Volume and Quark Mass Dependence of QCD

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Based on:
JB, Fischer, Isserstedt, Schaefer, PRD 104 (2021) 074035
and
JB, Fischer, Isserstedt (in preparation)

ITP Lunch Club Seminar, JLU Gießen
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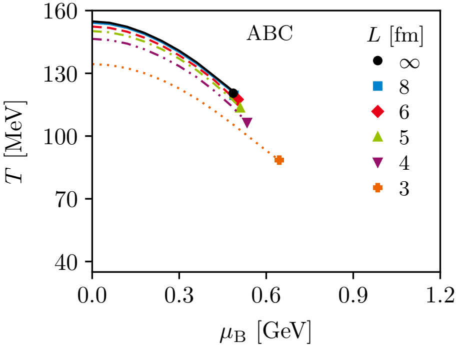
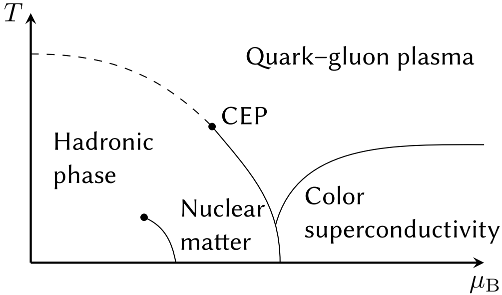
Outline

First Objective: QCD Phase Diagram in a Finite Volume



Motivation: Why Finite Volume?



- Goal of many experiments is to locate critical endpoint in QCD phase diagram
- "Fireball" of heavy-ion collisions has finite spatial extent
- Impact of volume effects on CEP is important for comparison between theory and experiment
- Cross-check between different theoretical approaches: lattice QCD (by construction formulated in a finite volume) vs. functional methods

Framework: Dyson–Schwinger Equations

Master DSE

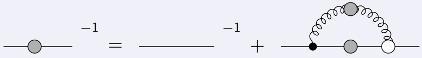
$$0 = \int \mathcal{D}\varphi \, \frac{\delta}{\delta\varphi} \exp(-\mathcal{S}[\varphi] + \langle \varphi, J \rangle) = \left\langle -\frac{\delta \mathcal{S}}{\delta\varphi} + J \right\rangle$$

- Quantum equations of motion of Euclidean n-point functions
- Non-perturbative, functional approach
- Obtained by taking appropriate number of functional derivatives of master DSE (and setting $J = 0$)
- Infinite tower of coupled, self-consistent equations $\rightarrow$ truncation needed
- Wide range of applications
  - Phase diagram, Columbia Plot, thermodynamics, ...
  - Together with BSEs: Hadron physics (spectroscopy, decays, ...)
  - Muon $g-2$, QED3, ...
  - ...

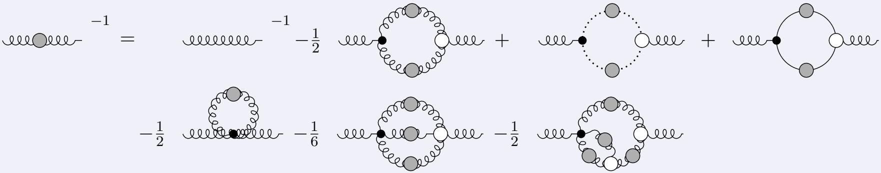
Reviews: Fischer, PPNP 105 (2019) 1
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

