluon plasma is the "most vortical fluid ever observed"

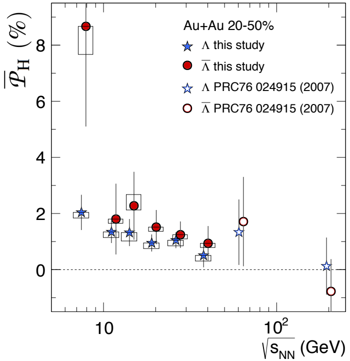
L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

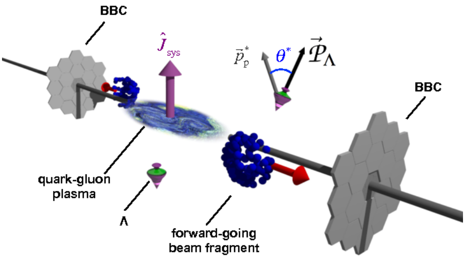
$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$

Great Red Spot of Jupiter $10^{-4}\,\mathrm{s}^{-1}$

- Weak decay of Λ hyperons: daughter particles emitted preferably along polarization direction $\boldsymbol{\mathcal{P}}$.
- Angular distribution of emitted momenta in hyperon rest frame
  B.I. Abelev, I. Selyuzhenkov. et al. (STAR), PRC76, 024915 (2007)

$$\frac{dN}{d\cos\theta_\star} = \frac{1}{2}\left(1 + \alpha|\boldsymbol{\mathcal{P}}|\cos\theta_\star\right)$$





L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

$\Longrightarrow$ Polarization of Λ hyperons along global angular momentum!

- Phenomenological calculations of polarization in literature: local equilibrium
  I. Karpenko, F. Becattini, EPJ C 77 (2017) 213

- Global (momentum integrated) polarization: good agreement with experimental data
  L. Adamczyk, et al. Nature 548 (2017) 62
  J. Adam, et al. Phys. Rev. C98 (2018) 014910

- Local polarization: opposite angular dependence
  F. Becattini, I. Karpenko, PRL 120 (2018) 1, 012302
  STAR collaboration, PRL 123 (2019) 13, 132301

  Recently important developments towards resolving this puzzle
  S. Y.F. Liu, Y. Yin, JHEP 07 (2021) 188
  B. Fu, S. Y.F. Liu, L. Pang, H. Song, Y. Yin, PRL 127 (2021) 14, 142301
  F. Becattini, M. Buzzegoli, A. Palermo, PLB 820 (2021) 136519
  F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, PRL 127 (2021) 27, 272302

- Dissipative corrections to polarization?
  $\Longrightarrow$ **dissipative spin hydrodynamics**

- Want: Dissipative spin hydrodynamics from microscopic theory
- Spin is quantum property
  $\Longrightarrow$ starting point: quantum field theory
- **Step 1: derive kinetic theory for spin-1/2 particles from quantum field theory**
  use Wigner function
  NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301; PRD104 (2021) 1, 016022
- **Step 2: derive hydrodynamic equations of motion from kinetic theory,**
  use method of moments
  NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)

Quantum kinetic theory?

- Kinetic theory: dynamics of system described by phase space distribution $f(x,p)$
  "probability to find particle at position x with momentum p"

- Equation of motion for $f(x,p)$: Boltzmann equation

$$p \cdot \partial f(x,p) = C[f]$$

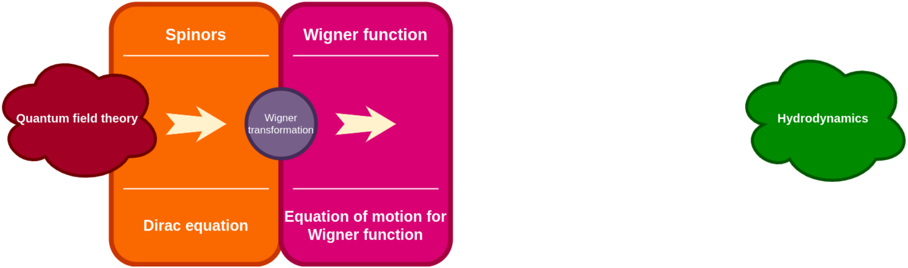
  collision term
  common starting point to derive hydrodynamics from microscopic theory

- **How to generalize to quantum field theory?**

- Issue: uncertainty relation, particle position and momentum not determined simultaneously

- How to define phase-space description (x,p)?

