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Faculty 02 - Economics and Business Studies
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Competition Policy and Strategy

Assignment 8

Aufgabe 8.1 (**Collusion - one-shot)**
Consider a Cournot oligopoly in which n firms offer a homogeneous good. The demand function in this market is $Q(p) = 110 - p$. All firms have a constant marginal costs of $c = 10$. Assume that firms simultaneously decide whether to engage in collusion that maximizes firms' joint profits or to behave competitively. The latter means that they respond optimally (*best responses*) to the quantity setting of the other firms.

a) Show that the profit of each firm in the case of collusion (index K) is $\pi_{i,K} = \frac{2500}{n}$ for $i = 1, ..., n$.

b) Show that a firm's profit is $\pi_{i,D} = \left(\frac{25(n+1)}{n}\right)^2$ when it deviates from the collusion (index D).

c) Show that the profit of a firm follows the function $\pi_{i,F} = \frac{1250(n+1)}{n^2}$ when another firm deviates from the collusion (index F).

d) Consider a Cournot duopoly in which firms can behave either competitively or according to collusion (*cooperate*). State the companies' payoff matrix and determine the Nash equilibrium. Interpret your result.
Note: To solve task d, use the indirect profit function you know in the Cournot model (index C): $\pi_{i,C} = \left(\frac{100}{n+1}\right)^2$.

Aufgabe 8.2 (**Collusion - repeated interaction)**
Start from the assumptions of task 8.1, but now firms play a repeated game for periods $t = 0, ..., T$. The demand function in this market is constant over time and is $Q(p) = 110 - p$ in each period. All firms have marginal costs of $c = 10$.

a) Show that the short-run incentive to deviate from the collusion for a firm is $\Delta\pi = \pi_D - \pi_K = 625[(n-1)/n]^2$. How does the deviation incentive change as the number of firms increases? Interpret your result.

Now assume that all firms have no knowledge regarding the point in time that marks the end of this market ($T \to \infty$).

b) Analyze whether firms can stabilize a collusion. Determine in general terms what the (long-run) future loss is if, as a consequence of the deviation, all firms return to the optimal quantities of the one-period game (one-shot Nash equilibrium) in comparison to the cooperation.

c) Determine for n firms the present value of profits in the competitive case V_C, in the collusive case V_K, and in the case of deviation V_D. Determine the critical discount factor $\delta_{crit}(n)$.

d) For the case of $n = 3$, determine the critical discount factor δ_{crit} , which determines whether the collusion is stable.

Assume that all companies know that this market will exist for exactly five more periods. After that, demand collapses because of an EU-wide prohibitive regulation that will then come into force.

e) In this case, can the companies stabilize a collusion non-cooperatively?

Aufgabe 8.3 (**Collusion – Competition Authority)**
Proceed from the specifications in task 8.2. Consider again the case where the firms are in Cournot competition. There is now a competition authority that, with probability $\rho = 0.2$, detects the cartel agreement and imposes a fine of F.
How do these two parameters affect the critical discount factor δ? Consider also the case of leniency. If a firm invokes leniency, the collusion is certain to be discovered, but the reporting firm will incur a reduced fine $f < F$. Determine the critical discount factor for this case as well. Interpret your results.

Aufgabe 8.4 (**Coll. in Bertrand Model with repeated interaction)**
Continue to consider the scenario outlined in 8.1 and 8.2. However, there are now n companies in a Bertrand competition.

a) Calculate profits in competition, profits in case of collusion, and profits in case of a deviation from collusion. What is the short term incentive to deviate?

b) Determine the critical discount factor δ_{crit} for the case of duopoly.