DSEs of QCD Propagators

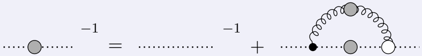
Quark Propagator



Gluon Propagator

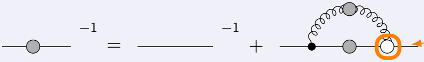


Ghost Propagator



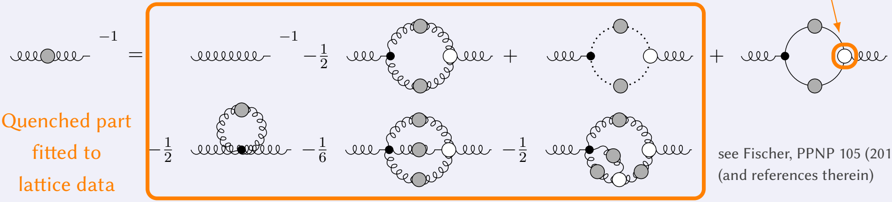
Truncation Scheme

Quark Propagator



Quark–gluon vertex ansatz

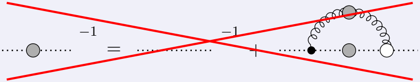
Gluon Propagator



Quenched part fitted to lattice data

see Fischer, PPNP 105 (2019) 1 (and references therein)

Ghost Propagator

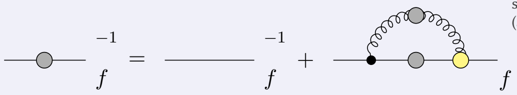


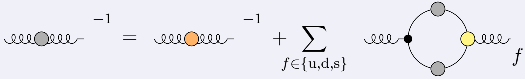
Not needed

Truncated Set of DSEs

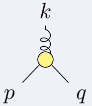
Truncated DSEs for Quarks and Gluons

see Fischer, PPNP 105 (2019) 1
(and references therein)





Quark–Gluon Vertex Ansatz



$$\Gamma_\mu^f(k,p,q) = \Gamma(k,p,q)\Gamma_\mu^{f,\mathrm{BC}}(p,q)$$ (Information about quarks)

Quenched Gluon Propagator

$$D_{\mu\nu}^{\mathrm{que}}(k) = D_{\mu\nu}^{\mathrm{que}}(k;T)$$ (Temperature-dependent fit to lattice data)

reference for lattice data: Fischer, Maas, Müller, EPJC 68 (2010) 165-181
Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037

Finite Volume Framework: Ansatz

- Feasible shape as ansatz: cube with edge length L:

$$\int_{\mathbb{R}^3} \mathrm{d}^3x \, \mathcal{L} \to \int_{[0,L]^3} \mathrm{d}^3x \, \mathcal{L}$$

- For quarks, free to choose between

$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = +\psi(\boldsymbol{x}) \quad \text{periodic boundary conditions (PBC)}$$
$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = -\psi(\boldsymbol{x}) \quad \text{antiperiodic boundary conditions (ABC)}$$

- For gluons, need PBC for kinematic reasons

→ Only discrete values possible in momentum space

Finite Volume Framework: Implications

→ Possible discrete momentum values given by:

Spatial Matsubara Modes

$$\omega_n^L = \begin{cases} 2n\pi/L & \text{for PBC}\,, \\ (2n+1)\pi/L & \text{for ABC}\,, \end{cases} \qquad n \in \mathbb{Z}$$

- Momentum integrals become sums

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, K(\boldsymbol{q}) \to \frac{1}{L^3} \sum_{\boldsymbol{n}\in\mathbb{Z}^3} K(\boldsymbol{q_n})\,,$$

where $\boldsymbol{q_n} := \sum_{i=1}^{3} \omega_{n_i}^L \boldsymbol{e}_i$ are allowed momentum vectors

Finite Volume Framework: Technical Obstacles

- Need angular information: rearrange into spheres of equal radius
- Problem: Number of points scales with third power of cutoff
- Observation: Outer spheres become increasingly dense

→ Solution:

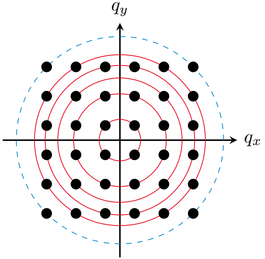


Figure: 2D ABC momentum grid

Continuum-improved Momentum Summation

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} K(\boldsymbol{q}) \to \frac{1}{L^3} \sum_{\boldsymbol{n} \in \mathbb{Z}^3}^{|\boldsymbol{q}_{\boldsymbol{n}}| < \Lambda_{\mathrm{vol}}} K(\boldsymbol{q}_{\boldsymbol{n}}) + \int_{|\boldsymbol{q}| > \Lambda_{\mathrm{vol}}} \frac{\mathrm{d}^3 q}{(2\pi)^3} K(\boldsymbol{q})$$