A path to dissipative spin hydrodynamics


Quantum field theory
Spinors
Wigner transformation
Wigner function
Dirac equation
Equation of motion for Wigner function
Hydrodynamics

- Wigner function for Dirac fields

$$W_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \left\langle : \bar{\psi}_\beta \left(x + \frac{y}{2}\right) \psi_\alpha \left(x - \frac{y}{2}\right) : \right\rangle$$

Wigner transformation of two-point function

- Dirac equation with general interaction

$$(i\hbar\gamma\cdot\partial - m)\,\psi(x) = \hbar\rho(x)$$

$\Longrightarrow$ Equation of motion for Wigner function

S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)

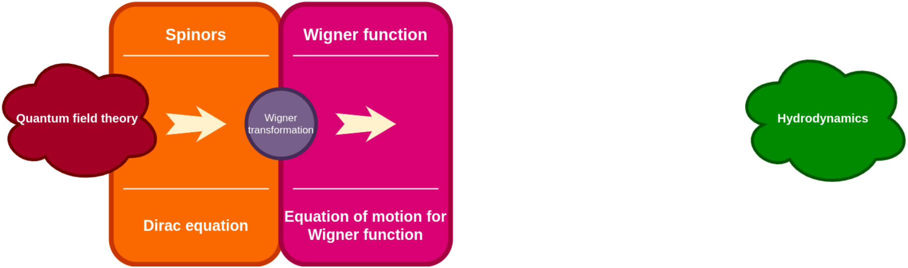
$$\left[\gamma\cdot\left(p + \frac{i\hbar}{2}\partial\right) - m\right] W = \hbar\,\mathcal{C}$$

with

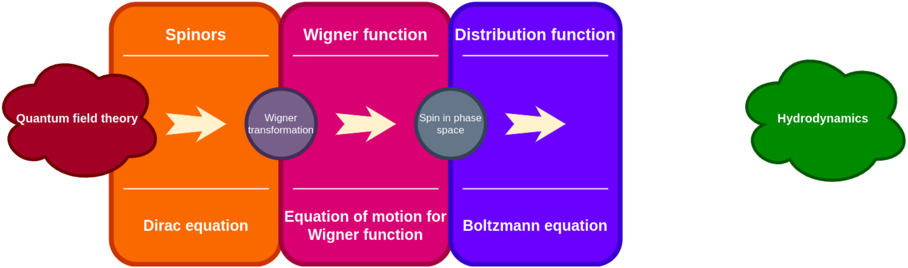
$$\mathcal{C}_{\alpha\beta} \equiv \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \left\langle : \bar{\psi}_\beta \left(x + \frac{y}{2}\right) \rho_\alpha \left(x - \frac{y}{2}\right) : \right\rangle$$

- Derive form of collision term from quantum field theory up to first order in $\hbar$

A path to dissipative spin hydrodynamics



A path to dissipative spin hydrodynamics



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- Idea: Phase-space description of spin dynamics, $(x,p) \to (x,p,\mathfrak{s})$
- Define distribution function in enlarged phase space

$$\mathfrak{f}(x,p,\mathfrak{s}) \equiv \frac{1}{2}\left[\frac{m}{p^2} p_\mu \mathrm{Tr}(\gamma^\mu W) - \mathfrak{s}_\mu \mathrm{Tr}(\gamma^\mu \gamma^5 W)\right]$$

- Equation of motion for Wigner function $\Longrightarrow$ Boltzmann-like equation for $\mathfrak{f}$

$$\boxed{p \cdot \partial \mathfrak{f}(x,p,\mathfrak{s}) = \mathfrak{C}[\mathfrak{f}]}$$

Collision term

- Calculate collision term from quantum field theory using Wigner function
- Take into account **nonlocal particle scatterings**

Nonlocal collision term

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- Find that off-shell terms cancel
  $\Longrightarrow$ Equation of motion for on-shell distribution function f

$$\delta(p^2 - m^2) p \cdot \partial f(x, p, \mathfrak{s}) = \mathfrak{C}_{\text{on-shell}}[f]$$

- Nonlocal collision term

$$\mathfrak{C}_{\text{on-shell}}[f] = \int d\Gamma_1 \, d\Gamma_2 \, d\Gamma' \, \mathcal{W} \, [f(x + \Delta_1, p_1, \mathfrak{s}_1) \\ \times f(x + \Delta_2, p_2, \mathfrak{s}_2) - f(x + \Delta, p, \mathfrak{s}) f(x + \Delta', p', \mathfrak{s}')]$$

  Particle positions displaced by

$$\Delta^\mu \equiv - \frac{\hbar}{2m(p \cdot \hat{t} + m)} \epsilon^{\mu\nu\alpha\beta} p_\nu \hat{t}_\alpha \mathfrak{s}_\beta$$

- **Interpretation:**
  Particles scatter with finite impact parameter
  $\Longrightarrow$ orbital angular momentum in microscopic collision
  $\Longrightarrow$ can be converted into spin

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- Local-equilibrium condition: collision term has to vanish

- Solution with nonlocal collision term

$$f_{eq}(x,p,\mathfrak{s}) = \frac{1}{(2\pi\hbar)^3} \exp\left[-\beta(x)\cdot p + \frac{\hbar}{4}\varpi_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right]$$

