Scheme Dependence of the

Chiral Phase Transition at High Densities

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[arXiv: 2206.13067]

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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Outline

1) General Objectives and Setup

2) Phase Diagram at High Density

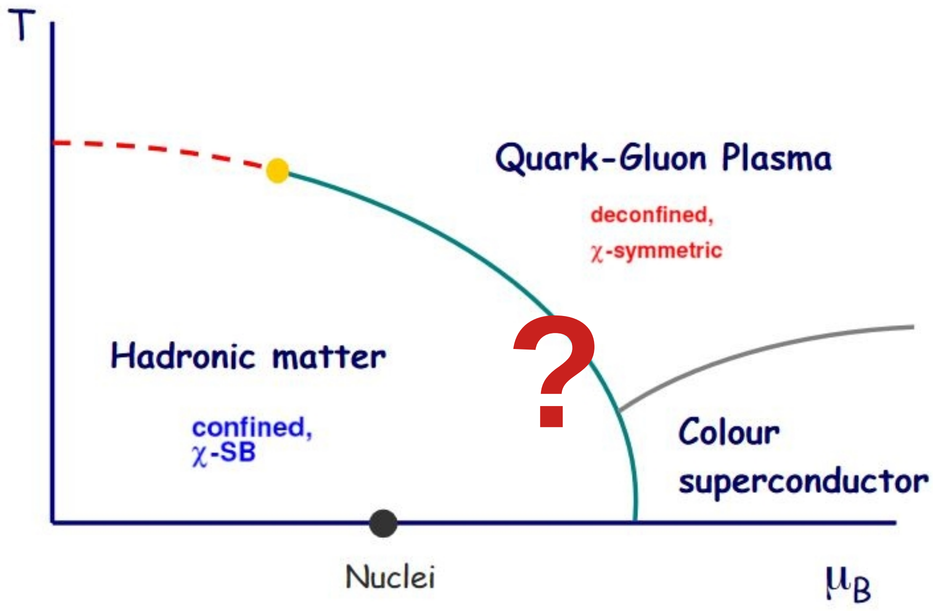
3) Regulator Scheme Dependence

4) Summary & Outlook

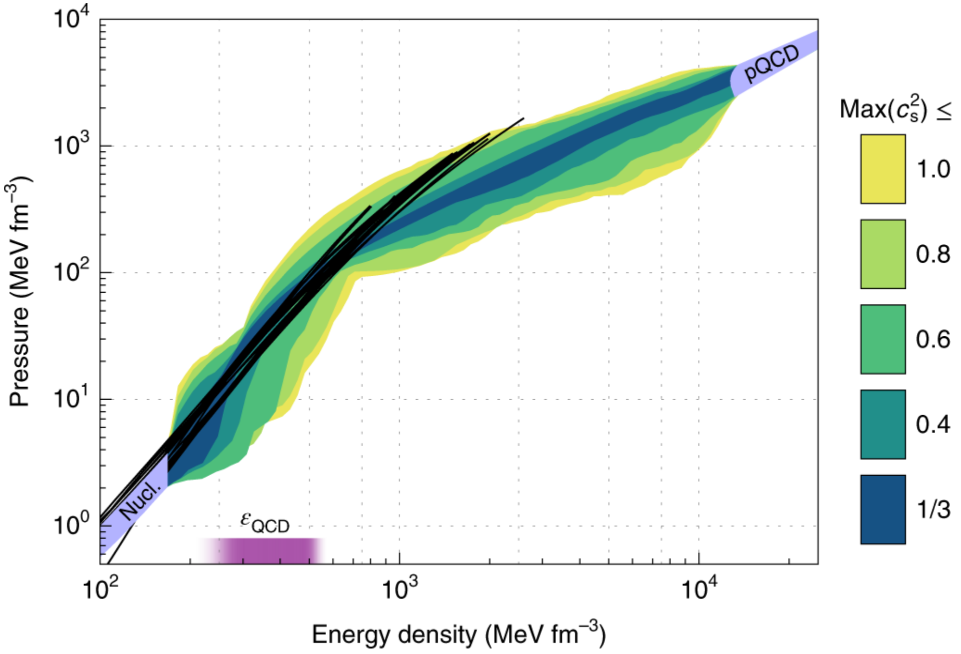
1) General Objectives and Setup

Objectives

[Blaizot (2004)]



[Annala et al.(2019)]



- QCD at low energies:
  - Chiral phase structure
  - Dynamical mass generation
  - Equation of State

- Especially interested in large μ_B and low T
  - neutron stars
  - lattice <u>not</u> applicable, use functional methods
  - Rely on effective models

Functional Renormalization Group

- Introduce an UV- and IR-regulation to our theory

  ➔ scale dependent effective action Γ_k

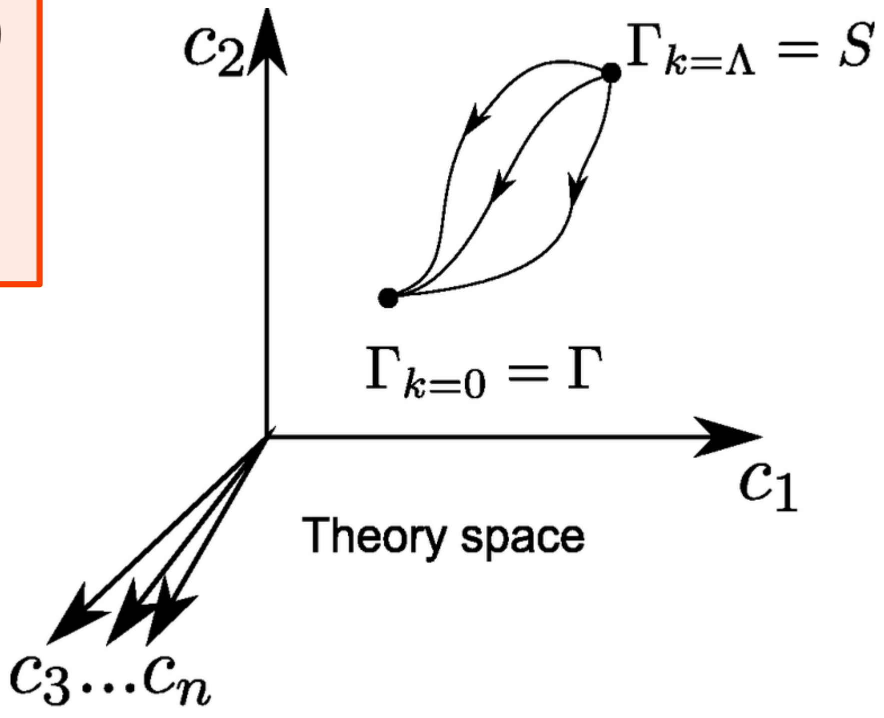
- Scale dependence of Γ_k

Wetterich equation: $(\partial_t = k\partial_k)$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ (\partial_t R_k) \left(\Gamma_k^{(1,1)} + R_k \right)^{-1} \right\}$$

Requirements for regulators:

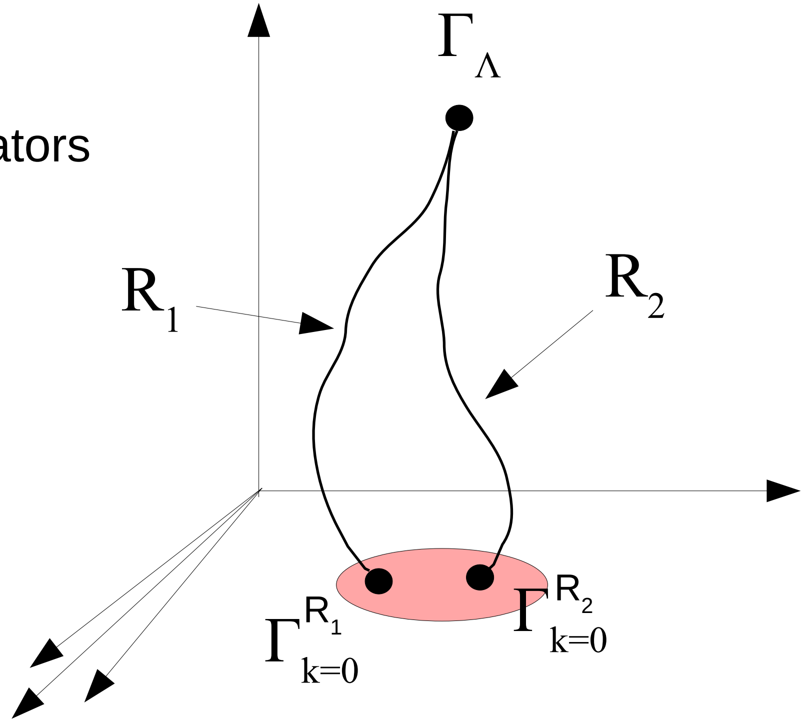
1. $\lim_{p^2/k^2 \to \infty} R_k(p^2) = 0$
2. $\lim_{p^2/k^2 \to 0} R_k(p^2) > 0$
3. $\lim_{k \to \infty} R_k(p^2) = \infty$



Truncation Errors

- Solving Wetterich eq. for full theory not possible
  - ➔ Use effective model / truncation
- Leads to truncation errors and regulator dependence of results
  - ➔ Choice of regulator becomes relevant
    - ↳ Different optimization criteria for regulators

      (Principle of minimum sensitivity,
      "Gap Criterion" [Litim(2000)],
      "Shortest Path" [Pawlowski(2007)])



Effective Action for QCD

➢Quark-meson model (N_f=2, N_c=3):

$$\phi = \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$$

$$\Gamma_k = \int_x \left\{ \underbrace{\bar{\psi} \left[Z_{\psi,k} \left(\not{\partial} - \mu\gamma_0 \right)}_{\text{kin. term}} + \underbrace{g_k \left(\sigma + i\gamma_5 \vec{\tau}\vec{\pi} \right) \psi \right]}_{\text{Yukawa interaction}} + \underbrace{\Omega_k \left(\phi^2 \right) + \frac{Z_{\phi,k}}{2} \left(\partial_\mu \phi \right)^2}_{\text{derivative expansion}} - \underbrace{c\sigma}_{\text{expl. sym. breaking}} \right\}$$

➢Various truncations:

Label	Scale Dep.
LPA	Ω_k
LPA+Y	Ω_k, g_k
LPA’	$\Omega_k, Z_{\psi,k}, Z_{\phi,k}$
LPA’+Y	$\Omega_k, g_k, Z_{\psi,k}, Z_{\phi,k}$

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kin. term | Yukawa interaction | derivative expansion | expl. sym. breaking