Inclusion of Temperature and Chemical Potential

- Finite temperature: bounded imaginary time to $[0, 1/T]$ and spin-statistics theorem together lead to

(Temporal) Matsubara Frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{for bosons}\,, \\ (2n+1)\pi T & \text{for fermions}\,, \end{cases} \qquad n \in \mathbb{Z}$$

- At finite T, energy integral becomes sum over Matsubara frequencies

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q_0}{2\pi} K(q_0) \to T \sum_{n=-\infty}^{\infty} K(\omega_n)$$

- Chemical potential corresponds to imaginary shift of energy

$$\omega_n \to \tilde{\omega}_n := \omega_n + \mathrm{i}\mu$$

QCD Phase Diagram

Order Parameter: Quark Condensate

$$\langle\overline{\psi}\psi\rangle_f \sim \sum_{\omega_n} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, \mathrm{Tr}\big[S_f(\omega_n + \mathrm{i}\mu, \boldsymbol{q})\big]$$

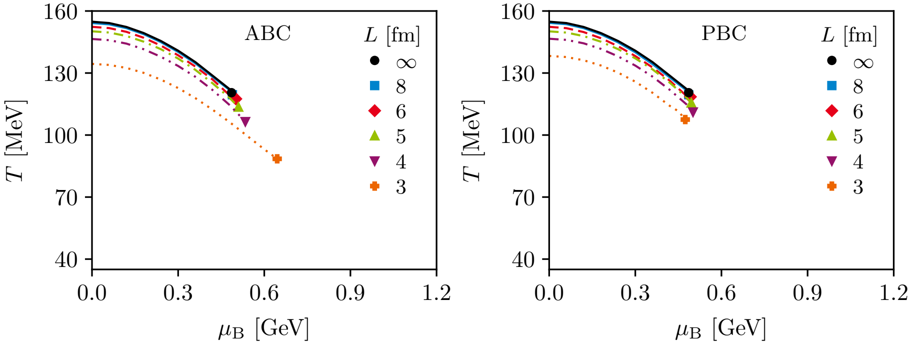
- $\langle\overline{\psi}\psi\rangle_f$ is divergent for $m_f > 0$. Therefore, define regularized condensate:

$$\Delta_{\mathrm{us}} := \langle\overline{\psi}\psi\rangle_{\mathrm{u}} - \frac{m_{\mathrm{u}}}{m_{\mathrm{s}}} \langle\overline{\psi}\psi\rangle_{\mathrm{s}}$$

Pseudocritical Temperature

$$T_{\mathrm{c}} := \arg\max_T \left| \frac{\partial \Delta_{\mathrm{us}}}{\partial T} \right|$$

Results: QCD Phase Diagram in a Finite Volume



- Consistent infinite-volume limit
- For decreasing L, pseudocritical temperature decreases and CEP (mostly) moves to higher μ
- Visible volume effects for $L \leq 4$ fm
- Very similar results for ABC and PBC above $L \geq 4$ fm

JB, Fischer, Isserstedt, Schaefer, PRD 104 (2021) 074035

Quark Number Fluctuations

Quark Number Fluctuations from QCD's Grand Potential

$$\chi_{ijk}^{\mathrm{uds}} = -T^{(i+j+k)-4} \frac{\partial^{i+j+k}}{\partial\mu_{\mathrm{u}}^{i}\partial\mu_{\mathrm{d}}^{j}\partial\mu_{\mathrm{s}}^{k}}\,\Omega$$

- Grand potential is not accessible by DSEs $\rightarrow$ quark number densities are:

Quark Number Density from Propagator

$$-\frac{\partial}{\partial\mu_f}\Omega = \rho_f \sim \sum_{\omega_n}\int\frac{\mathrm{d}^3 q}{(2\pi)^3}\,\mathrm{Tr}\big[\gamma_4 S_f(\omega_n + \mathrm{i}\mu, \boldsymbol{q})\big]$$

- Neglect off-diagonal terms, i.e., mixed derivatives:

Quark Number Fluctuations from QCD's Grand Potential

$$\chi_n^f = T^{n-4}\frac{\partial^{n-1}}{\partial\mu_f^{n-1}}\,\rho_f$$

Baryon Number Fluctuations

Baryon Number Fluctuations from Quark Number Fluctuations

$$\chi_n^{\mathrm{B}} \approx \frac{1}{3^n}\left(2\chi_n^{\mathrm{u}} + \chi_n^{\mathrm{s}}\right)$$

- Relation to cumulants of baryon number distribution:

$$C_n^{\mathrm{B}} = V\,T^3\,\chi_n^{\mathrm{B}}$$

- Directly linked to moments of baryon number distribution:

$$\sigma_{\mathrm{B}}^2 = C_2^{\mathrm{B}}\,, \quad S_{\mathrm{B}} = C_3^{\mathrm{B}}(C_2^{\mathrm{B}})^{-3/2}\,, \quad \kappa_{\mathrm{B}} = C_4^{\mathrm{B}}(C_2^{\mathrm{B}})^{-2}\,, \quad \ldots$$

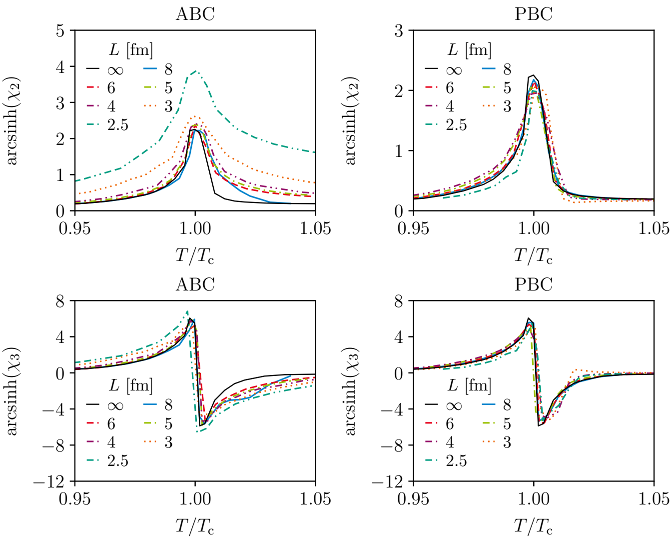
- Ratios relate theoretical and experimental quantities:

$$\chi_3^{\mathrm{B}}/\chi_2^{\mathrm{B}} = S_{\mathrm{B}}\sigma_{\mathrm{B}}\,, \quad \chi_4^{\mathrm{B}}/\chi_2^{\mathrm{B}} = \kappa_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2\,, \quad \ldots$$

→ Explicit volume dependence drops out!

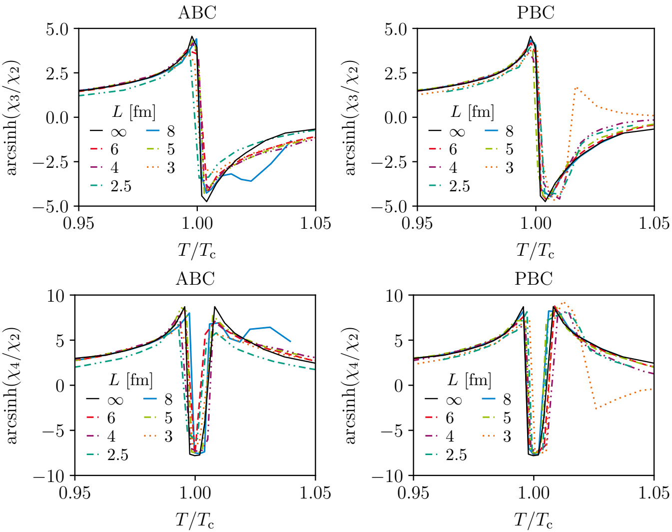
Reviews: Luo, Xu, Nucl. Sci. Tech. 28 (2017) 112
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, Phys. Rep. 853 (2020) 1