β_μ fluid velocity/temperature, $\varpi_{\mu\nu} \equiv -(1/2)\partial_{[\mu}\beta_{\nu]}$ thermal vorticity,
$\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha \mathfrak{s}_\beta$ dipole-moment tensor
$a^{[\mu}b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$

with $\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0 \Longrightarrow$ global equilibrium, $p\cdot\partial f_{eq} = 0$

- Equilibrium polarization

$$\mathcal{A}^\mu = \frac{1}{(2\pi\hbar)^3}\frac{\hbar}{4m}\epsilon^{\mu\nu\alpha\beta}p_\nu\varpi_{\alpha\beta}e^{-\beta\cdot p} + \mathcal{O}(\hbar^2)$$

$\Longrightarrow$ Fluid polarized along thermal vorticity through nonlocal collision term

Beyond global equilibrium

- So far: kinetic theory
  Assumptions: $\hbar$ expansion
  system sufficiently dilute, particles free between collisions

$$l_{\text{int}} \ll l_{\text{mfp}}$$

interaction range mean free path

- Still: kinetic theory is microscopic theory, challenging to solve

- Derive effective theory valid on macroscopic scales
  $\Longrightarrow$ hydrodynamics

- Additional assumption: gradients in system sufficiently small

$$l_{\text{mfp}} \ll l_{\text{hydro}}$$

mean free path hydrodynamic scale ($\sim$ inverse gradients)

- Relativistic hydrodynamics based on **conservation equations** for energy-momentum tensor and charge current

$$\partial_\mu T^{\mu\nu} = 0 \;, \qquad \partial_\mu N^\mu = 0$$

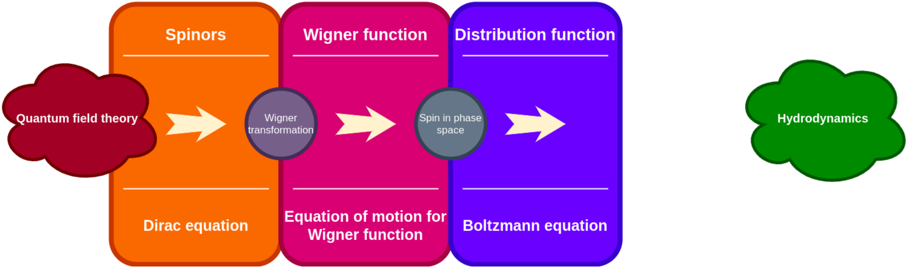
- Spin hydrodynamics: promote spin tensor to additional dynamical quantity
  W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, PRC **97**, no. 4, 041901 (2018)
  W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, PRD **97**, no. 11, 116017 (2018)
  W. Florkowski, F. Becattini, and E. Speranza, APB **49**, 1409 (2018)

- Equation of motion: conservation of **total** angular-momentum tensor

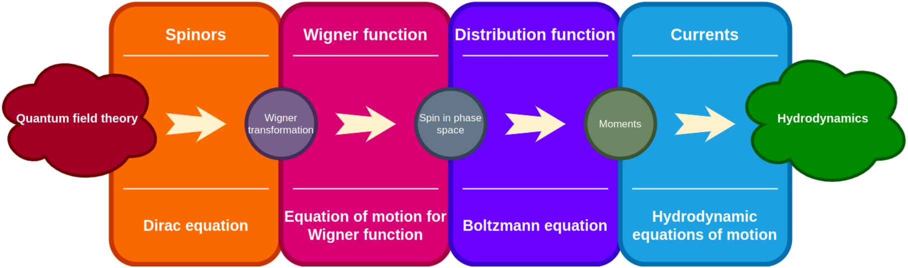
$$J^{\lambda,\mu\nu} = \underbrace{x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}}_{\text{orbital part}} + \underbrace{\hbar S^{\lambda,\mu\nu}}_{\text{spin tensor}}$$

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \qquad \Longrightarrow \qquad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

A path to dissipative spin hydrodynamics



A path to dissipative spin hydrodynamics



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E. Speranza, NW, EPJA 57 (2021) 5, 155
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- Charge current, energy-momentum tensor and spin tensor can be expressed as integrals of distribution function from microscopic definitions
- Apply Noether's theorem to Dirac Lagrangian $\Longrightarrow$ canonical currents

$$T_C^{\mu\nu} = \int d\Gamma\, p^\nu \left[p^\mu \left(1 - \frac{\hbar^2}{4m^2}\partial^2 \right) + \frac{\hbar}{2}\Sigma_{\mathfrak{s}}^{\mu\lambda}\partial_\lambda \right] f(x,p,\mathfrak{s}) + \frac{\hbar^2}{m}\int d^4p\, p^\nu D_{\mathcal{V}}^\mu + \mathcal{O}(\hbar^3)\,,$$

$$S_C^{\lambda,\mu\nu} = \frac{1}{2}\int d\Gamma\, \left(p^\lambda \Sigma_{\mathfrak{s}}^{\mu\nu} + p^\mu \Sigma_{\mathfrak{s}}^{\nu\lambda} + p^\nu \Sigma_{\mathfrak{s}}^{\lambda\mu} \right) f(x,p,\mathfrak{s})$$