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LPA'	Ω_k, $Z_{\psi,k}$, $Z_{\phi,k}$
LPA'+Y	Ω_k, g_k, $Z_{\psi,k}$, $Z_{\phi,k}$

➢Restrict ourselves to LPA:

$$g_k \equiv g\,, Z_{\Psi,k} = Z_{\Phi,k} \equiv 1$$

➔ Optimized regulator:

$$R_k^{flat,3d} = (k^2 - \vec{p}^2)\,\Theta(k^2 - \vec{p}^2)$$

Effective Potential

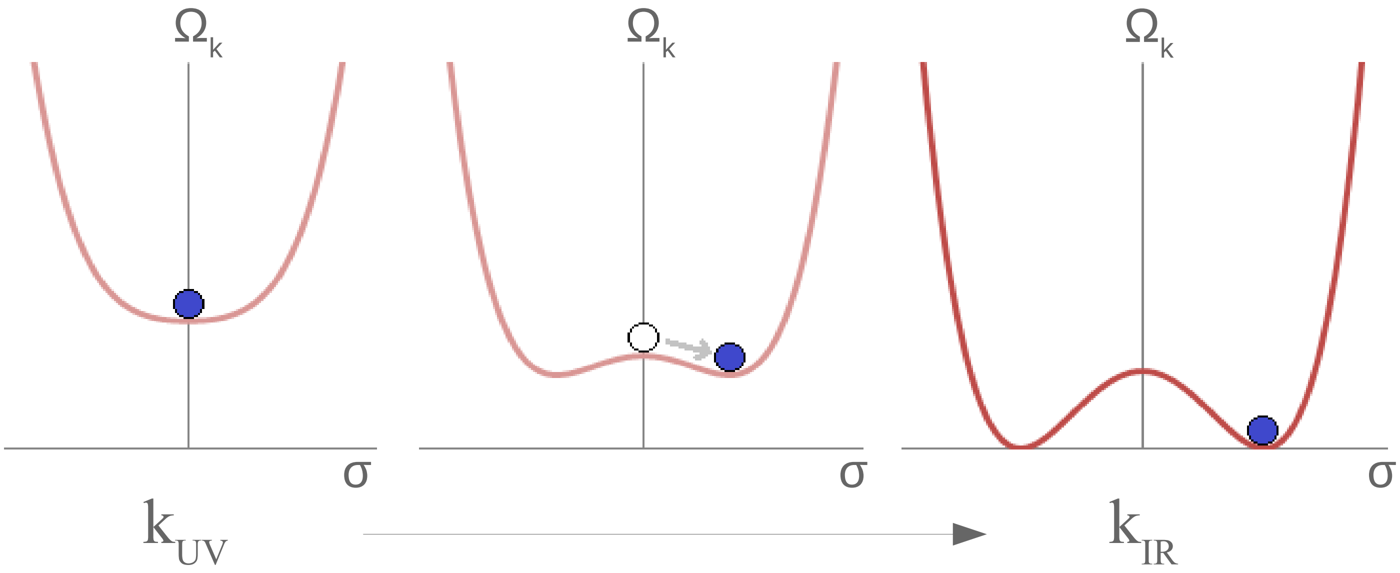
$$\phi = \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$$

- UV-Ansatz for the effective potential: $\Omega_\Lambda = a_1 \phi^2 + \frac{a_2}{2} \phi^4$

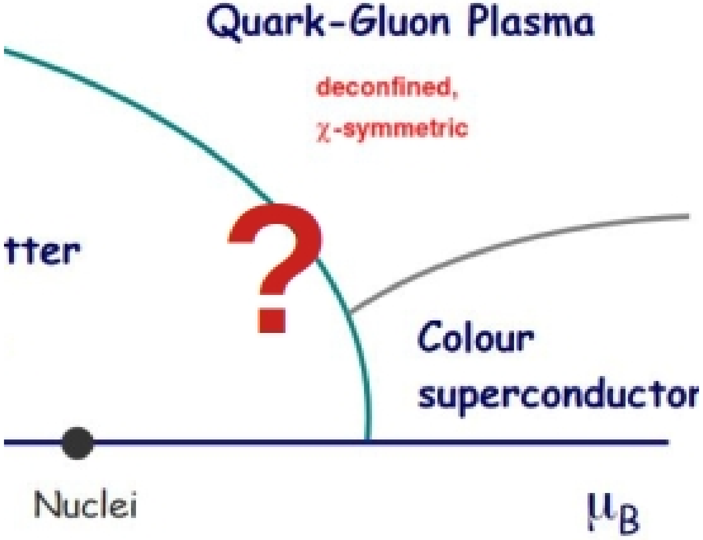
- Parameter fixing (a_1,a_2,c and g) to get correct vacuum values

$$\begin{aligned} \bar{m}_\sigma &= 550\,\mathrm{MeV} \\ \bar{m}_\pi &= 138\,\mathrm{MeV} \\ \bar{m}_\psi &= 300\,\mathrm{MeV} \\ f_\pi &= 93\,\mathrm{MeV} \end{aligned}$$

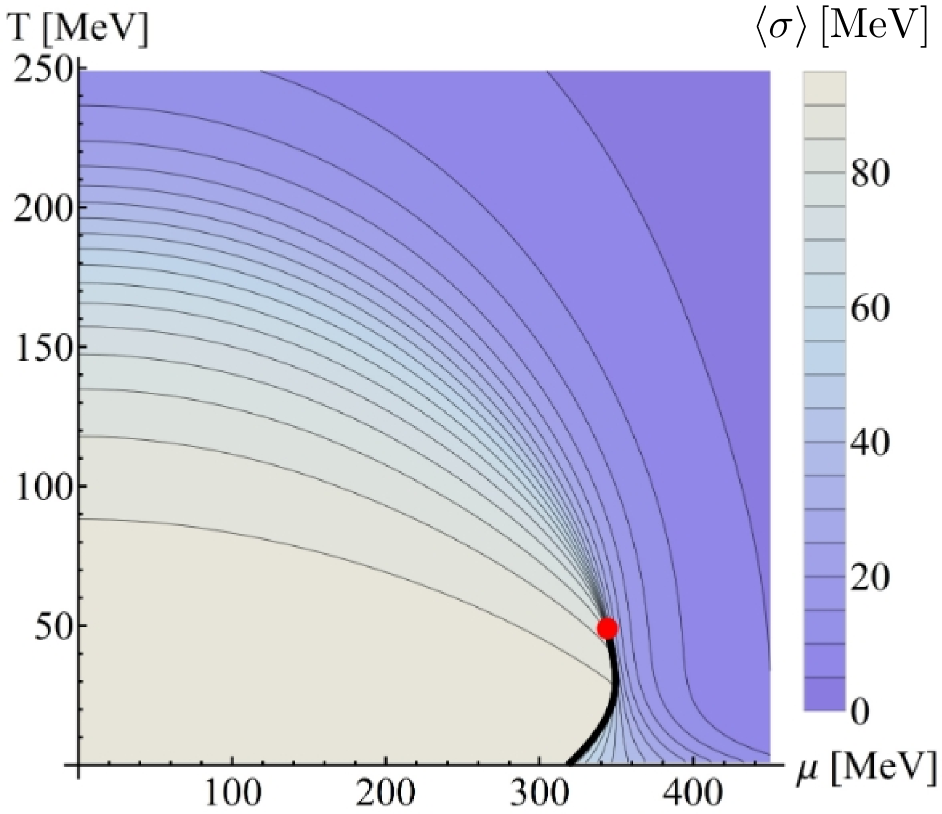
- Spontaneous symmetry breaking:



2) Phase Diagram at High Density

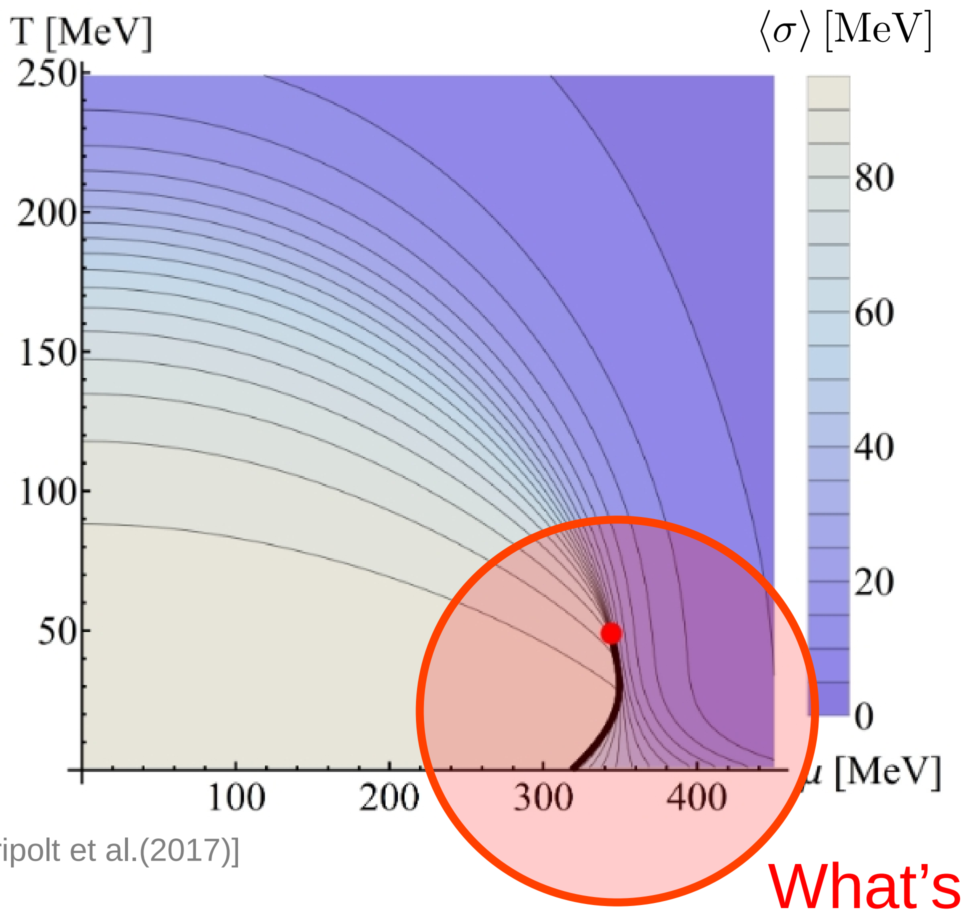


Chiral Phase Transition



[Tripolt et al.(2017)]