Results: Baryon Number Fluctuations at $\mu = \mu_c$



- Visible volume effects (especially for ABC)

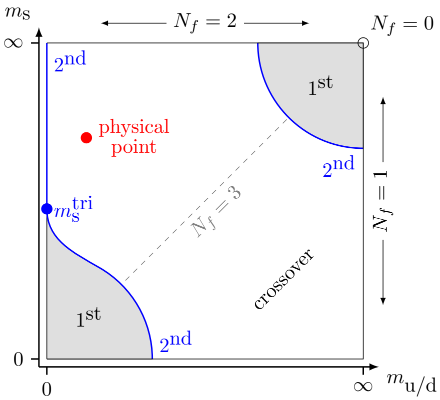
Results: Baryon Number Fluctuation Ratios at $\mu = \mu_c$



- (Essentially) independent of system size → no implicit volume dependence

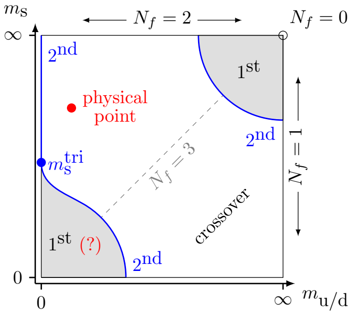
JB, Fischer, Isserstedt (in preparation)

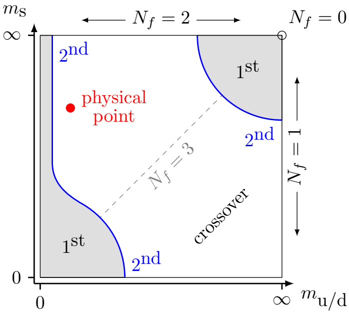
Second Objective: Columbia Plot and (Up Quark) Chiral Limit



Motivation: Columbia Plot(s)

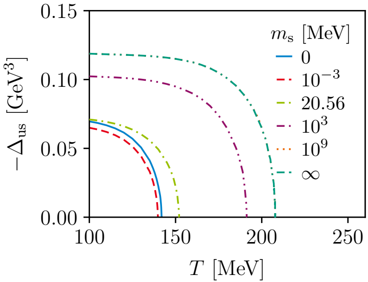
for reference on upper right corner in DSE framework, see Fischer, Luecker, Pawlowski, PRD 91 (2015) 014024

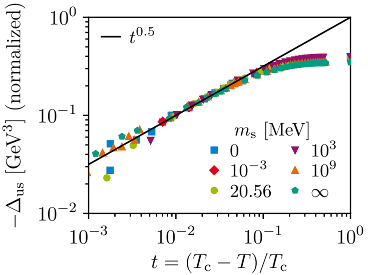


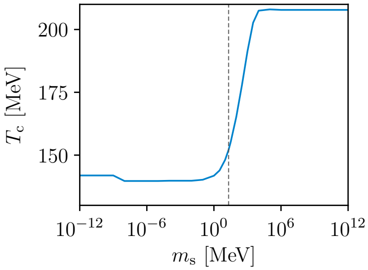


- Two different scenarios for Columbia Plot: anomalously broken (left) or restored (right) $U_A(1)$-symmetry
- Existence of first order region in lower left corner (of left scenario) is not yet clear see Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141
- Chiral limit is difficult for lattice QCD but no conceptual problem for our framework