$D_{\mathcal{V}}^\mu$: interaction contribution

canonical energy-momentum tensor not symmetric for free fields or in global equilibrium

$\Longrightarrow$ spin tensor not conserved

$\Longrightarrow$ not consistent with physical picture

E. Speranza, NW, EPJA 57 (2021) 5, 155

- Microscopic definition of currents not unique

  $\Longrightarrow$ **pseudo-gauge transformations**
  F.W. Hehl, Rept. Math. Phys. 9, 55 (1976)

$$\begin{aligned} T^{\mu\nu} &\to T^{\mu\nu} + \frac{\hbar}{2}\partial_\lambda(\Phi^{\lambda,\mu\nu} + \Phi^{\nu,\mu\lambda} + \Phi^{\mu,\nu\lambda}) \, , \\ S^{\lambda,\mu\nu} &\to S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu} + \hbar\,\partial_\rho Z^{\mu\nu\lambda\rho} \end{aligned}$$

  leave invariant equations of motion and global charges

$$P^\mu \equiv \int d\Sigma_\lambda \, T^{\lambda\mu} \, , \qquad J^{\mu\nu} \equiv \int d\Sigma_\lambda \, J^{\lambda,\mu\nu}$$

  global spin different in different pseudo-gauges
  $\Longrightarrow$ different splitting of total angular momentum into orbital part and spin

- Idea: find pseudo-gauge with clearer physical interpretation
  $\Longrightarrow$ Hilgevoord-Wouthuysen (HW) pseudo-gauge
  J. Hilgevoord, S.A. Wouthuysen, Nuclear Physics 40, 1 (1963)

NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301
NW, D. Wagner, E. Speranza, 2204.01797 (2022)

- Generalize HW pseudo-gauge transformation to interacting case

$$
\begin{aligned}
T^{\mu\nu}_{HW} &= \int d\Gamma\, p^\mu p^\nu f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)\ , \\
S^{\lambda,\mu\nu}_{HW} &= \int d\Gamma\, p^\lambda \left(\frac{1}{2}\Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{4m^2} p^{[\mu}\partial^{\nu]}\right) f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)
\end{aligned}
$$

- Equations of motion

$$
\partial_\mu T^{\mu\nu}_{HW} = \int d\Gamma\, p^\nu\, \mathfrak{C}[f] = 0\ ,
$$

$$
\hbar\,\partial_\lambda S^{\lambda,\mu\nu}_{HW} = \int d\Gamma\, \frac{\hbar}{2}\Sigma^{\mu\nu}_{\mathfrak{s}}\, \mathfrak{C}[f] = T^{[\nu\mu]}_{HW}
$$

p^μ: collisional invariant
$\Sigma^{\mu\nu}_{\mathfrak{s}}$: collisional invariant only for local collisions

Physical interpretation

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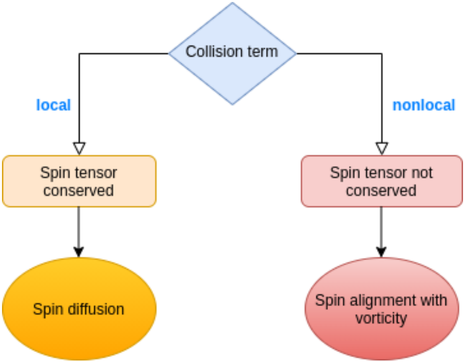
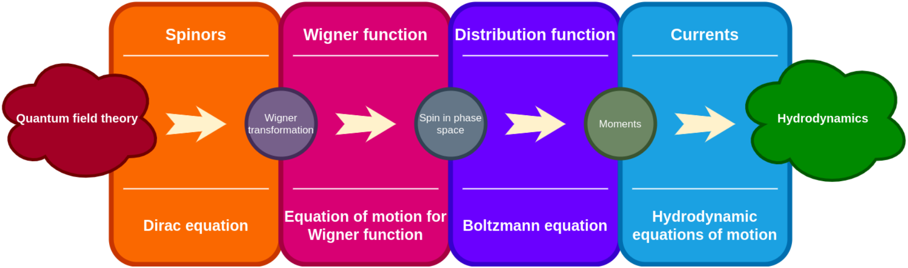
- Local collision term $\Longrightarrow$ spin conserved
- Nonocal collision term
  $\Longrightarrow$ conversion between orbital angular momentum and spin until global equilibrium is reached



Dissipative spin hydrodynamics



- Dissipative case: conservation laws do not provide closed system of equations
- **Have:** spin kinetic theory from quantum field theory
  $\Longrightarrow$ derive additional equations of motion for dissipative spin hydrodynamics from kinetic theory
- Use method of moments
  G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047
- Generalize to include spin degrees of freedom