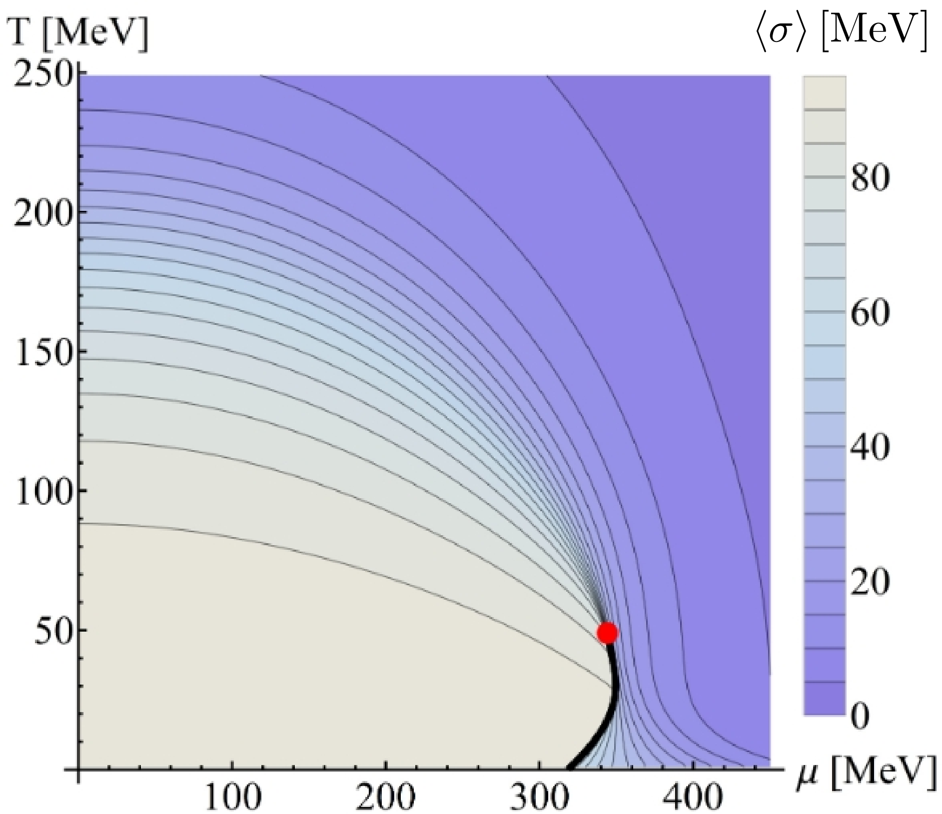
Chiral Phase Transition



[Tripolt et al.(2017)]

What's going on here ?

Chiral Phase Transition



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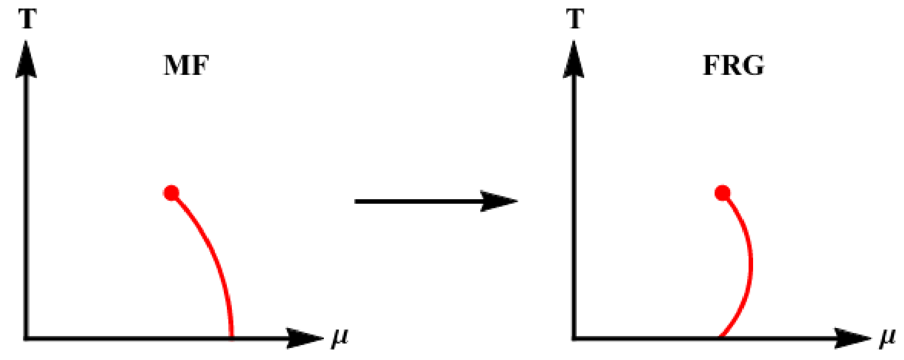
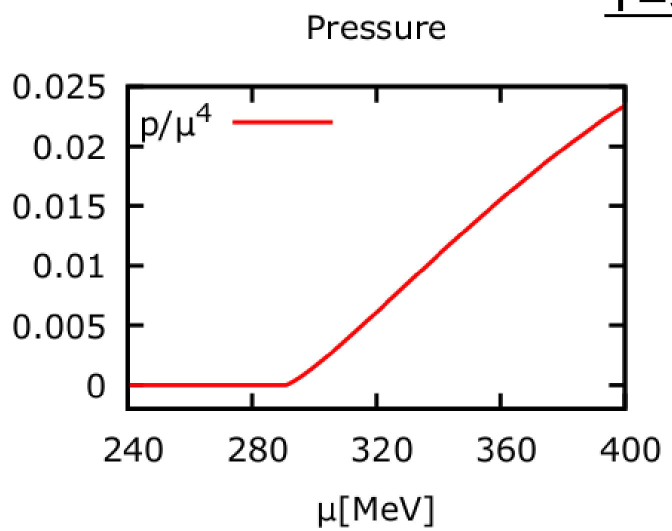
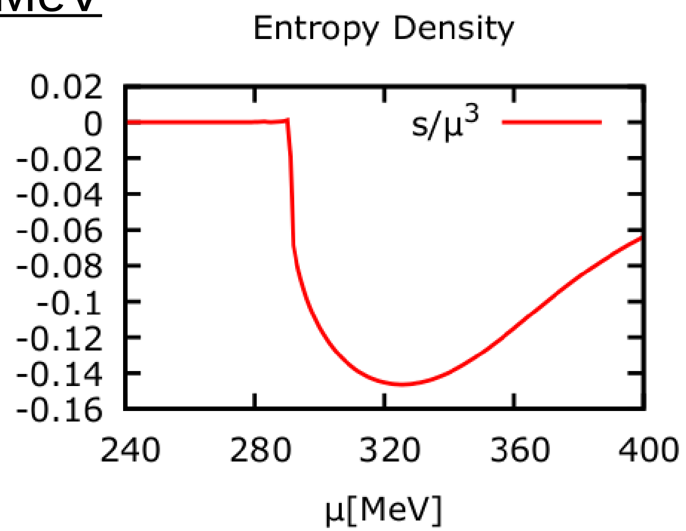


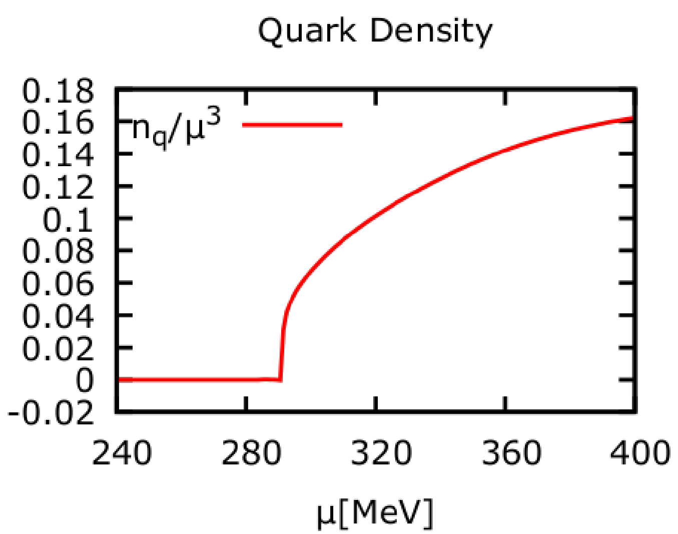
- Strange back-bending in the phase diagram
- Only small first order transition with residual condensate
- Not found in mean field (MF) calculations

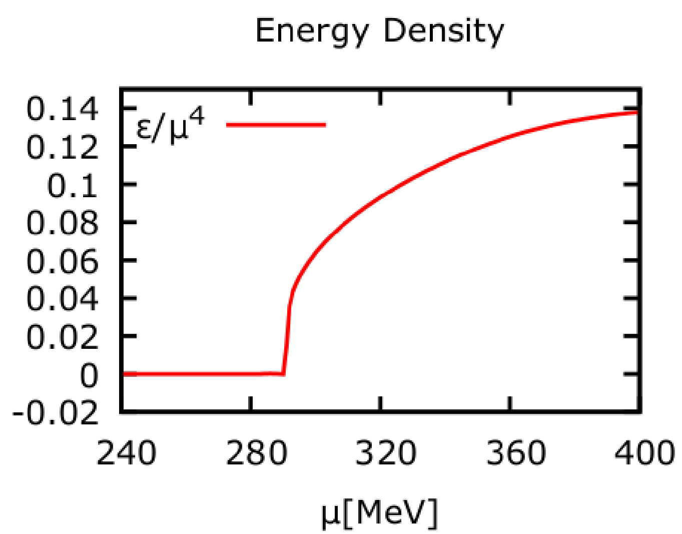
Thermodynamics

T=5 MeV









Thermodynamics

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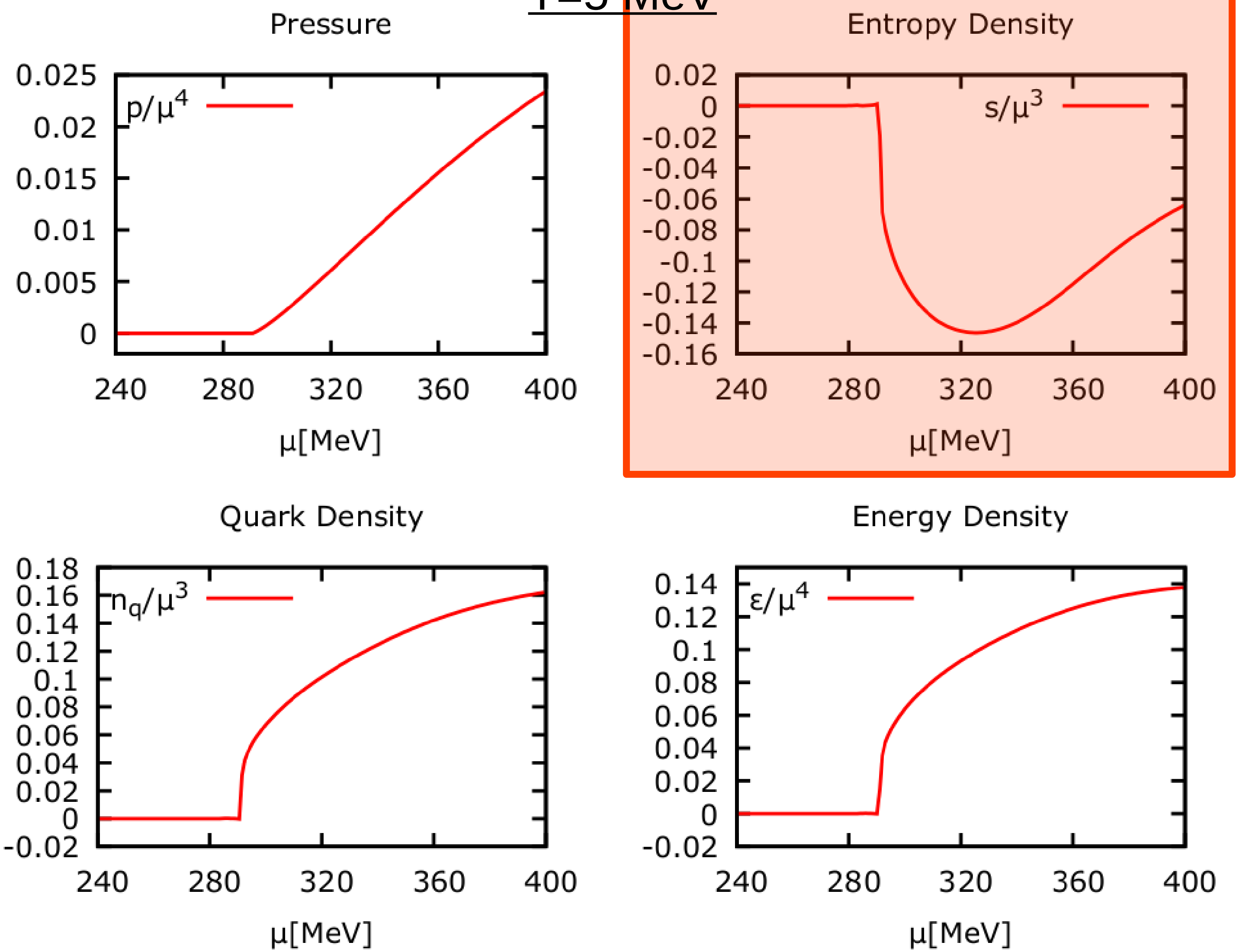


