

# Competition Policy and Strategy

## Assignment 3

### Exercise 3.1 (Productive Efficiency)

Consider a firm that produces a good (quantity  $q$ , price  $p$ ) using capital (quantity  $K$ , price  $r$ ) and labor (quantity  $L$ , price  $w$ ). The technology can be described by the production function  $q = F(K, L) = K^{\frac{2}{3}}L^{\frac{1}{3}}$ . Both factors of production are variable. Assume that there is perfect competition in all relevant markets, i.e., the firm is a price taker in both factor and product markets.

- What is the type of function called that follow the structure of above defined production function? What characterizes the returns to scale of functions of this kind?
- State the cost minimization problem of the firm and derive the first order conditions of the cost minimization using the Lagrange method.
- Calculate the conditional factor demand functions  $K(w, r, q)$  and  $L(w, r, q)$  as well as the cost function  $C(w, r, q)$  based on the first order conditions derived in task b).

*Remark on references: You can read up on the contents of this exercise in a Mikroeconomics textbook, for example, Jehle, G.A. & Reny, P.J., "Advanced Microeconomic Theory"*

### Exercise 3.2 (Dynamic Efficiency)

Consider a duopolistic market for a homogeneous good. Demand is defined as follows:

$$q(p) = 1680 - 4p$$

- In a benchmark situation both firms have identical costs per unit produced ( $c_0 = 9$ ). Derive the market equilibrium for both, Cournot as well as Bertrand competition, respectively. Comment on the differences between both situations.
- Assume now that both firms are in a Cournot competition. Assume further that at most one firm may invest into a new production technology. This new technology reduces production costs to  $c_1 = 6$  per unit for exactly one period. What is the maximum any firm would be willing to pay for this new technology? Consider two different cases: First, only one firm has access to the investment into new technology. Second, the investment into new technology is available to both firms.
- Continue to assume that that at most one firm may employ the new technology for exactly one period, such that  $c_1 = 6$  for this firm. What is a firms willingness to pay for such a technology in case of Bertrand competition?
- Compare the results from tasks a) - c).

**Exercise 3.3 (Economies of Scale / Economies of Scope)**

Consider a three firms, each operating in a different industry (*industry 1, 2, and 3*). Each firm has its unique cost function, specified as follows.

- a) The cost function of the firm operating in *industry 1* can be described as  $C(q) = 900 + 3q + q^2$ . Determine the interval of output quantity  $q$  for which this firm shows *Economies of Scale* in its production. Interpret your results.
- b) The cost function of the firm operating in *industry 2* can be described as  $C(q) = 70 + 4q$ . Show that this firm has *Economies of Scale* in its production for  $q > 0$ . What is the term used to describe markets in which companies typically have such a cost structure? Interpret your results and compare them with the results from task a).
- c) The cost function of the firm operating in *industry 3* can be described as  $C(q_A, q_B) = 2q_A + q_B - 3q_Aq_B$ . Check whether there are *Economies of Scope* for this company in the production of good A and good B. Interpret your results.