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G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047

- Hydrodynamics: evolution of system determined by conservation laws
- Dissipative hydrodynamics: conservation laws do not provide closed set of equations to determine all components of conserved currents
- Idea: derive additional equations of motion from kinetic theory, Boltzmann equation

$$p \cdot \partial f(x,p) = C[f]$$

distribution function   collision term

- Expand distribution function in moments of p^μ, irreducible with respect to the Little group of fluid velocity
- Obtain equations of motion for those moments from Boltzmann equation
- Employ truncation procedure to close system of moment equations

Decomposition of currents

NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)

- Decompose currents with respect to fluid velocity u^μ

$$
\begin{aligned}
N^\mu &= n u^\mu + n^\mu \,, \\
T^{\mu\nu}_{\text{sym}} &= \epsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0 + \Pi) + u^{(\mu} h^{\nu)} + \pi^{\mu\nu} \,, \\
S^{\lambda,\mu\nu} &= u^\lambda \tilde{\mathfrak{N}}^{\mu\nu} + \Delta^\lambda_\alpha \tilde{\mathfrak{P}}^{\alpha\mu\nu} + 2u_{(\alpha}\tilde{\mathfrak{H}}^{\lambda)\mu\nu\alpha} + \tilde{\mathfrak{Q}}^{\lambda\mu\nu} \\
&\quad + \frac{\hbar}{2m}\partial^{[\nu}\left[\epsilon_0 u^{\mu]} u^\lambda - \Delta^{\mu]\lambda}(P_0 + \Pi) + \pi^{\mu]\lambda}\right]
\end{aligned}
$$

$$
\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu, \;\; \tilde{A}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} A_{\alpha\beta}, \;\; a^{(\mu} b^{\nu)} \equiv a^\mu b^\nu + a^\nu b^\mu
$$

- 4+10+24 degrees of freedom $\leftrightarrow$ 1+4+6 equations of motion
  $\Longrightarrow$ need additional equations of motion to close system of equations
- All components can be related to moments of distribution function, e.g., spin-energy tensor

$$
\tilde{\mathfrak{N}}^{\mu\nu} \equiv -\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta} u_\alpha \langle E_p^2 \, \mathfrak{s}_\beta \rangle
$$

$$
E_p \equiv p \cdot u
$$

$\Longrightarrow$ derive additional equations of motion from kinetic theory

Equilibrium and scale ordering

NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)

- Nonlocal collision term vanishes only in global equilibrium
  $\Longrightarrow$ How to define "local" equilibrium as starting point of hydrodynamic expansion?
- Issue about local equilibrium:
  Nonlocality scale of collision term smaller than scale on which dissipation happens
- Treat vorticity on different scale than other gradients
  S. Li, M. Stephanov, H.-U. Yee, PRL127 (2021) 8, 082302
- Impose ordering of scales

$$\Delta < \ell_{\text{mfp}} \ll \ell_{\text{hydro}}\ , \qquad \Delta \ll \ell_{v} < \ell_{\text{hydro}}$$

needed for molecular chaos — allows for definition of local equilibrium

with

$$\textit{expansion in spin degrees of freedom}\quad \frac{\Delta}{\ell_v} \sim \frac{\ell_{\text{mfp}}}{\ell_{\text{hydro}}}\quad \textit{hydrodynamic expansion}$$

Δ: nonlocality of microscopic collision, ℓ_{mfp}: mean free path
ℓ_{hydro}: scale of inverse dissipative gradients, ℓ_v: scale of inverse vorticity

- Local equilibrium:
  neglect $\mathcal{O}(\Delta/\ell_{\text{hydro}})$ $\Longrightarrow$ Nonlocal collision term vanishes if spin potential equal to thermal vorticity

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- Expand distribution function up to first order in $\hbar$ and gradients

$$f = f_{\text{eq}} + \delta f = f_{0p}\left[1 + \frac{\hbar}{4}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu} + \phi_p + \mathfrak{s}\cdot\zeta_p\right],$$

equilibrium, deviations from equilibrium

Zeroth-order distribution function

$$f_{0p} \equiv \frac{1}{(2\pi\hbar)^3}\, e^{-\beta_0 u\cdot p + \alpha_0}$$

inverse temperature β_0, chemical potential α_0,
spin potential: thermal vorticity + dissipative corrections

$$\Omega_{\mu\nu} = \varpi_{\mu\nu} + \mathcal{O}\left(\frac{1}{l_{\text{hydro}}}\right)$$

- Define dissipative irreducible moments

$$\rho_n^{\mu_1\cdots\mu_l} \equiv \langle E_p^n\, p^{\langle\mu_1}\dots p^{\mu_l\rangle}\rangle_\delta$$
$$\tau_n^{\mu,\mu_1\cdots\mu_l} \equiv \langle E_p^n\, \mathfrak{s}^\mu\, p^{\langle\mu_1}\dots p^{\mu_l\rangle}\rangle_\delta$$