➔ Entropy densities are negative!

↳ Unphysical result but consistent with Clausius-Clapeyron:

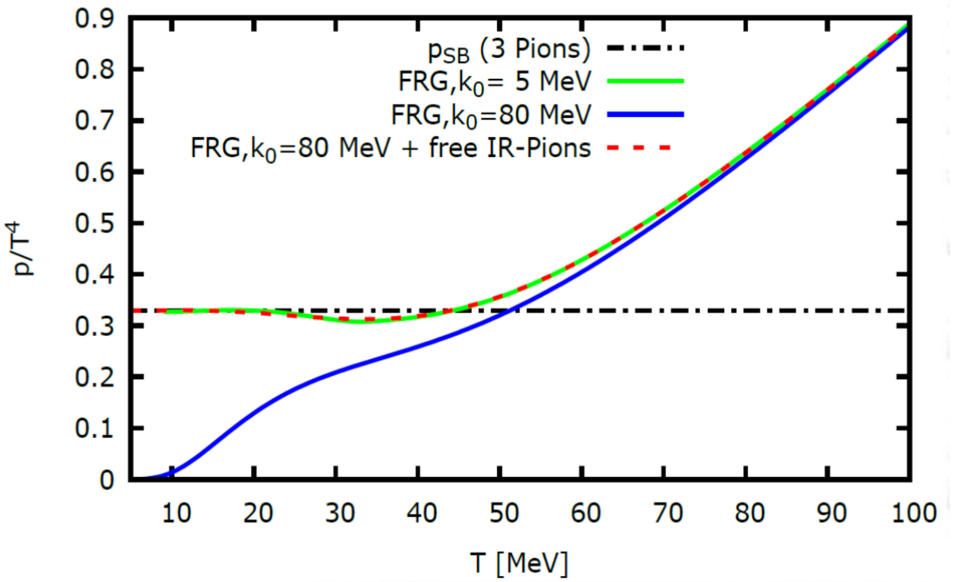
$$\frac{dT_c}{d\mu_c} = -\frac{\Delta n}{\Delta s}$$

Missing Physics Due to Finite Infrared Cutoff?

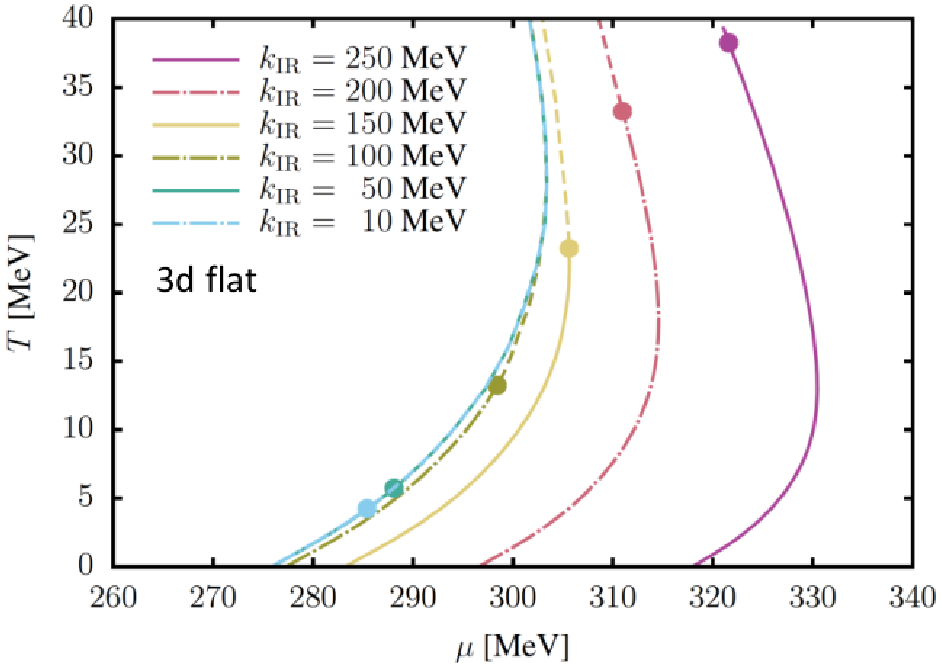
- Example pressure in the chiral limit:

  At low temperatures the infrared cutoff $k_{IR}=k_0$ results in large errors

  Maybe it's similar for back-bending?



- Chiral phase transition for various k_{IR}:



- Back-bending found over wide range of IR-scales
- Transition line freezes out at scales around k=100 MeV
  - ↳ Not caused by missing IR physics
- CEP sensitive to choice of k_{IR}

Potential Reasons

- Signal for physics which are not captured in this model
  - Inhomogeneous Phases:
    - Assumption of spatial homogeneity wrong?
  - Color Superconductivity:
    - Attractive diquark channel could lead to Cooper pairing and a supercond. phase

      (comp. NJL studies)

- Missing DoF which might be vital for the thermodynamics in this region:
  - Vector mesons
  - Diquarks
  - ...
- Truncation artifact

  → Check scheme dependence: Use different regulators

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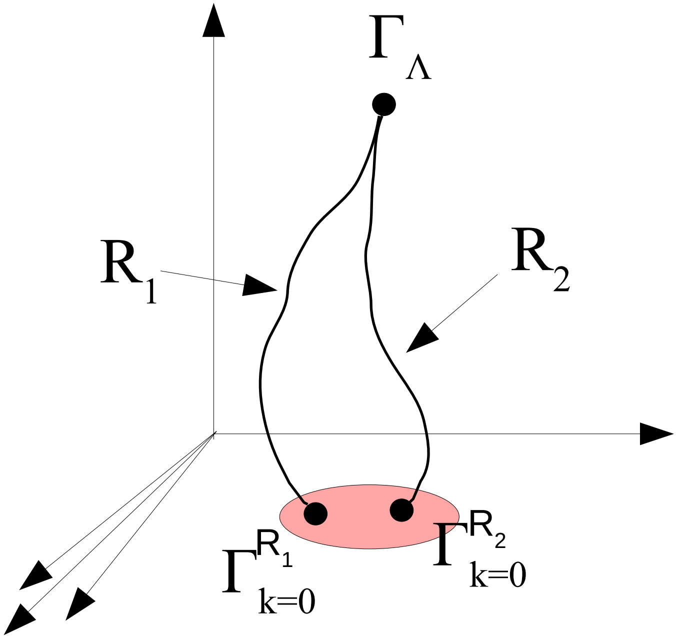
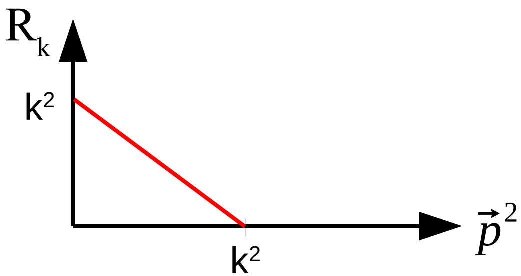
3) Regulator Scheme Dependence



Regulator Choices



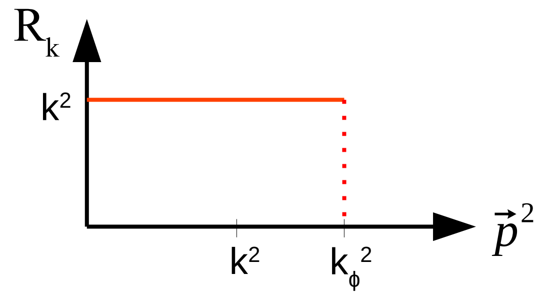
- Flat "Litim" regulator:

$$R_k^{flat,3d} = (k^2 - \vec{p}^2)\,\Theta(k^2 - \vec{p}^2)$$

➔ Replaces (3-)momentum in loop propagators:

$$G_k^{-1} = p_0^2 + k^2 + m^2$$

➔ Optimized for LPA, "usual" choice



- Mass-like regulator:

$$R_k^{mass,3d} = k^2\,\Theta(k_\phi^2 - \vec{p}^2)$$

➔ Momentum structure unchanged:

$$G_k^{-1} = p_0^2 + \vec{p}^2 + k^2 + m^2$$

➔ Not obtained via any optimization criterion

Pole Proximity in the Vacuum

- Observation: Vacuum flows for small σ values run alongside pion pole
  - Example flat regulator:

$$E_\pi = \sqrt{k^2 + 2\Omega'_k}$$

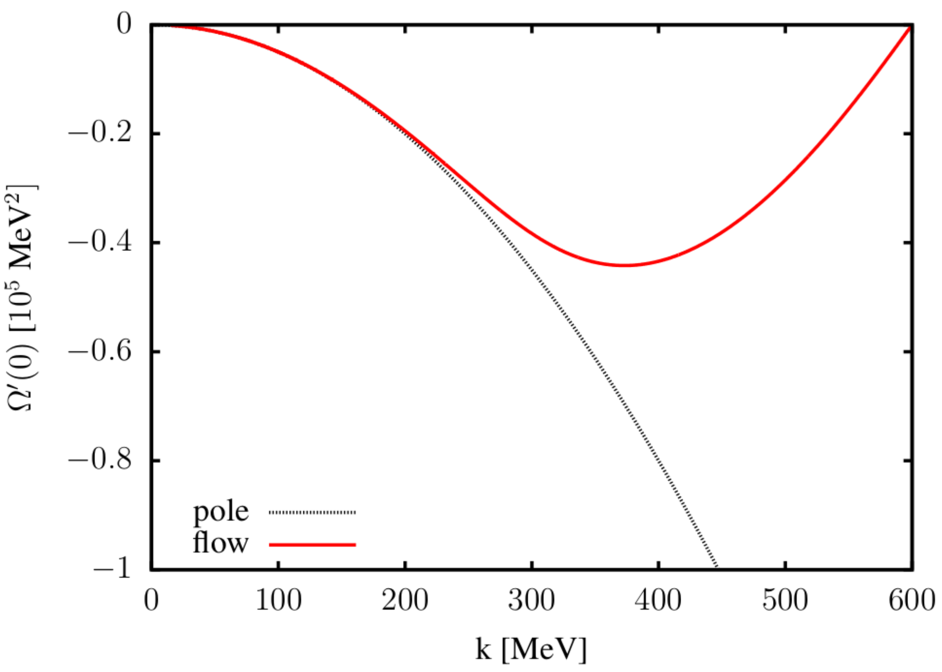
$$\partial_t \Omega_k^{\text{vac}}\big|_{\sigma^2=0} = \frac{k^5}{12\pi^2}\left(\frac{4}{E_\pi} - \frac{24}{k}\right)$$

➔ At low scales: $E_\pi / k \sim 0.01$

  - Flows with mass-like regulators much closer to the pole:

$$E_\pi / k \sim 10^{-150} (\text{estimated})$$

➔ vacuum calculations and usual way of parameter fixing unfeasible



"New" Parameter Fixing

- Silver Blaze: For T=0, $\mu<\mu_c$ observables are independent of μ

  ➔ use this property to extrapolate f_π and m_σ to the vacuum

[Berges et al.(1997)]

- Use alternative approach for parameter fixing:

k_ϕ

Approximated LPA' flow eq's at large scales

→partial IR fixed point

k_χ

- Solve full LPA flow starting at fixed point
- Tune k_ϕ and k_χ to adjust IR-values

k_{IR}

At large scales $k>k_\chi$:
Neglect mesonic contributions

- Simple flows for Yukawa coupling and wavefunction renormalizations at σ=0:

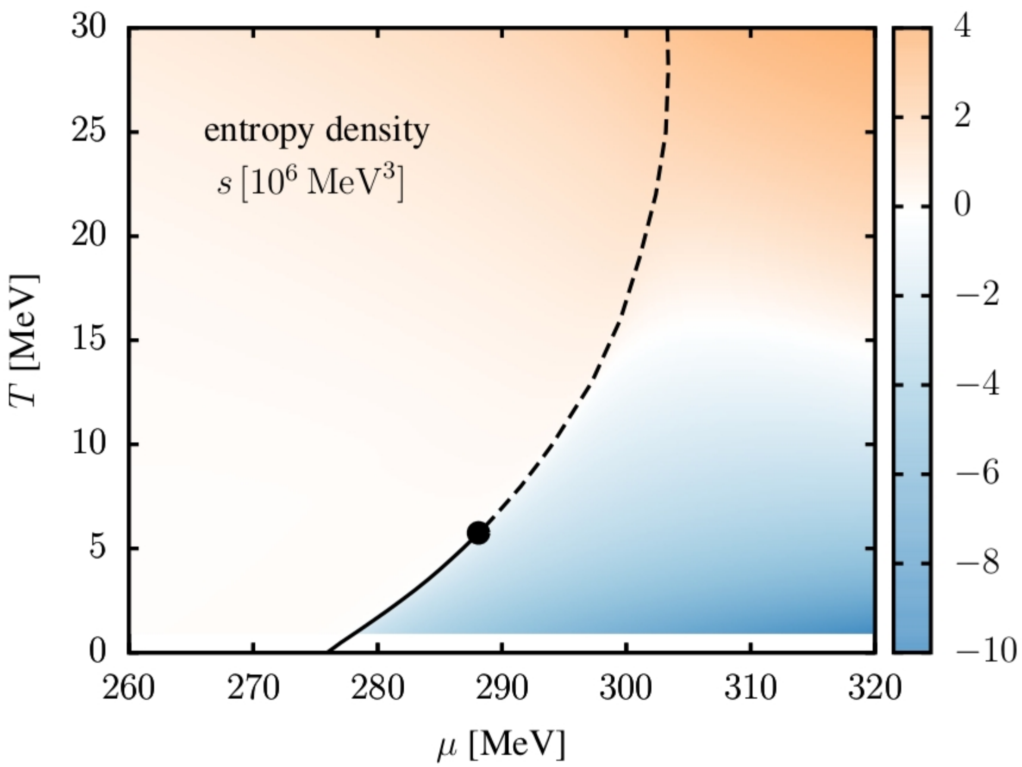
$$\partial_t g_k^2 = \eta_{\phi,k} g_k^2 \qquad \eta_{\phi,k} = \frac{N_c g_k^2}{8\pi^2}$$

$$\eta_{\psi,k} = 0$$

- IR fixed points for coefficients n≥2 in the field expansion of the dimensionless potential

Main Results

- Flat regulator:



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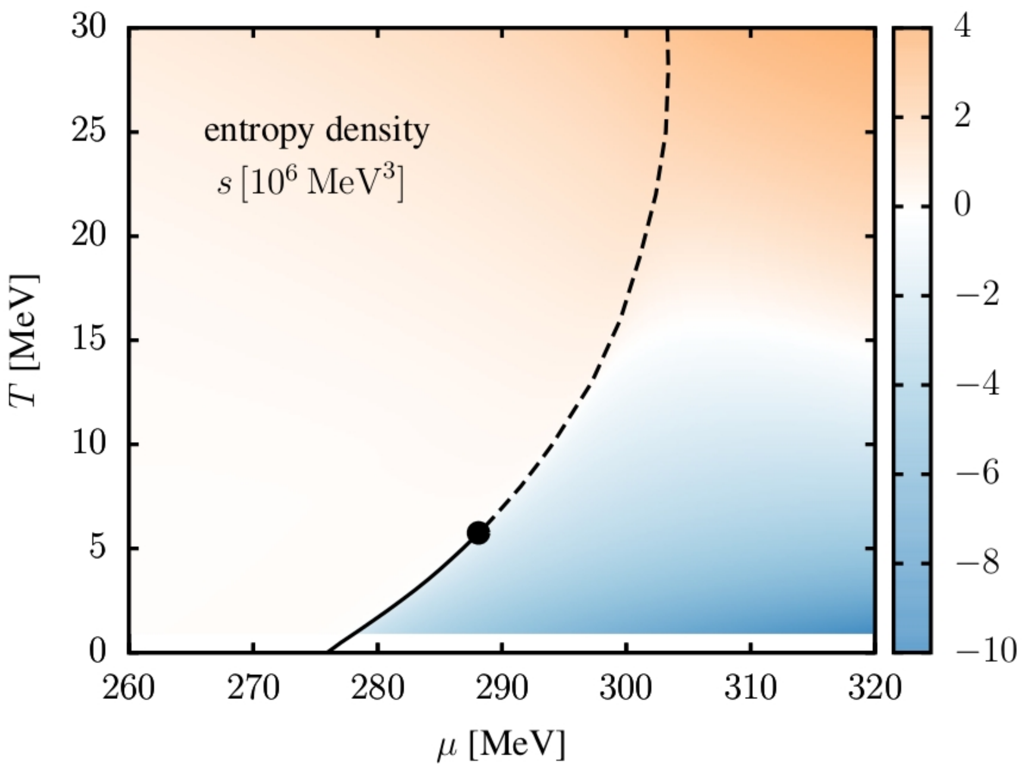
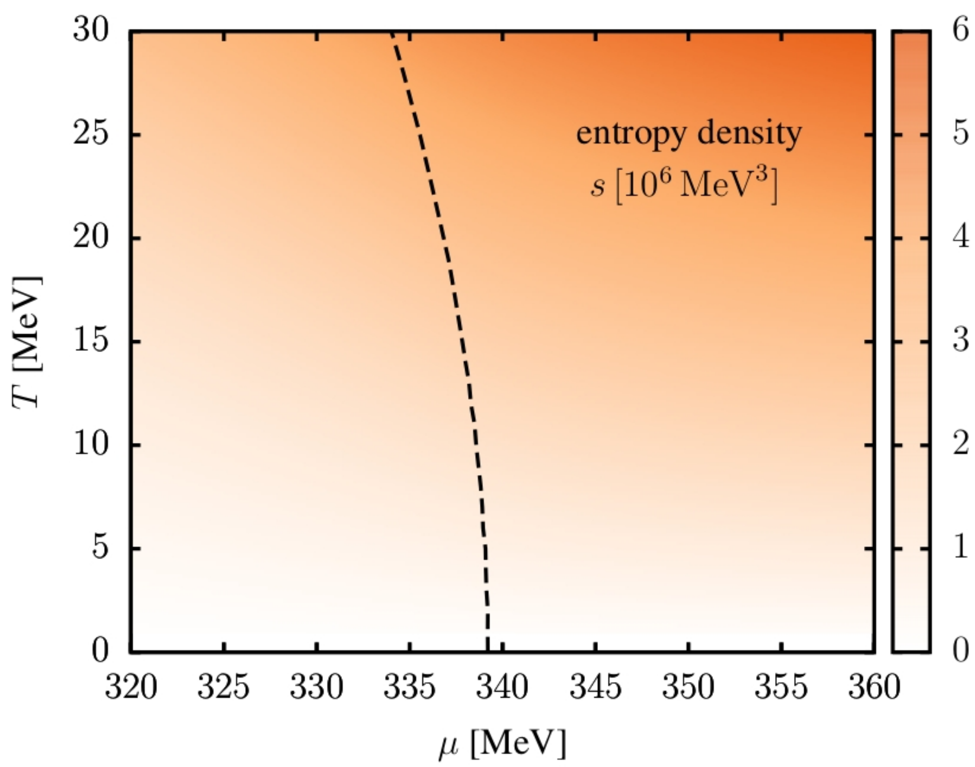


- Mass-like regulator:



→No back bending/ negative entropy densities with mass-like regulator!

→Very strong scheme dependence. Why?