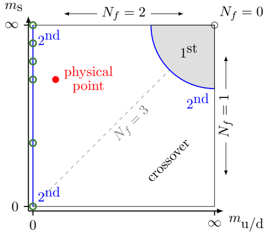
Results: Condensate and Critical Scaling in Chiral Limit







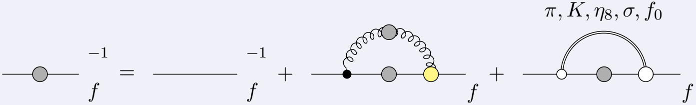
Resulting Columbia Plot:



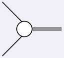
Meson Backcoupling Ansatz

- Improve truncation in chiral limit: long-range correlations in vertex become important → add meson backcoupling diagram to quark DSE

Modified Quark DSE



Bethe–Salpeter Amplitudes



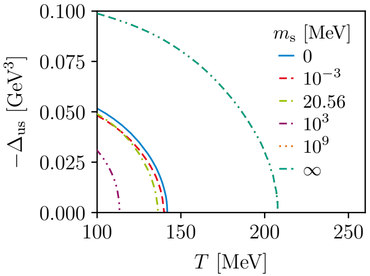
(Obtained from quarks: Goldberger–Treiman-like relations)

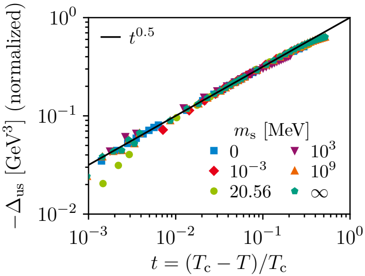
Free Meson Propagator

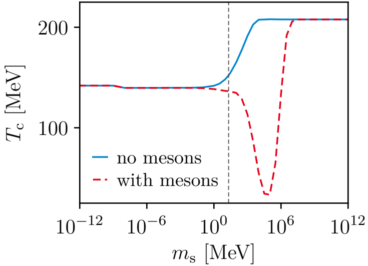
(Mass from Gell-Mann–Oakes–Renner fit)

details on meson backcoupling: Fischer, Müller, PRD 84 (2011) 054013

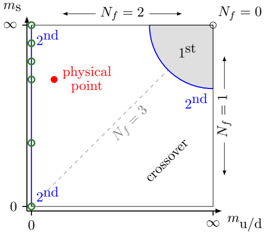
Results for Meson Backcoupling



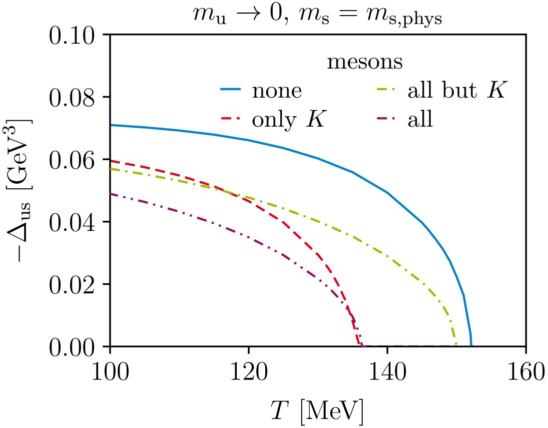




Resulting Columbia Plot:



Meson Backcoupling: Influence of kaons on T_c



- Drop in T_c is almost exclusively caused by kaons while shape of condensate is modified by other mesons

Conclusion and Outlook

Conclusion:

- Studied finite-volume effects on QCD phase diagram using DSEs beyond rainbow–ladder truncation for ABC and PBC
- Crossover line and CEP exhibit visible volume effects for $L \leq 4\,\mathrm{fm}$
- Baryon number fluctuations show volume dependence, ratios do not
- Second order phase transition across whole left edge of Columbia Plot (both with and without meson backcoupling)
- Kaon backcoupling causes drastic decrease of critical temperature for intermediate strange quark masses

Outlook:

- Implement QCD scaling to meson backcoupling
- Investigate imaginary chemical potentials
- Study finite-volume effects in chiral limit