$A^{\langle\mu_1\cdots\mu_n\rangle}$: traceless, symmetric projection orthogonal to u^μ

- δf can be expressed in basis of dissipative moments

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- Matching conditions (Landau frame)

$$u_\mu N^\mu = u_\mu N^\mu_{eq} \, , \qquad u_\mu T^{\mu\nu}_{\mathsf{sym}} = u_\mu T^{\mu\nu}_{\mathsf{sym},eq}$$

- Additional matching condition in presence of spin

$$u_\lambda J^{\lambda,\mu\nu} = u_\lambda J^{\lambda,\mu\nu}_{eq}$$

$\Longrightarrow$ Defines spin potential near local equilibrium

- Conservation laws for N^μ, $T^{\mu\nu}$, and $J^{\lambda,\mu\nu}$

$\Longrightarrow$ equations of motion for thermodynamic potentials α_0, β_0, u^μ, $\Omega^{\mu\nu}$

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- Boltzmann equation $\Longrightarrow$ exact equations of motion for spin moments $\tau_n^{\mu,\mu_1\cdots\mu_l}$, e.g.,

$$
\begin{aligned}
&\dot{\tau}_r^{\langle\mu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\rangle} = \\
&\left[\xi_r^{(0)}\theta + \frac{G_{2(r+1)}}{D_{20}}\Pi\theta - \frac{G_{2(r+1)}}{D_{20}}\pi^{\lambda\nu}\sigma_{\lambda\nu} - \frac{G_{3r}}{D_{20}}\partial\cdot n\right]\omega_0^\mu - \frac{\hbar}{2m}I_{(r+1)1}\Delta_\lambda^\mu\nabla_\nu\tilde{\Omega}^{\lambda\nu} \\
&- \frac{\hbar}{2m}\tilde{\Omega}^{\langle\mu\rangle\nu}\left[I_{(r+1)1}I_\nu - I_{(r+2)1}\frac{\beta_0}{\epsilon_0+P_0}\left(-\Pi\dot{u}_\nu + \nabla_\nu\Pi - \Delta_{\nu\lambda}\partial_\rho\pi^{\lambda\rho}\right)\right] \\
&+ r\,\dot{u}_\nu\tau_{r-1}^{\langle\mu\rangle,\nu} + (r-1)\sigma_{\alpha\beta}\tau_{r-2}^{\langle\mu\rangle,\alpha\beta} - \Delta_\lambda^\mu\nabla_\nu\tau_{r-1}^{\lambda,\nu} - \frac{1}{3}\left[(r+2)\tau_r^{\langle\mu\rangle} - (r-1)m^2\tau_{r-2}^{\langle\mu\rangle}\right]\theta \\
&- \frac{\hbar}{2m}I_{(r+1)0}\epsilon^{\mu\nu\alpha\beta}u_\nu\dot{\Omega}_{\alpha\beta}
\end{aligned}
$$

$\nabla^\mu \equiv \Delta_\nu^\mu\partial^\nu$, $\theta \equiv \nabla\cdot u$, $\sigma^{\mu\nu} \equiv \nabla^{\langle\mu}u^{\nu\rangle}$, $\omega_0^\mu \equiv -(1/2)\epsilon^{\mu\nu\alpha\beta}u_\nu\Omega_{\alpha\beta}$, $I^\mu \equiv \nabla^\mu\alpha_0$

- Collision term

$$
\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_n\rangle} \equiv \int d\Gamma\, E_p^{r-1}\mathfrak{s}^\mu p^{\langle\mu_1}\cdots p^{\mu_n\rangle}\mathfrak{C}[f]
$$

Truncation procedure and dynamical moments

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- Infinite number of coupled equations $\Longrightarrow$ need truncation procedure
- **Idea** (Israel-Stewart): Approximate moments which do not appear in conservation laws by those which do appear
- **Conventional hydrodynamics:**

$$n^\mu \equiv \rho_0^\mu, \qquad h^\mu \equiv \rho_1^\mu, \qquad \Pi \equiv -\frac{m^2}{3}\rho_0, \qquad \pi^{\mu\nu} \equiv \rho_0^{\mu\nu}$$

particle diffusion | heat flux | bulk viscous pressure | shear stress

$\Longrightarrow$ 14-moment approximation

- **Spin hydrodynamics:** additional dynamical moments from spin tensor

$$\mathfrak{n}^\nu \equiv \tau_2^\nu, \qquad \mathfrak{p}^\mu \equiv \tau_0^\mu, \qquad \mathfrak{z}^{\lambda\mu} \equiv \tau_1^{(\langle\mu\rangle,\lambda)}, \qquad \mathfrak{q}^{\lambda\mu\nu} \equiv \tau_0^{\nu,\mu\lambda}$$

spin energy | spin pressure | spin diffusion | spin stress

$\Longrightarrow$ 14+24-moment approximation

- Dynamical spin moments determined by form of spin tensor
  $\Longrightarrow$ Pseudo-gauge dependence enters!