Possible Explanation

What causes the back-bending? What has changed when we switched regulators?

- <u>At large chemical potentials:</u>

  Fermi momentum p_F acts as infrared cutoff, e.g. fermionic contribution for mass-like regulator:

$$\partial_t \Omega_k = \int_{p_F}^{k_\phi} \mathrm{d}p \; \mathrm{I}\left(p^2\right) \qquad p_F = \sqrt{\mu^2 - k^2 - m_\psi^2}$$

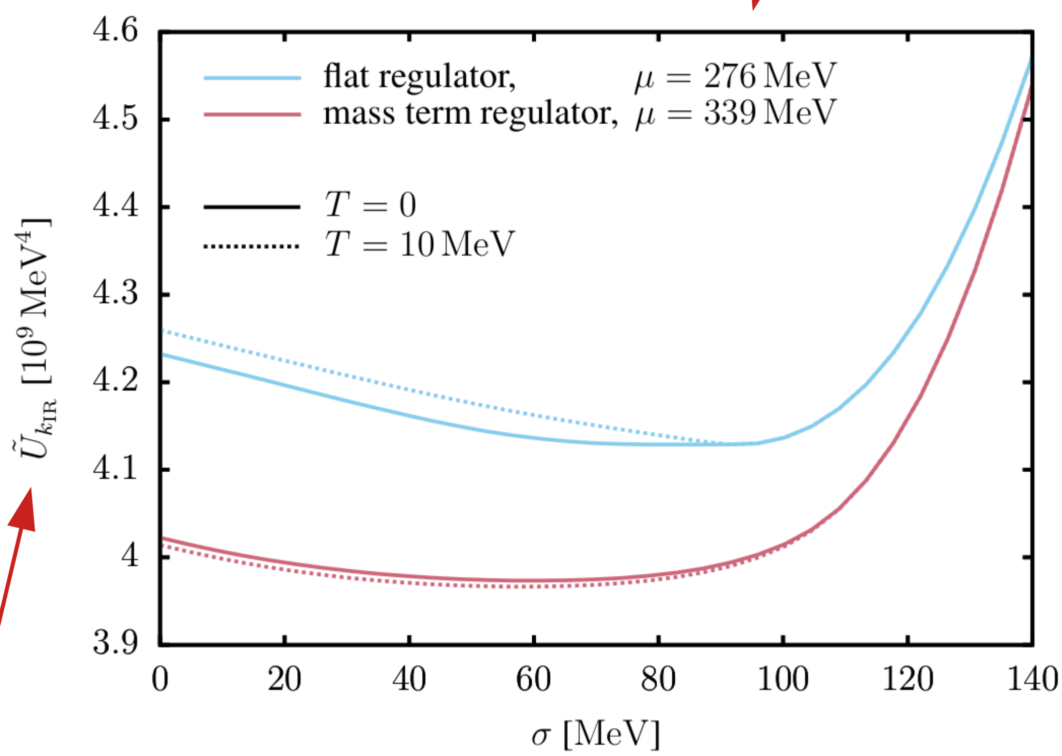
  ➔ Fermions decouple from flow

- Decoupling changes almost arbitrarily with choice of regulator, truncation can't compensate this

  ↪ Truncation artifacts!

Closer Look at the Effective Potential

crit. values at T=0



explicit sym. breaking included

Mass-like regulator:

- Small variation between T=0 and T=10 MeV
- Potential pushed **downwards** for increasing temperature

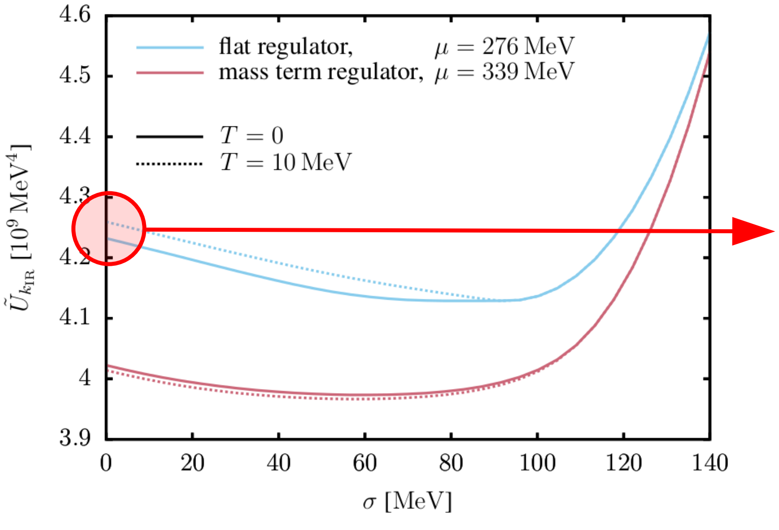
  ↪ chiral symmetry restoration

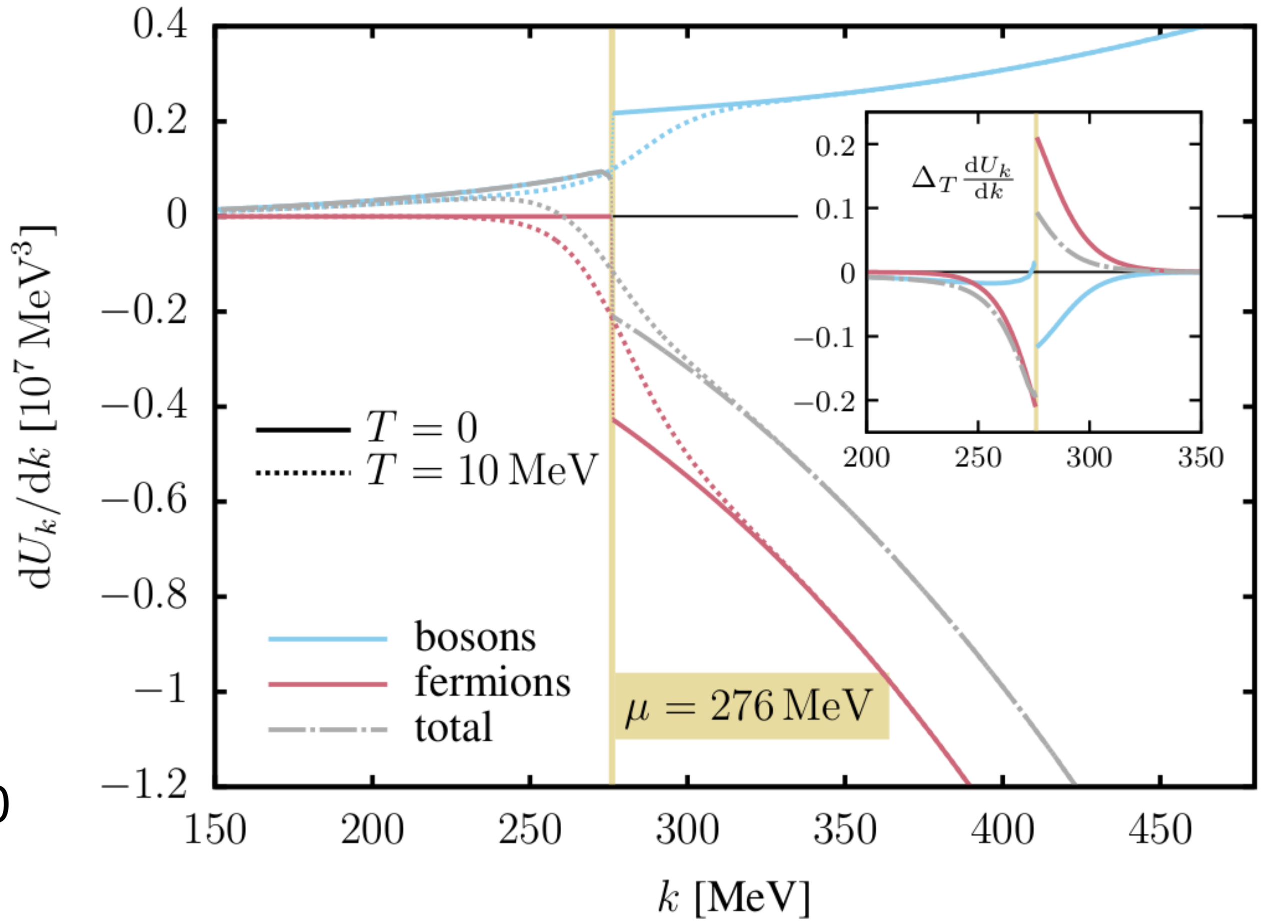
Flat regulator:

- Stronger temperature dependence for small fields
- Potential pushed **upwards**

  ↪ chiral symmetry breaking increases with T

Flows with Flat Regulators





- Flat regulator:
  - Instant decoupling for T=0
  - Strong T-dependence
  - Symmetry restoration at low scales decreases with T

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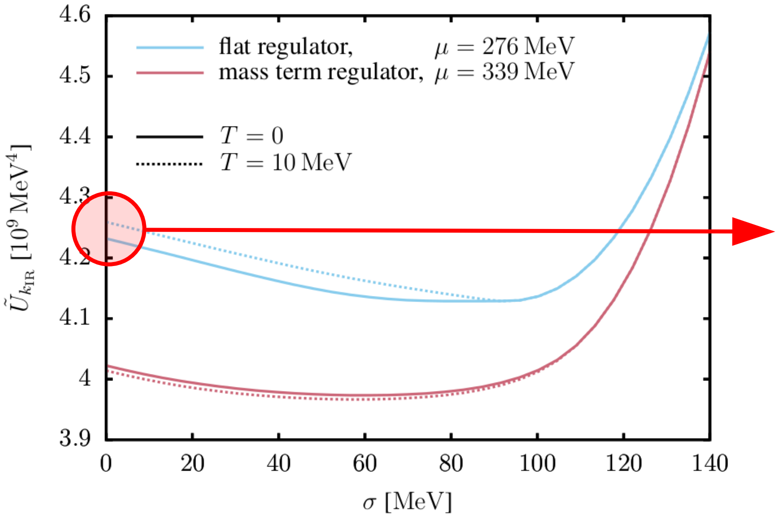
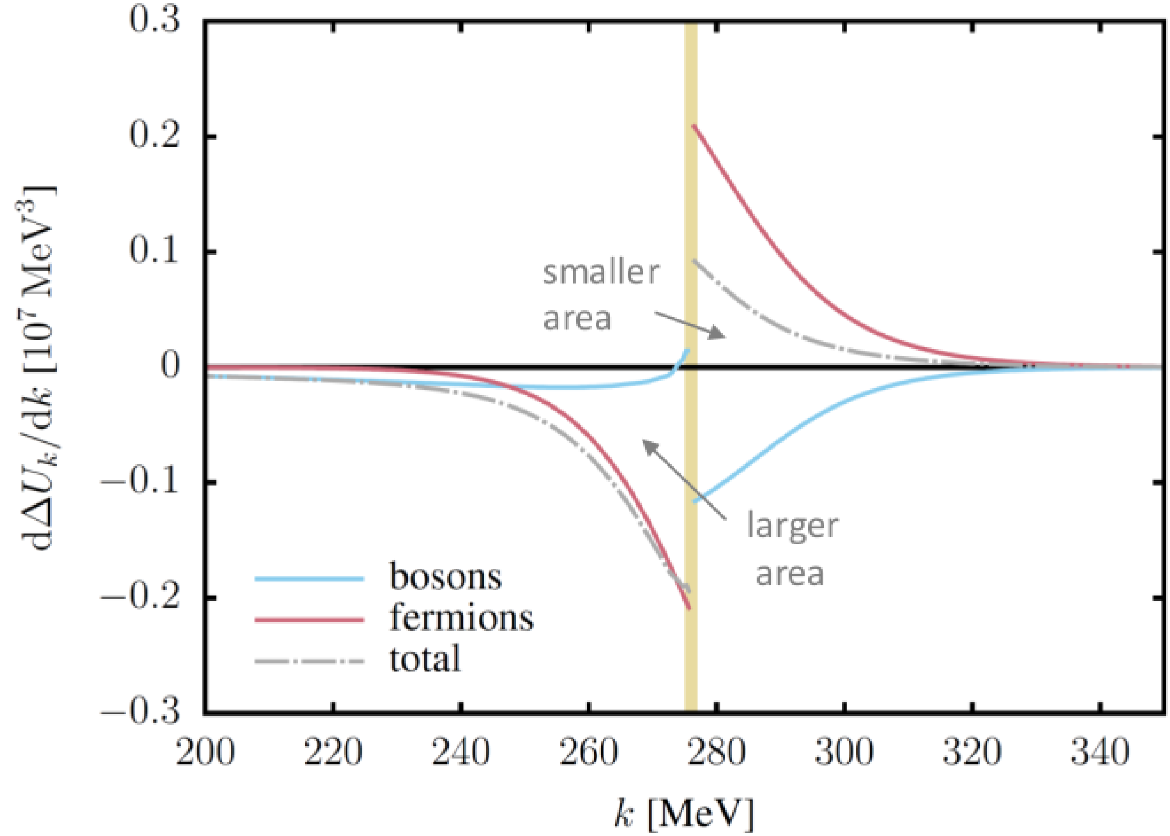
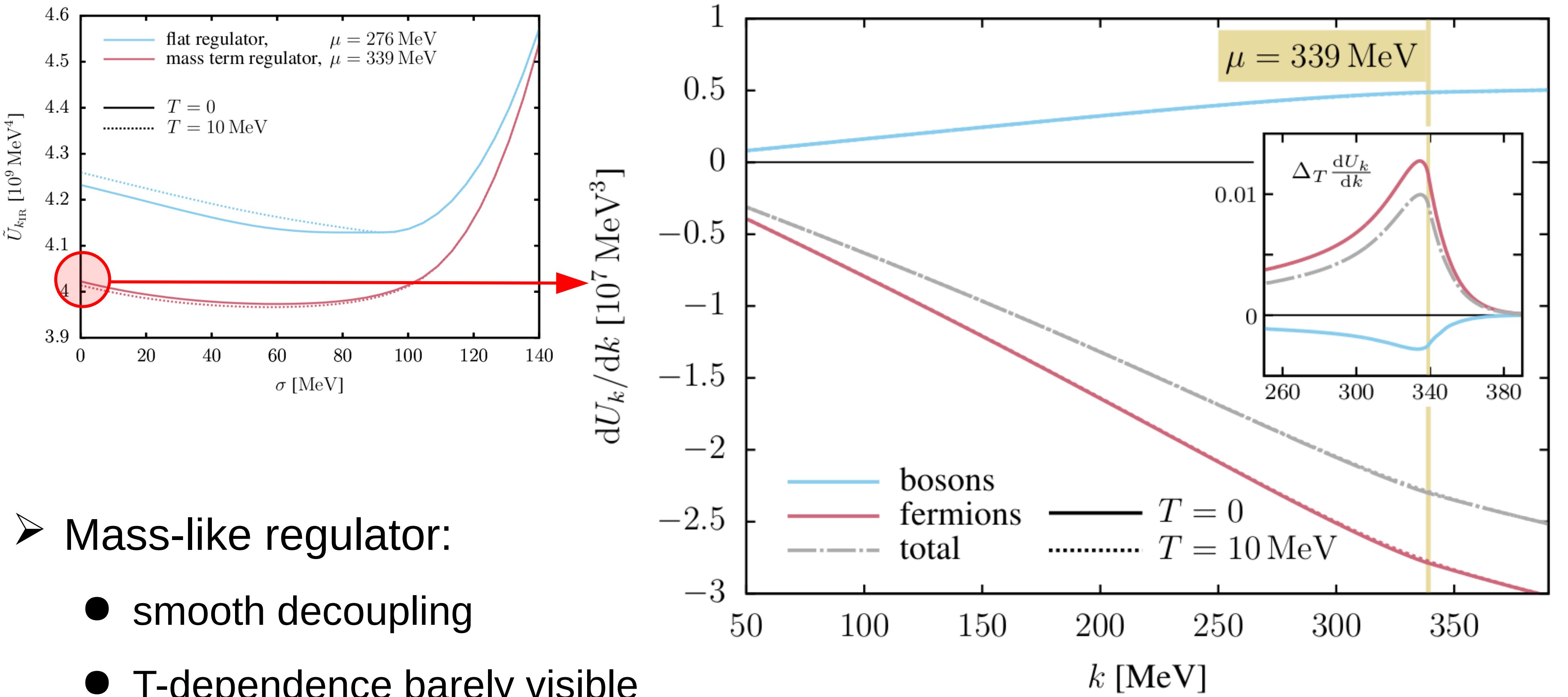




- Flat regulator:
  - Instant decoupling for T=0
  - Strong T-dependence
  - Symmetry restoration at low scales decreases with T
  - Bosonic contributions cause net increase in symmetry breaking when temperature is raised

Flows with Mass-like Regulators



- Mass-like regulator:
  - smooth decoupling
  - T-dependence barely visible
  - sym. breaking decreases with temperature for all scales

Fazit Fermion Decoupling

- Flat regulator:
  - Abrupt decoupling leads to strong T-dependencies
  - Backcoupling into bosonic flow leads to asymmetry and increased symmetry breaking at finite T
- Mass-like regulator:
  - Smooth decoupling, nearly unperturbed by regulator
  - Symmetry restoration when temperature is raised

4) Summary and Outlook

4) Summary

- ➢ Investigated truncation artifacts in the FRG framework
- ➢ Results with flat regulator at large densities:
  - ➔ phase transition shows a strange back-bending
  - ➔ negative entropy densities
- ➢ Mass-like regulator: **No** back-bending, entropy remains **positive**
- ➢ <u>Likely reason</u>: Regulator induced change in the fermion decoupling causes artifacts

4) Outlook

- Optimize setup to allow access to full range of temperatures and chemical potentials

- Find solutions/regulators for more advanced truncations, e.g. LPA' or higher order derivative expansions

- Effects on neutron star equation of state and mass-radius relations?

    - see [arXiv:1910.11929] and [arXiv:2007.07394] for first FRG works on those topics

Backup Slides

Why are we using 3d-regulators?

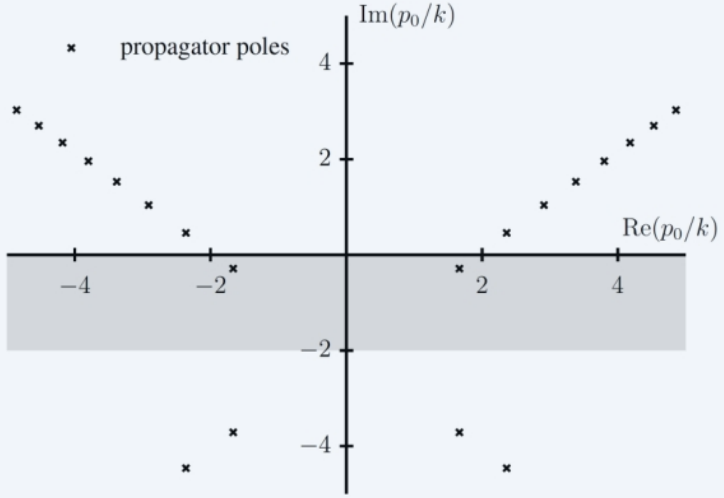
- Downside: Dimensionally reduced regulators break O(4) symmetry

- Upsides:
  - Matsubara summation can be performed analytically
  - Easiest way to prevent regulators from breaking Silver Blaze symmetry

4d fermionic regulators: $R_k^F = i\not{p}\, r^F(p^2/k^2)$

- Necessary for Silver Blaze: $R_k^F(p,\mu) = R_k^F(\tilde{p},0) \text{ with } \tilde{p} = (p_0 + i\mu,\, \vec{p})$