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- General form of collision terms

$$\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_l\rangle} = \sum_{n=0}^{N_l} B_{rn}^{(l)} \tau_n^{\mu,\langle\mu_1\cdots\nu_l\rangle} + \int [d\Gamma]\, \mathcal{W}\, E_p^{r-1} p^{\langle\mu_1} \cdots p^{\mu_l\rangle} \mathfrak{s}^\mu f_{0p} f_{0p'}$$
$$\times \left[-\frac{\hbar}{4} (\Omega_{\alpha\beta} - \varpi_{\alpha\beta}) \Sigma_{\mathfrak{s}}^{\alpha\beta} + \frac{1}{2} \partial_{(\beta}\beta_{\alpha)} \Delta^\beta p^\alpha \right]$$

dissipative spin moments
difference between spin potential and thermal vorticity
from nonlocal collision term, thermal shear,

- Invert matrix $B_{rn}^{(l)}$ to obtain final form of equations of motion

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Obtain equations of motion of all dynamical spin moments, e.g.,

$$
\begin{aligned}
&\tau_{\mathfrak{q}} \Delta^{\mu}_{\rho} \Delta^{\nu\lambda}_{\alpha\beta} \frac{d}{d\tau} \mathfrak{q}^{\langle\rho\rangle\alpha\beta} + \mathfrak{q}^{\langle\mu\rangle\nu\lambda} \\
=&-\boldsymbol{\mathfrak{d}^{(2)} \beta_0 \sigma_{\rho}{}^{\langle\nu} \epsilon^{\lambda\rangle\mu\alpha\rho} u_\alpha} + \mathfrak{K}^{(2)}_{\Omega I} \tilde{\Omega}^{\langle\mu\rangle\langle\nu} I^{\lambda\rangle} + \mathfrak{K}^{(2)}_{\nabla\Omega} \Delta^{\nu\lambda}_{\alpha\beta} \nabla^{\alpha} \tilde{\Omega}^{\mu\beta} - \mathfrak{K}^{(2)}_{\omega\sigma} \sigma^{\nu\lambda} \omega_0^{\mu} \\
&- \mathfrak{K}^{(2)}_{\Omega\Pi} \tilde{\Omega}^{\langle\mu\rangle\langle\nu} \left(-\Pi \dot{u}^{\lambda\rangle} + \nabla^{\lambda\rangle} \Pi - \Delta^{\lambda\rangle}_{\alpha} \partial_{\beta} \pi^{\alpha\beta} \right) + \mathfrak{g}^{(2)}_1 \mathfrak{z}^{\mu\langle\nu} F^{\lambda\rangle} + \mathfrak{g}^{(2)}_2 \mathfrak{z}^{\mu\langle\nu} I^{\lambda\rangle} \\
&+ \mathfrak{g}^{(2)}_3 \Delta^{\mu}_{\rho} \Delta^{\nu\lambda}_{\alpha\beta} \nabla^{\beta} \mathfrak{z}^{\rho\alpha} + \mathfrak{g}^{(2)}_4 \mathfrak{q}^{\langle\mu\rangle\nu\lambda} \theta + \mathfrak{g}^{(2)}_5 \mathfrak{q}^{\langle\mu\rangle\rho\langle\nu} \sigma^{\lambda\rangle}_{\rho} + 2\tau_{\mathfrak{q}} \mathfrak{q}^{\langle\mu\rangle\rho\langle\nu} \omega^{\lambda\rangle}_{\rho} \\
&+ \mathfrak{g}^{(2)}_6 \mathfrak{p}^{\langle\mu\rangle} \sigma^{\nu\lambda} - 6\mathfrak{g}^{(2)}_7 \mathfrak{q}^{\rho\mu}{}_{\rho} \sigma^{\nu\lambda} + \mathfrak{g}^{(2)}_8 F^{\mu} \mathfrak{z}^{\langle\nu\lambda\rangle} + \mathfrak{g}^{(2)}_9 \mathfrak{p}^{\langle\nu} \nabla^{\lambda\rangle} u^{\mu} + \mathfrak{g}^{(2)}_{10} \mathfrak{q}^{\rho\langle\nu}{}_{\rho} \nabla^{\lambda\rangle} u^{\mu}
\end{aligned}
$$

relaxation time, obtained from local collision term

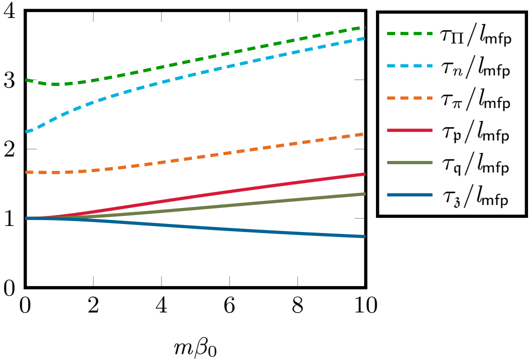
transport coefficients

Navier-Stokes limit, **from nonlocal collision term**

and similar equations for $\mathfrak{p}^{\langle\mu\rangle}$ and $\mathfrak{z}^{\lambda\mu}$