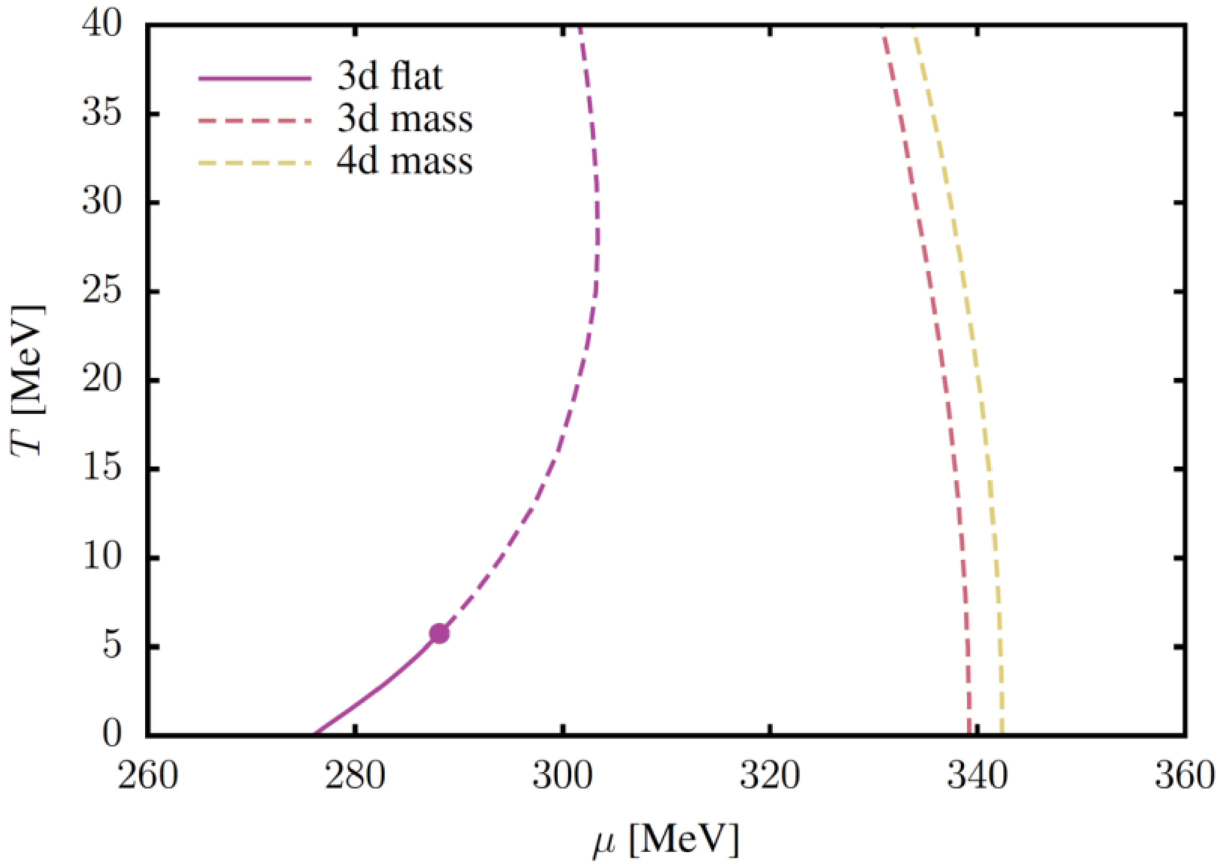
- Silver Blaze violation still possible due to regulator induced complex poles, e.g. with exponential reg.:



- Discussion and Solutions:
  [Pawlowski,Strodhoff (2015)]

Why are we using 3d-regulators?

- Mass-like regulators allow us to circumvent this problems
- Only small quantitative differences found:

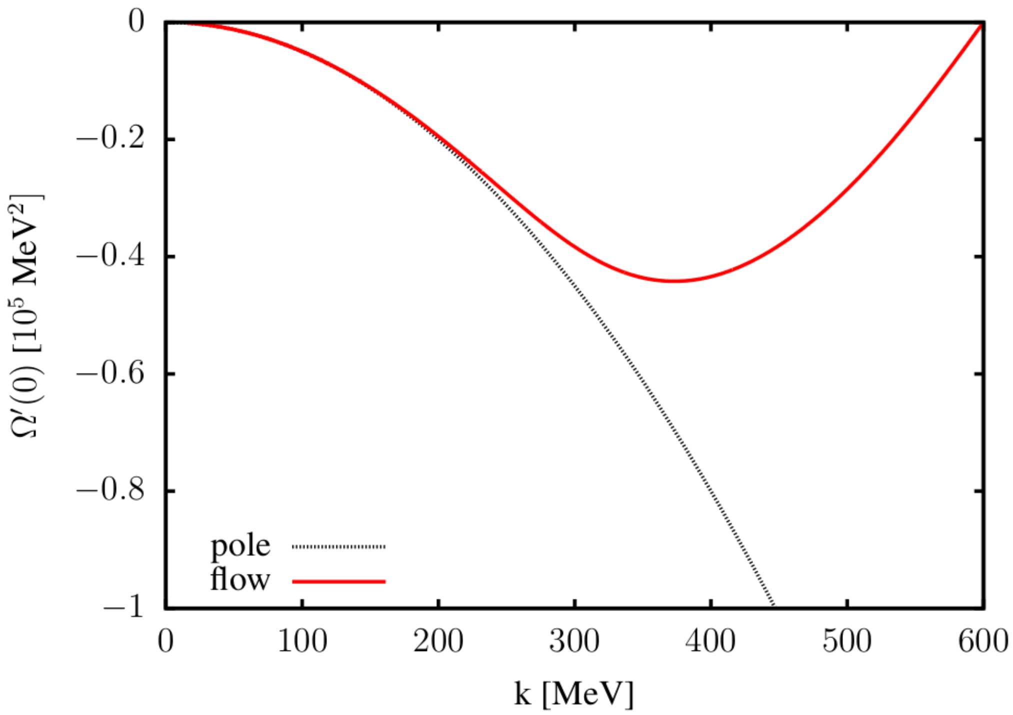


→ Minor impact of dimensional reduction on results

→ Especially: No connection between back-bending and 3d-regulators

Pole Proximity: Estimation

➢ Reminder: Vacuum flow runs close to the pole at E_π=0:

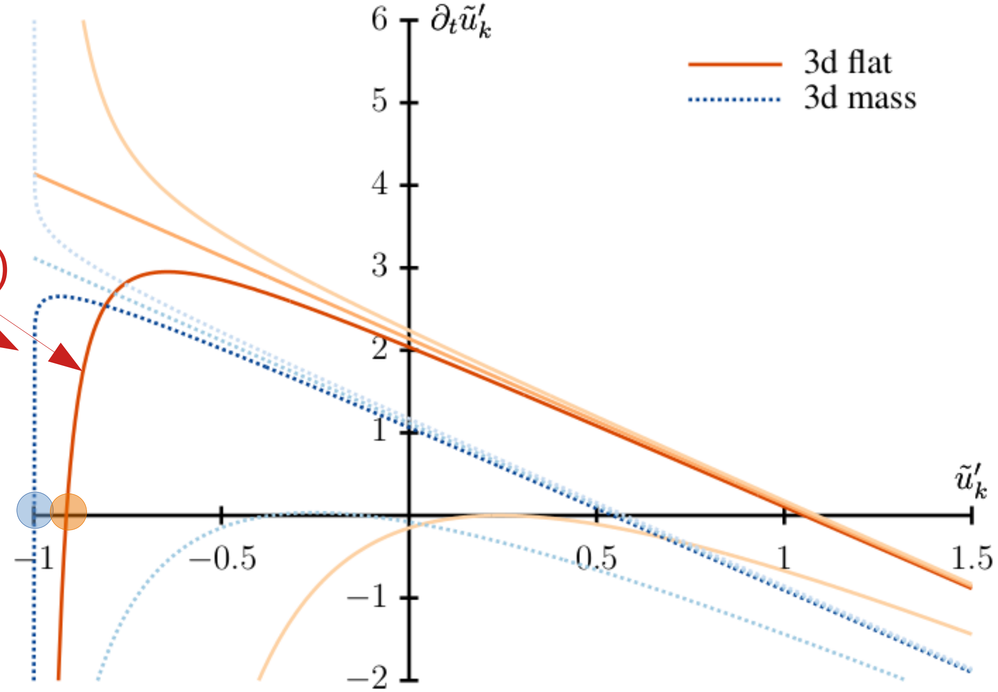


How can we estimate the distance to this pole for a given regulator?

Pole Proximity: Estimation

- For convenience define $\tilde{u}'_k = 2U'_k(0)/k^2$ ⟶ Pole at $\tilde{u}'_k = -1$
- Examine it's flow equation for different (fixed) values U'' :

Relevant cases (small U''>0)



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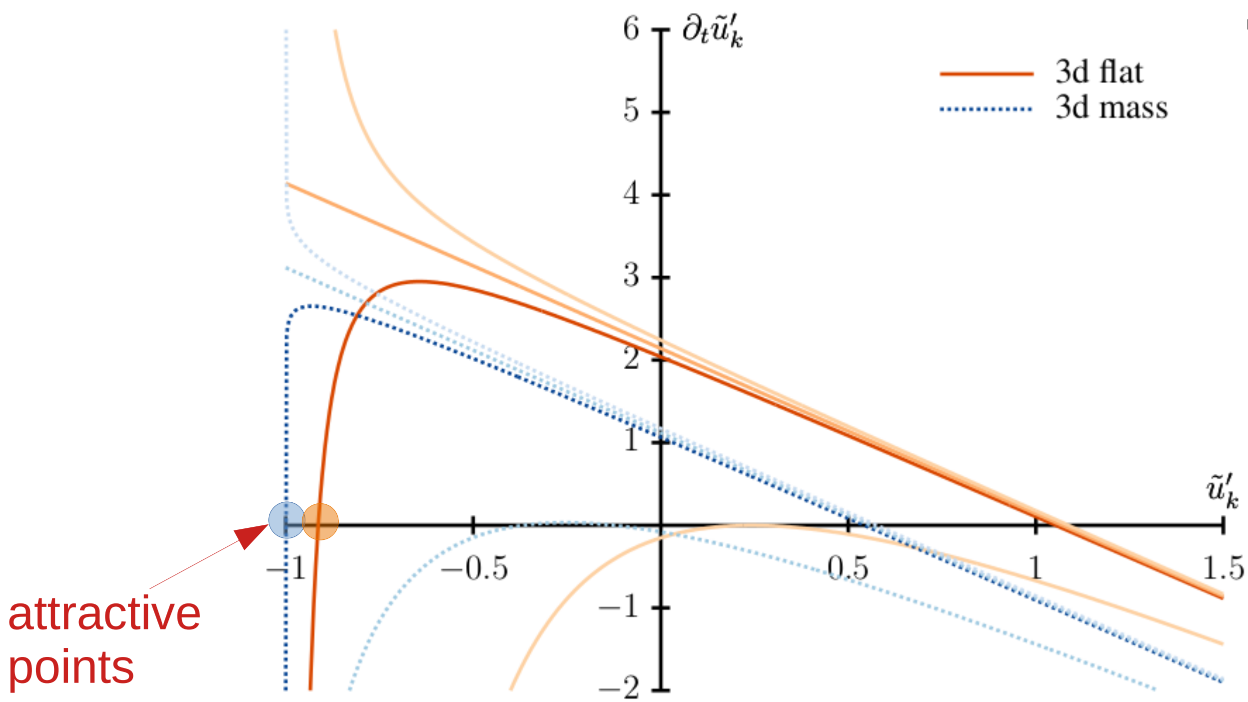

➔ Relevant cases have <u>attractive stationary point</u> near pole

↪ no fixed point (depends on U'')

↪ Explains behavior observed in the flow

↪ Setting $\partial_t \tilde{u}'_k = 0$ and expanding in $\delta u = \tilde{u}'_k + 1$ allows to approximate distance between pole and attractive point