$\Longrightarrow$ closed set of relaxation equations

- Calculate relaxation times for spin moments in dependence of $m\beta_0$
- Spin relaxation times of same order as (though slightly smaller than) those for spin-independent dissipative moments (Π, n^μ, $\pi^{\mu\nu}$)



NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)

- Observable in heavy-ion collisions: Pauli-Lubanski vector
  F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, AP338, 32 (2013)
  E. Speranza, NW, EPJA 57 (2021) 5, 155

$$\Pi^\mu(p) = \frac{1}{2\mathcal{N}} \int d\Sigma_\lambda p^\lambda dS\, \mathfrak{s}^\mu f(x,p,\mathfrak{s})$$

- Equilibrium:

$$\Pi^\mu_{\text{eq}}(p) = -\frac{\hbar}{4m\mathcal{N}} \int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\mu\nu}(E_p u_\nu + p_{\langle\nu\rangle})$$

- Express dissipative corrections through dynamical spin moments:

$$\delta\Pi^\mu(p) = \frac{1}{2\mathcal{N}} \left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2}\right) \int d\Sigma_\lambda p^\lambda \Bigg\{ \chi_\mathfrak{p} \mathfrak{p}^{\langle\nu\rangle} - 6\chi_\mathfrak{n} \mathfrak{q}^{\rho\nu}{}_\rho + \mathfrak{x}_\mathfrak{p} u^\nu \mathfrak{z}^\lambda{}_\lambda$$
$$+ \left[\chi_\mathfrak{z} \mathfrak{z}^{\nu\alpha} + \left(\mathfrak{x}_\mathfrak{q} \mathfrak{q}^{\lambda\alpha}{}_\lambda + \mathfrak{x}_\mathfrak{p} \mathfrak{p}^{\langle\alpha\rangle}\right) u^\nu\right] p_{\langle\alpha\rangle} + \left(\chi_\mathfrak{q} \mathfrak{q}^{\langle\nu\rangle\alpha\beta} + \mathfrak{x}_\mathfrak{z} u^\nu \mathfrak{z}^{\langle\alpha\beta\rangle}\right) p_{\langle\alpha} p_{\beta\rangle} \Bigg\}$$

Navier-Stokes limit

NW, D. Wagner, E. Speranza, D. H. Rischke, 2203.04766 (2022)

- So far: kept terms up to second order in equations of motion
  $\Longrightarrow$ transient hydrodynamics

- Now: keep only first-order terms $\Longrightarrow$ Navier-Stokes limit

- No additional dynamical quantities,
  everything can be expressed in terms of α_0, β_0, u^μ, $\Omega^{\mu\nu}$ and their derivatives

- Full expression of Pauli-Lubanski vector lengthy

- From nonlocal collision term: contributions independent of spin potential

$$\delta\Pi^\mu(p) \simeq \frac{1}{2\mathcal{N}}\left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2}\right)\int d\Sigma_\lambda p^\lambda \chi_\sigma \sigma_\rho{}^{\langle\alpha}\epsilon^{\beta\rangle\nu\tau\rho} u_\tau p_{\langle\alpha} p_{\beta\rangle} + \ldots$$

$\Longrightarrow$ contribution from shear to local polarization, vanishes after momentum integration
Effects of shear important for description of local Λ polarization
S. Y.F. Liu, Y. Yin, JHEP 07 (2021) 188
B. Fu, S. Y.F. Liu, L. Pang, H. Song, Y. Yin, PRL 127 (2021) 14, 142301
F. Becattini, M. Buzzegoli, A. Palermo, PLB 820 (2021) 136519
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, PRL 127 (2021) 27, 272302

Conclusions and outlook

- Derived second-order spin hydrodynamics for spin-1/2 particles from kinetic theory using method of moments
- Dissipative corrections to Pauli-Lubanski vector
  $\Longrightarrow$ depend on all dynamical spin moments
  $\Longrightarrow$ Navier-Stokes limit: contributions from spin potential and shear
  $\Longrightarrow$ Need numerical implementation to compare to experiment
- Numerical simulation possible only if equations are causal and stable
  $\Longrightarrow$ Causality and stability analysis needed
- Derived kinetic theory for spin-1 particles
  D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation
  $\Longrightarrow$ Derive spin-1 hydrodynamics
- Include electromagnetic fields
  $\Longrightarrow$ Dissipative spin magnetohydrodynamics